Rate Equations Three-Level System

Introduction: Consider the 3-level scheme 2.3.1-1. Under which condition can the upper pump level 3 be eliminated, so that the 3-level system can be replaced by an effective 2-level system?

1) Write down a complete set of rate equations for the 3-level system. Assume that the states are not degenerate and that no laser field is present (which can be ensured for example by blocking one of the mirrors in a laser setup).

2) Specialize the rate equations for the steady state. To simplify the future discussion please replace the corresponding quantities by the expressions given below. Explain, why these replacements are reasonable from the physics point of view. Explain the physical meaning of these quantities.

- give all rates in units of γ_{21}, e.g. $g_{32} = \gamma_{32}/\gamma_{21}$.
- give all population densities in units of the absolute population density n_{tot}, e.g. $n'_1 = n_1/n_{tot}$.
- replace all relative populations n'_1, n'_2, and n'_3 by the following set of populations: relative inversion n', relative total population n'_0 of level 1 and 2, and relative population n'_3 of level 3.
- Replace the relative decay rates g_{32} and g_{31} by $g_{32} = \eta \cdot g_3$ and $g_{31} = (1-\eta) \cdot g_3$. What is the physical meaning of η und g_3?

3) Solve the reformulated rate equations for steady state.
Hint: Use $n_{tot} = n_1 + n_2 + n_3$ to have three independent equations.

4) Under which condition can the population in level 3 be neglected? Hint: a series expansion might be useful.

5) Under which condition is inversion achieved?
To simplify the math, introduce the following quantities: $x = \frac{W_{13}}{\gamma_{21}}/g_3$ and $y = \eta \cdot g_3$.
What is the physical meaning of x and y?
How does this inversion condition look like if the approximation is applied which has already been used to eliminate level 3? Express the threshold condition in terms of pump rate W_{13} and transition rate γ_{21}.

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