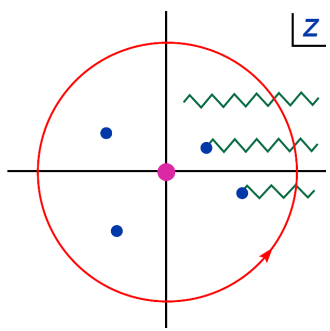
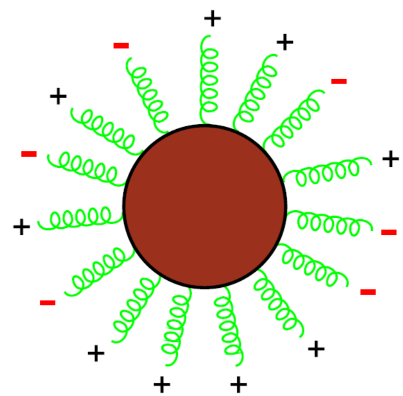
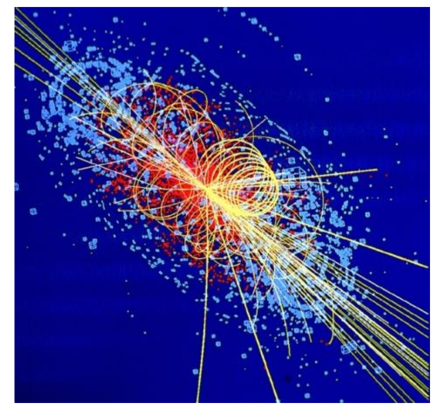
Precise Theory for the Energy Frontier







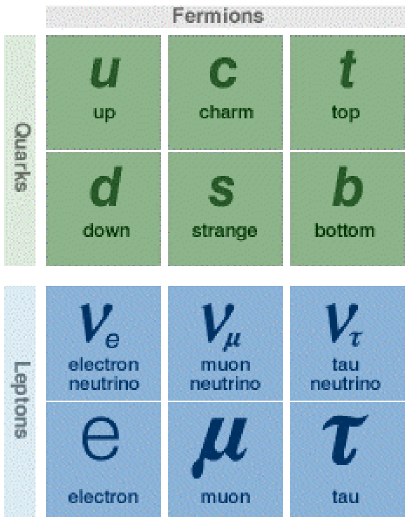
Lance Dixon (SLAC)

Opening Symposium for “Mass, Spectra, Symmetry: Particle Physics in the Era of the LHC”

Berlin, 28 September 2009

A Remarkable Theory: The Standard Model

Matter – spin 1/2



QCD – modern theory of the strong force – binds quarks together into protons, plus many more hadrons

Forces – spin 1*

Massless - infinite range

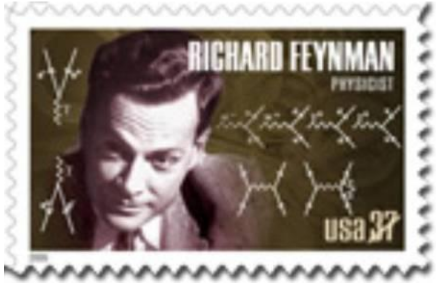
Mass ~ 100 GeV Why?

unstable, made at particle accelerators

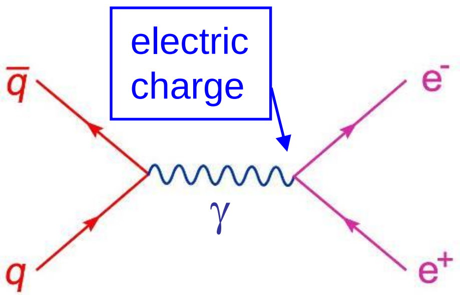
stable, everyday matter

*Gravity (spin 2) is **very weak** at the particle level – ignore it here

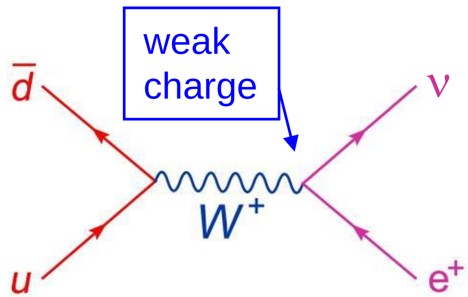
The Three Forces



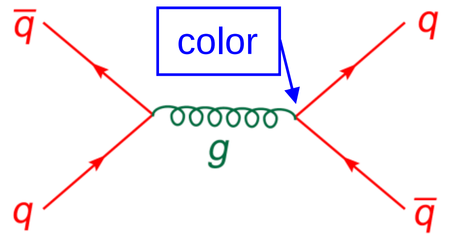
Feynman diagrams let us visualize them



electromagnetism



weak



strong

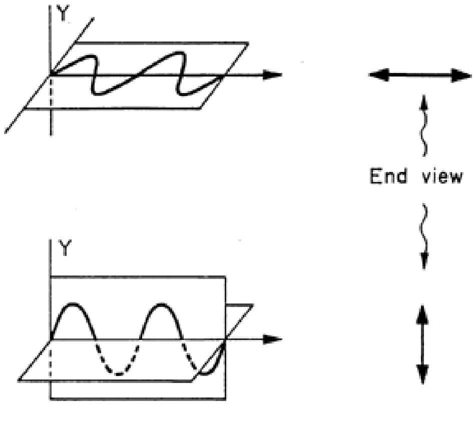
Many similarities between the 3 forces, but one **big** difference:

• photon and gluon* are **massless particles**
→ travel **long range,** at speed of light

• *W* and *Z* particles are **very massive**: ~100 times mass of proton
→ they can only have influence over ~1/100 of a proton radius!

Something Is Missing: $2 \neq 3$

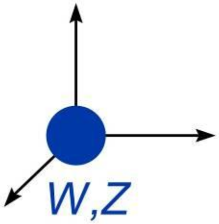
2 is the number of ways light (photons) can be polarized. Cross 2 pairs of polarized sunglasses to see this.



Linearly polarized light

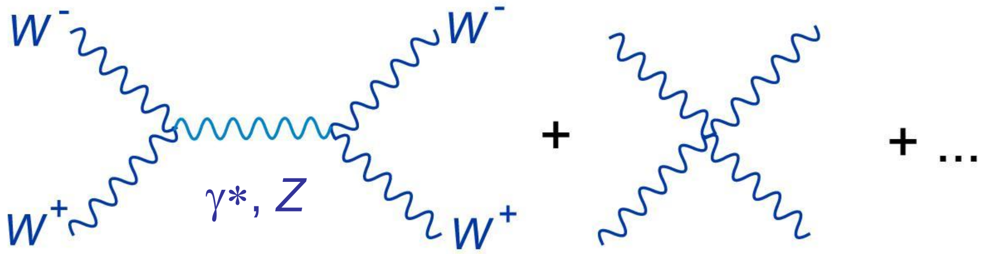
W & *Z* just like photons, except they have **mass**. **Massive** particles can be stationary (**at rest**).

For a particle at rest,
all **3** directions of space (x,y,z) are **equivalent**

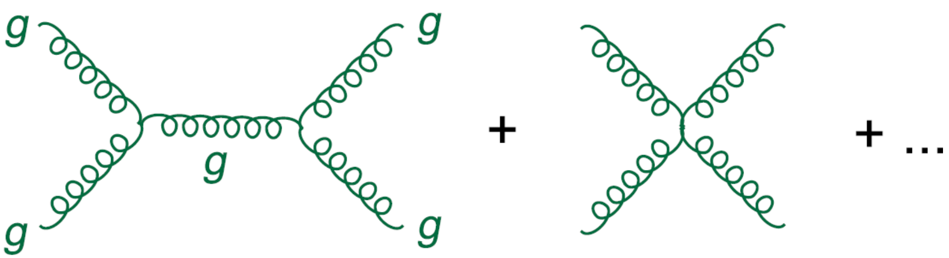


there must be **3** ways to polarize *W* and *Z* bosons
→ **where does the extra polarization come from?**

Vector Bosons Also Self-Interact



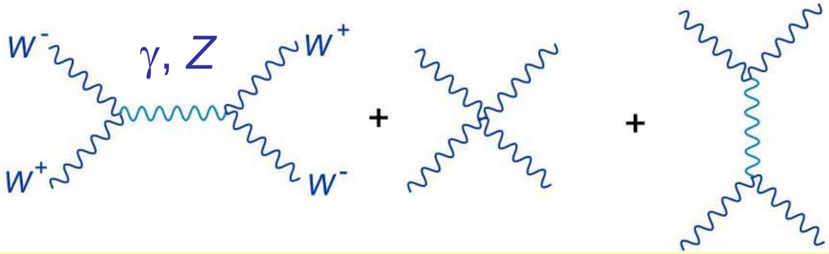
electroweak



QCD

Something Is Missing (II)

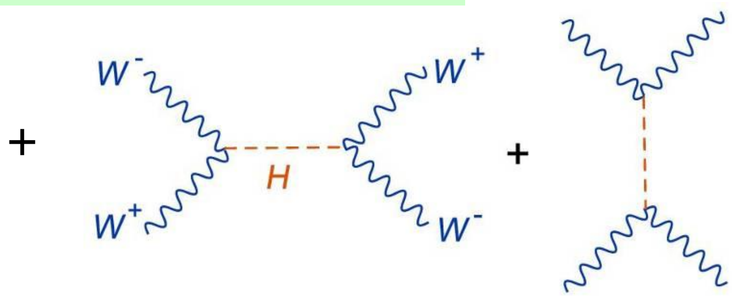
Weak self-interactions by themselves would **violate unitarity** at energies well above the weak boson masses**:**



$\Rightarrow$ probability $>$ 100%

Higgs (Anderson, Brout, Englert, Guralnik, Hagen & Kibble) realized long ago (~1964) that a single scalar spin 0 particle could fix this problem

The Higgs boson *H*



$\Rightarrow$ probability $<$ 100%

Higgs boson can also give **mass** to all fermions, not just *W* and *Z*

Hunt for the Higgs

- Higgs boson invented in 1964.
- Experimental searches only began around 1980, really picked up steam in the 1990s and 2000s

  (LEP → Tevatron)
- Search is difficult: Higgs doesn't talk to particles it doesn't give much mass to – and those are the stable particles we know how to collide!

$m_u = 0.003$	$m_c = 1.3$	$m_t = 184$	
$m_d = 0.006$	$m_s = 0.12$	$m_b = 5.0$	(in units of m_p)
$m_e = 0.0005446$	$m_\mu = 0.1126$	$m_\tau = 1.894$	

What If Higgs Is Wrong, or Incomplete?

- Lot of reasons to believe that other "new physics" is lurking nearby.
- Related to hierarchy problem from quadratic divergences in simplest Higgs model:

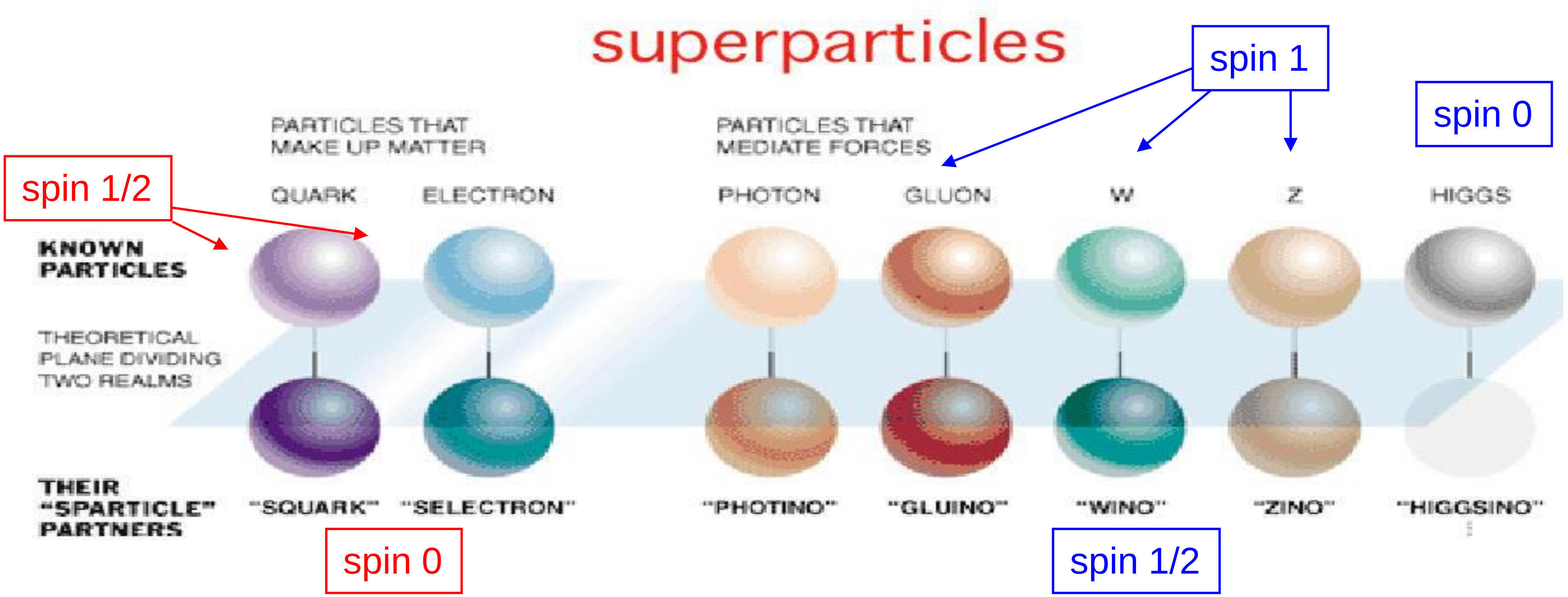
$$\Rightarrow \quad m_H^2 \propto \Lambda^2$$

- Also, SM Higgs accomodates, but does not explain, patterns of fermion masses m_f
- No SM candidate for **dark matter**
- But what exactly is the new physics?

No-one really knows.

One Possibility: Supersymmetry

- Symmetry between fermions (matter) and bosons (forces)
- Predicts that for every elementary particle we have already seen there is another one we will see soon!
- Solution to the hierarchy problem:
  fermion + boson corrections to Higgs mass cancel
- One particle can be **dark matter**
- **But is it right?**



New Physics Around the Corner

We expect new physics at the 100 GeV – 1 TeV mass scale, associated with electroweak symmetry breaking.
At the very least, a Higgs boson (or something like it).

- Supersymmetry predicts a host of new massive particles in this mass range, including a **dark matter candidate**

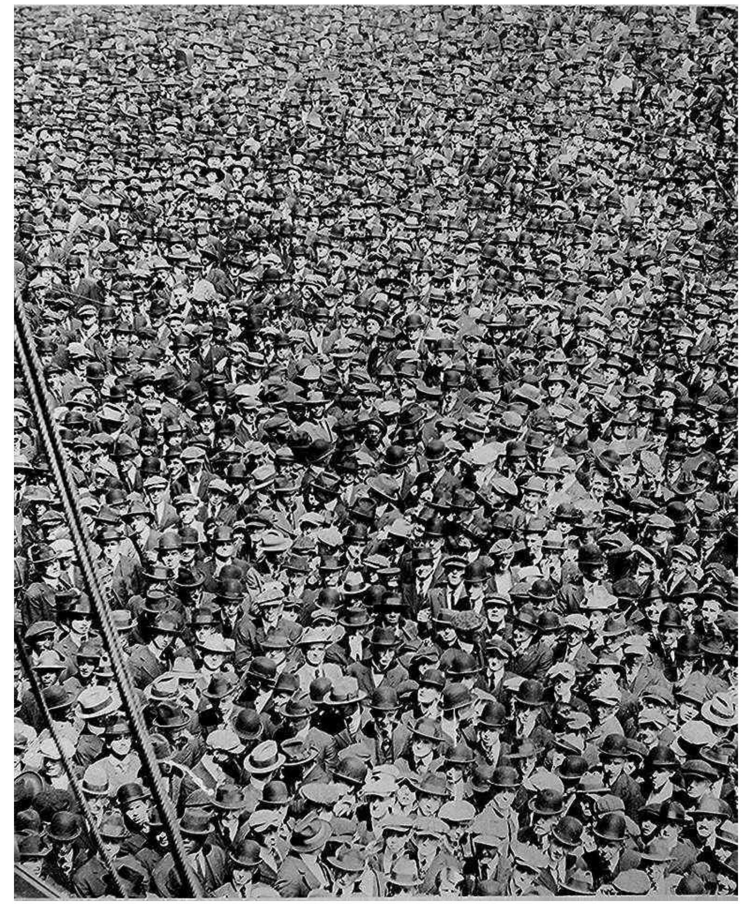
- Many other theories of electroweak scale $m_{W,Z}$ = 100 GeV make similar predictions:
    - **new dimensions of space-time**
    - **new forces**
    - etc.

How to sort them all out?

Signals vs. Backgrounds

electron-positron colliders
– small backgrounds

vs.



proton colliders
– large backgrounds

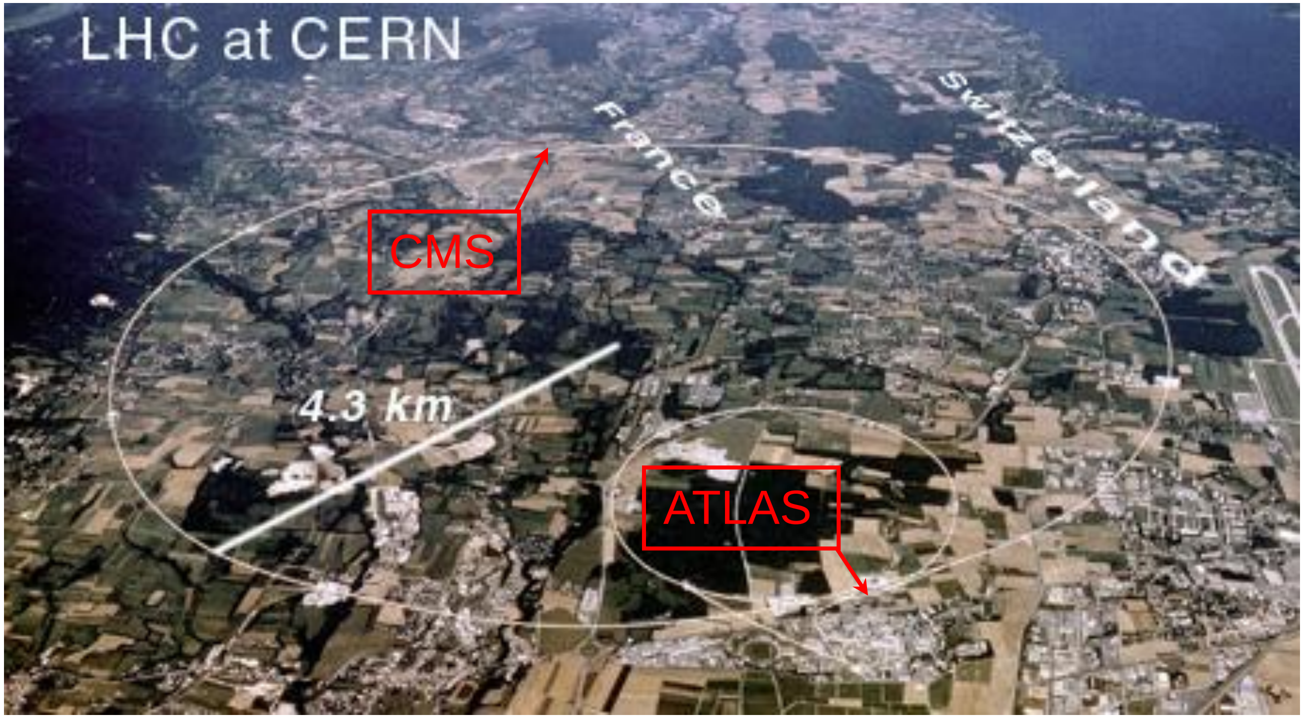
The Energy Frontier Is at Proton Colliders

Tevatron, Fermilab, Illinois
Run II: 2001 → 2011?

- collides protons with antiprotons
- energy = 10 times best e^+e^- LEP2

- protons = bags of strongly interacting quarks and gluons
- collisions make hundreds of strongly-interacting particles
- backgrounds large

The Large Hadron Collider

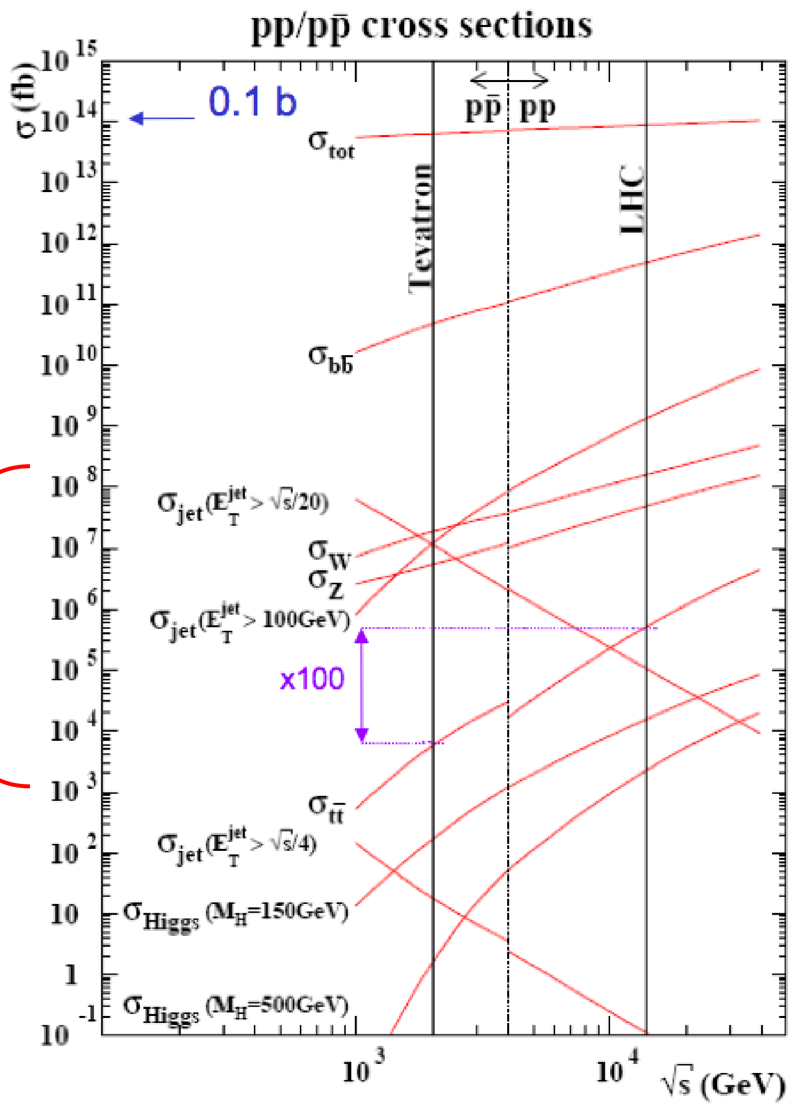


- Proton-proton collisions at **7→10→14 TeV** center-of-mass energy, 3.5→5→7 times greater than previous (Tevatron)
- Luminosity (collision rate) 10—100 times greater

- New window into physics at the shortest distances – opening this year!

Tevatron & LHC Are QCD Machines

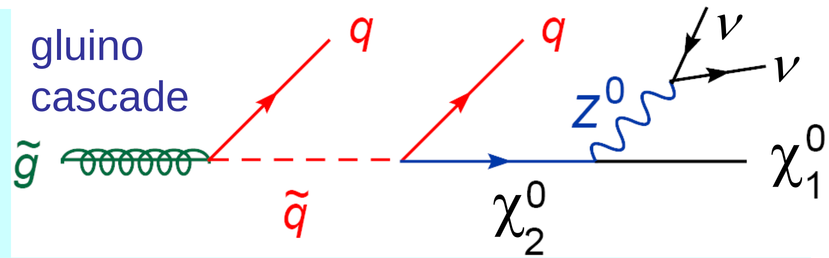
Need precise understanding of “old physics” that looks like new physics



← new physics?

Signals and Backgrounds

- New particles – whether from
  - supersymmetry
  - extra dimensions
  - new forces
  - Higgs boson(s)



typically decay into old particles:
quarks, gluons, charged leptons, neutrinos, photons,
*W*s & *Z*s (which in turn decay to leptons, …)

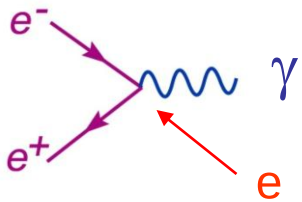
- Kinematic signatures not always clean (*e.g.* mass bumps) if neutrinos, or other escaping particles present

- Need **precise Standard Model backgrounds** for a variety of multi-particle processes, to maximize potential for new physics discoveries

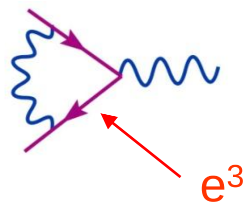
How to Make Precise?

- We can (essentially) only compute reaction rates as a perturbative expansion in small parameters (couplings)

QED



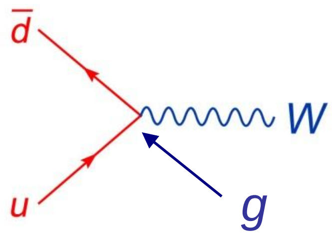
+



+ …

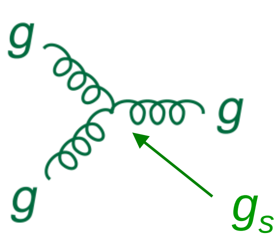
$$\frac{e^2}{4\pi} \equiv \alpha = \frac{1}{137}$$

weak



$$\frac{g^2}{4\pi} = \frac{\alpha}{\sin^2\theta_W} = \frac{1}{30}$$

QCD



$$\frac{g_s^2}{4\pi} = \alpha_s \approx \frac{1}{10}$$

Asymptotic Freedom

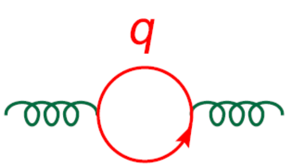
Gross, Wilczek, Politzer (1973)

Gluon self-interactions make QCD more calculable at high energies

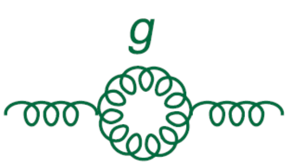
Quantum fluctuations of massless virtual particles polarize vacuum

QED: electrons screen charge (e larger at short distances)

QCD: gluons anti-screen charge (g_s smaller at short distances)



> 0



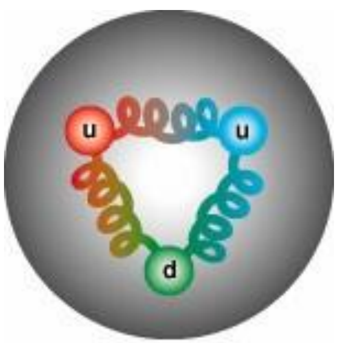
< 0 $\longrightarrow$ $\alpha_s(\mu)$

$$\frac{d\alpha_s(\mu)}{d\ln\mu^2} \equiv \beta(\alpha_s) = \left[-\frac{11}{3}N_c + \frac{2}{3}N_f\right]\frac{\alpha_s^2}{4\pi}$$

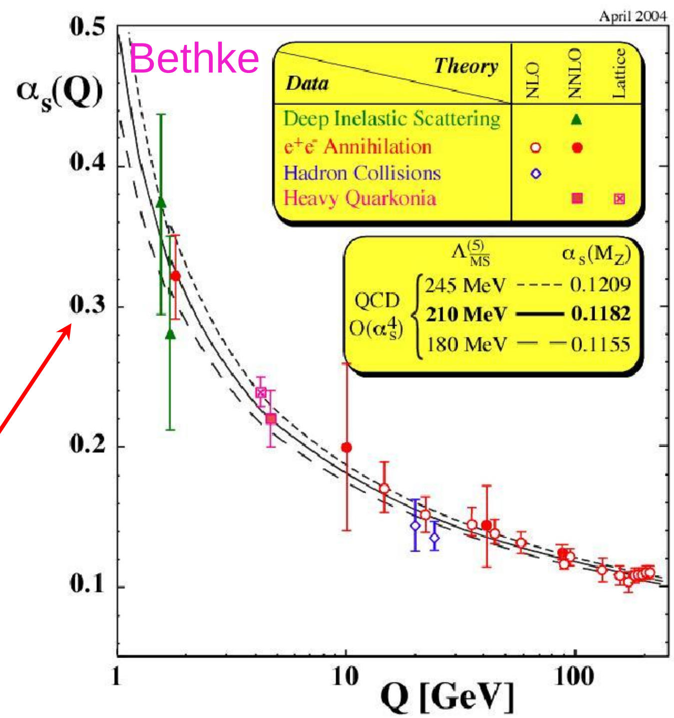
For N_c=3, N_f =5 or 6, gluons win

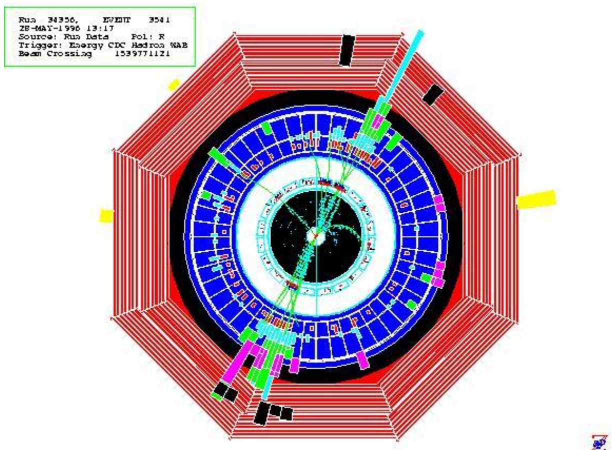
Asymptotic Freedom (cont.)

Running of α_s is only *logarithmic*,
slow at short distances (large Q or μ).



confining



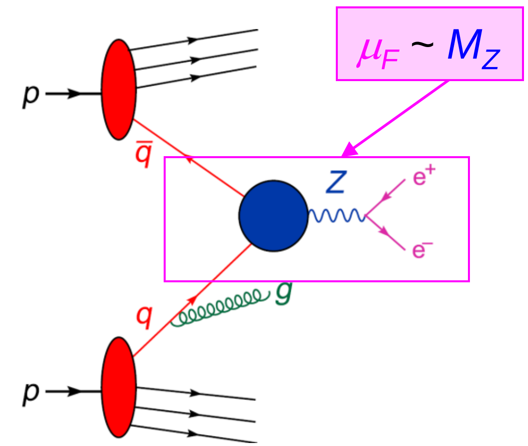


calculable

QCD Factorization & Parton Model

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- Sampled "one at a time" in hard collisions.

→ QCD-improved parton model



suitable final state

Parton distribution function

factorization scale ("arbitrary")

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Partonic cross section, computable in perturbative QCD

partonic CM energy^2

renormalization scale ("arbitrary")

Partonic Cross Section in Perturbation Theory

$$\hat\sigma(\alpha_s,\mu_F,\mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat\sigma^{(0)} + \frac{\alpha_s}{2\pi}\hat\sigma^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2\hat\sigma^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

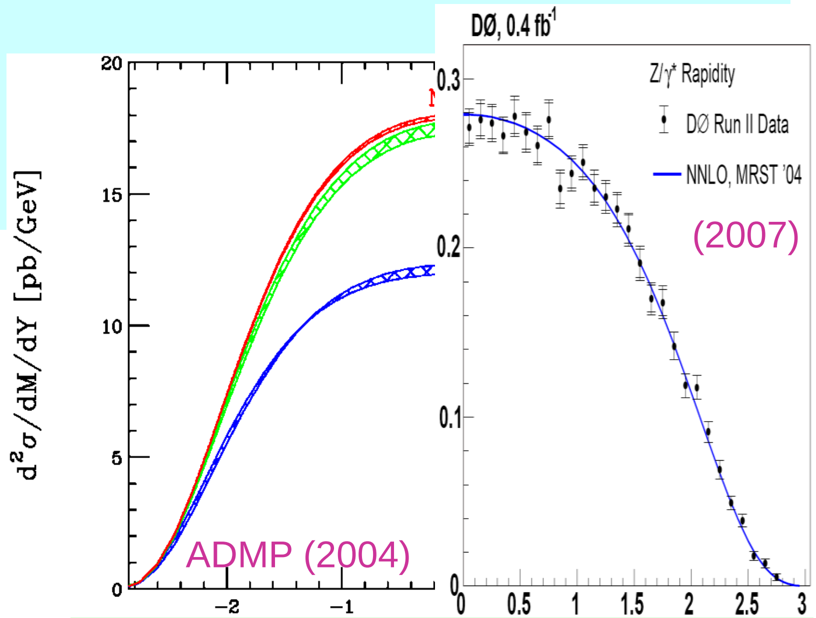
LO NLO NNLO

Problem: Leading-order, tree-level predictions only qualitative due to **poor convergence** of expansion in $\alpha_s(\mu)$ (setting $\mu_R = \mu_F = \mu$)

Example: *Z* production at Tevatron
Distribution in rapidity *Y*

$$Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

$$\frac{d\sigma}{dY} \quad \text{has} \quad n_\alpha = 0$$



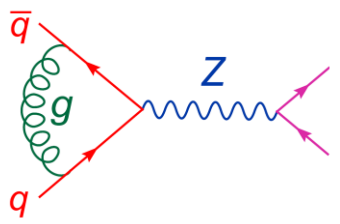
still ~50% corrections, LO → NLO

by NNLO, a precision observable

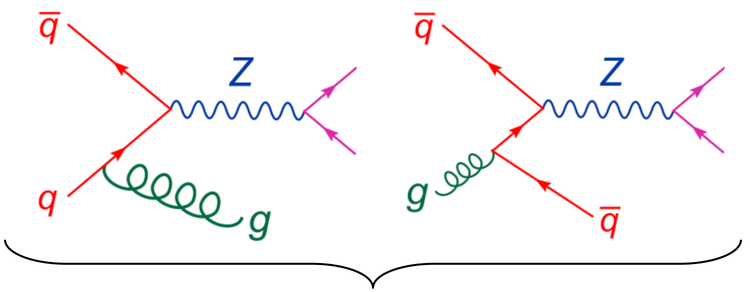
Need for Loop Amplitudes

- NLO corrections require one-loop amplitudes, as well as tree-level amplitudes with one additional parton.
- Both terms are infrared divergent; use dimensional regularization with $D = 4 - 2\epsilon$
- After adding terms, renormalizing *q(x)*, all $1/\epsilon$ poles cancel.
- Simplest example – *Z* production:

$\hat{\sigma}^{(1)}$



1 loop



tree + 1 parton

Lack of Loop Amplitudes

At NLO, the bottleneck for more complex processes is the lack of availability of one-loop amplitudes.

$$\sigma(n \text{ jets}) = [\alpha_s(\mu)]^n \{A + \alpha_s(\mu) B + \alpha_s^2(\mu) C + \cdots\}$$

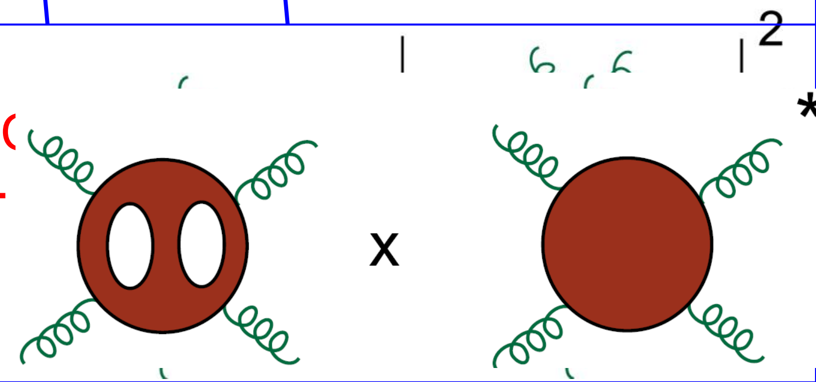
state of the art:

NNLO

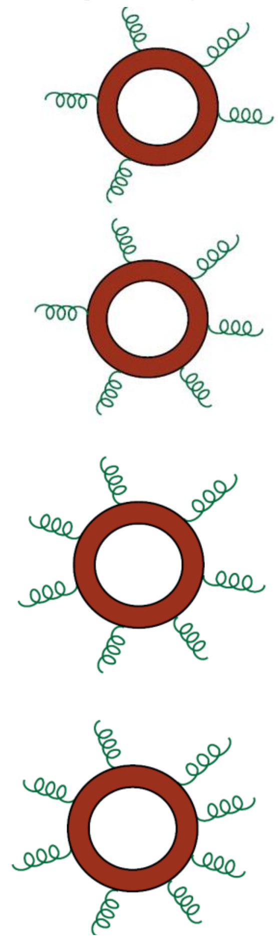
NLO = 2-loop x tree* + …

LO

n=2 *n*=3



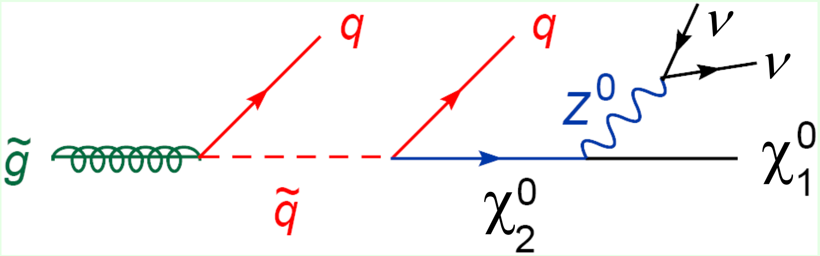
Strong growth in difficulty at one loop (NLO) with number of final-state objects



# of jets	# 1-loop Feynman diagrams (gluons only)
3	810
4	10,860
5	168,925
6	3,017,490

Background to Search for Supersymmetry

- **Cascade from gluino to neutralino** (dark matter, escapes detector)

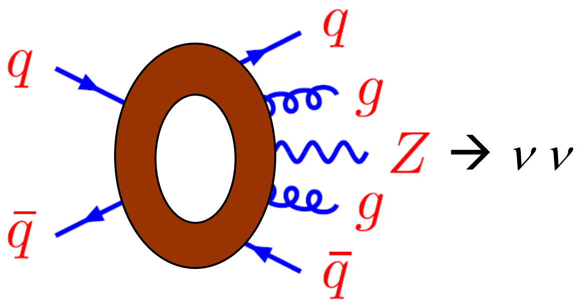


- **Signal: missing energy + 4 jets**
- SM background from *Z* + 4 jets,
  Z → neutrinos

Current state of art for *Z* + 4 jets:
ALPGEN, based on **LO tree** amplitudes
→ normalization still quite uncertain

- Motivates goal of

$$pp \to Z + 4 \text{ jets at NLO}$$



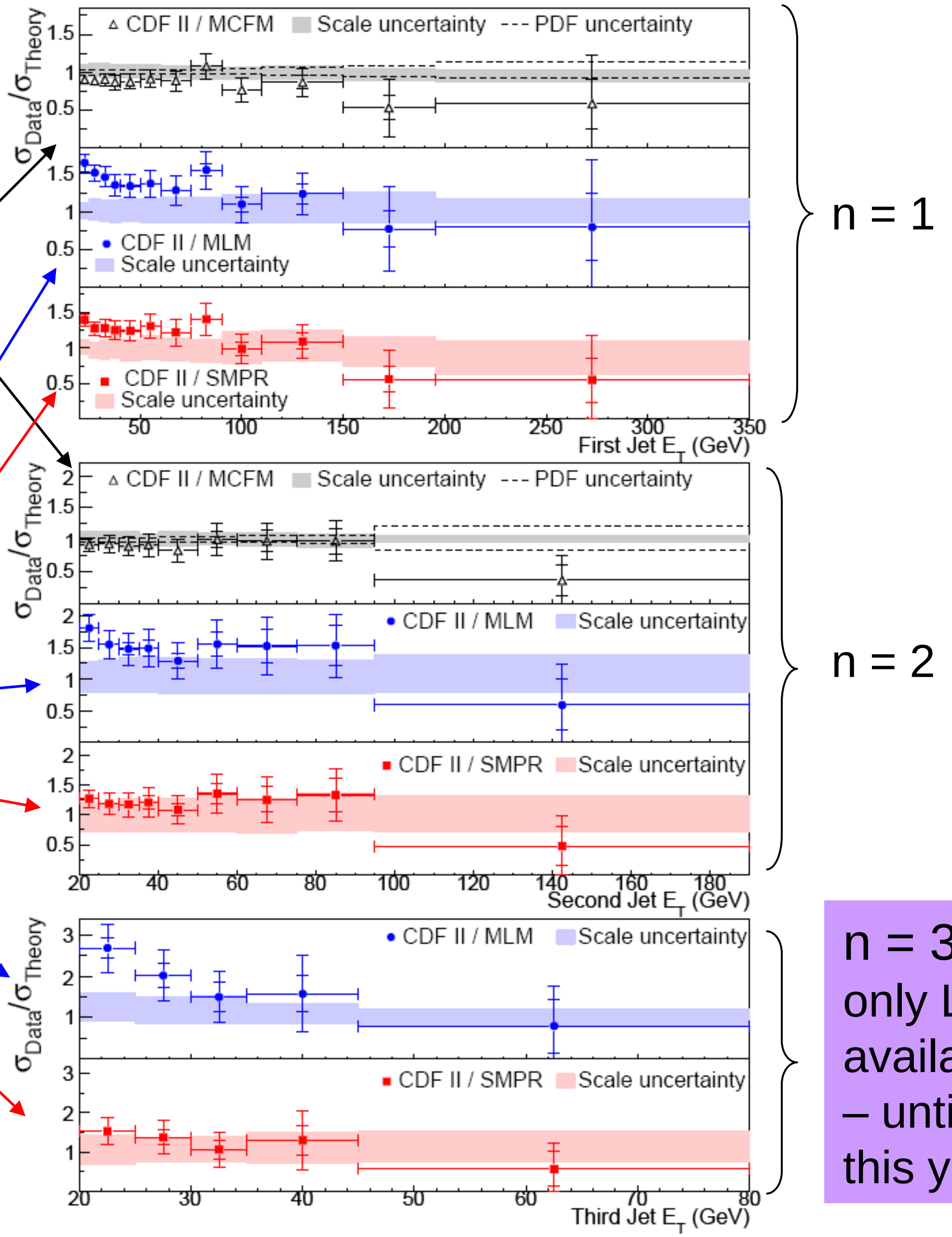
1 leg beyond state-of-art

Tevatron W + n jets Data

CDF, 0711.4044 [hep-ex]

NLO (MCFM)

LO with different matching schemes



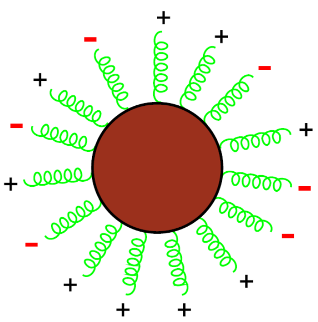
n = 3 only LO available – until this year

% uncertainty

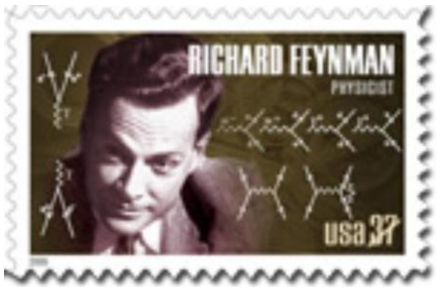
number of jets	LO	NLO
1	16%	7%
2	30%	10%
3	42%	11%

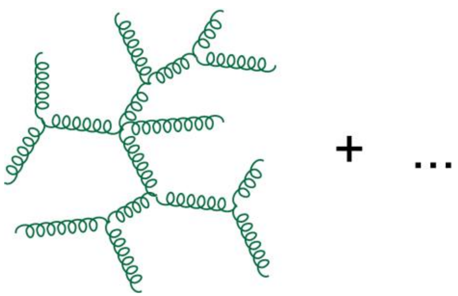
A Better Way to Compute?

- Backgrounds (and many signals) require detailed understanding of scattering amplitudes for many ultra-relativistic ("massless") particles – especially quarks and gluons of QCD



- **Feynman** told us how to do this – in principle





- However, **Feynman diagrams**, while very general and **powerful**, are not optimized for these processes
- There are more efficient methods for multi-gluon + quark processes!

Feynman Diagrams Not Obsolete

- Many state-of-art NLO calculations based on them, such as:
- $p\bar{p} \to W b\bar{b},\ m_b \neq 0$ — Higgs background at Tevatron
  Febres Cordero, Reina, Wackeroth, hep-ph/0606102
- $pp \to t\bar{t}$ jet — SUSY background at LHC
  Dittmaier, Uwer, Weinzierl, hep-ph/0703120, 0810.0452
- $pp \to WW$ jet — Higgs (+ jet) background at LHC
  Dittmaier, Kallweit, Uwer, 0710.1577; 0908.4124
  Campbell, Ellis, Zanderighi, 0710.1832
- $pp \to t\bar{t}\, b\bar{b}$ — Higgs (+ $t\bar{t}$) background at LHC
  Bredenstein et al., 0807.1248, 0905.0110

Remembering a Simpler Time...

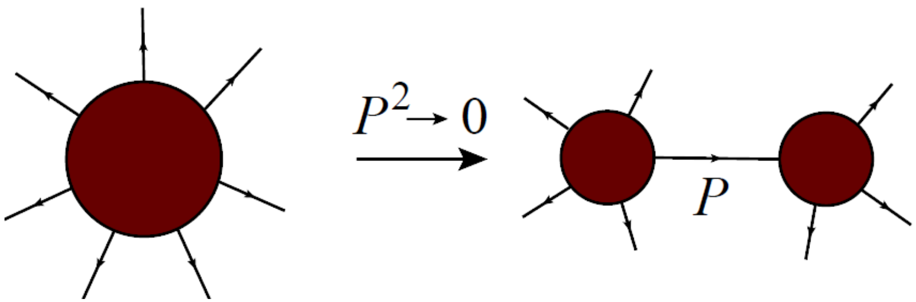
The 1960s

• In the 1960s there was no QCD, no Lagrangian or Feynman rules for the strong interactions

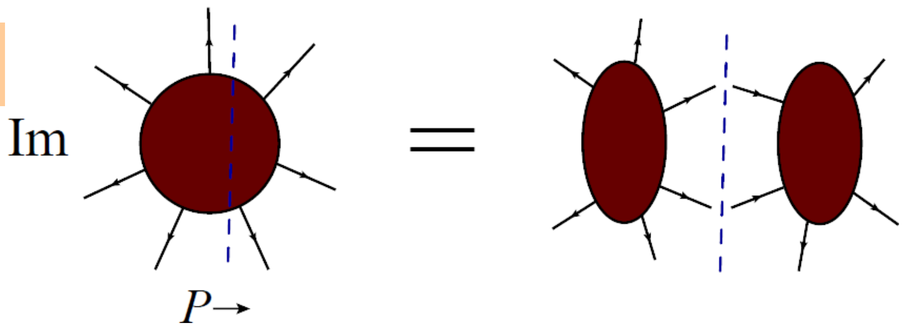
The Analytic *S*-Matrix

Bootstrap program for strong interactions:
Reconstruct scattering amplitudes **directly** from **analytic properties**

- Poles



- Branch cuts



Chew, Mandelstam;
Eden, Landshoff,
Olive, Polkinghorne;
Veneziano;
Virasoro, Shapiro;
… (1960s)

Analyticity fell out of favor in 1970s with the rise of QCD & Feynman rules

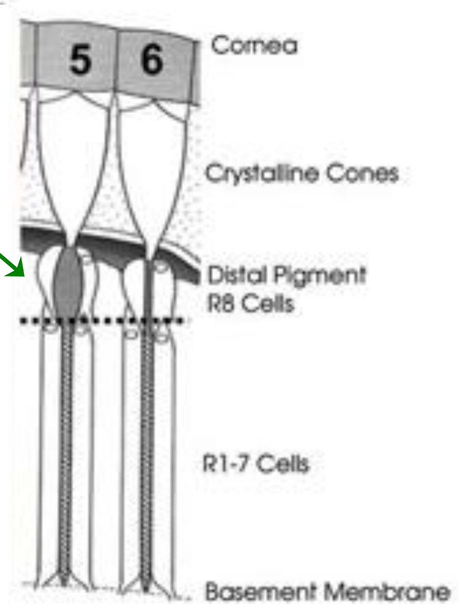
Ironically, it has now been **resurrected** for computing amplitudes for **perturbative** QCD – as an alternative to Feynman diagrams!

The Tail of the Mantis Shrimp

- Reflects left and right circularly polarized light differently

- Led biologists to discover that its eyes have differential sensitivity
- It communicates via the helicity formalism

$\lambda/4$ plate



"It's the most private communication system imaginable. No other animal can see it."

- Roy Caldwell (U.C. Berkeley)

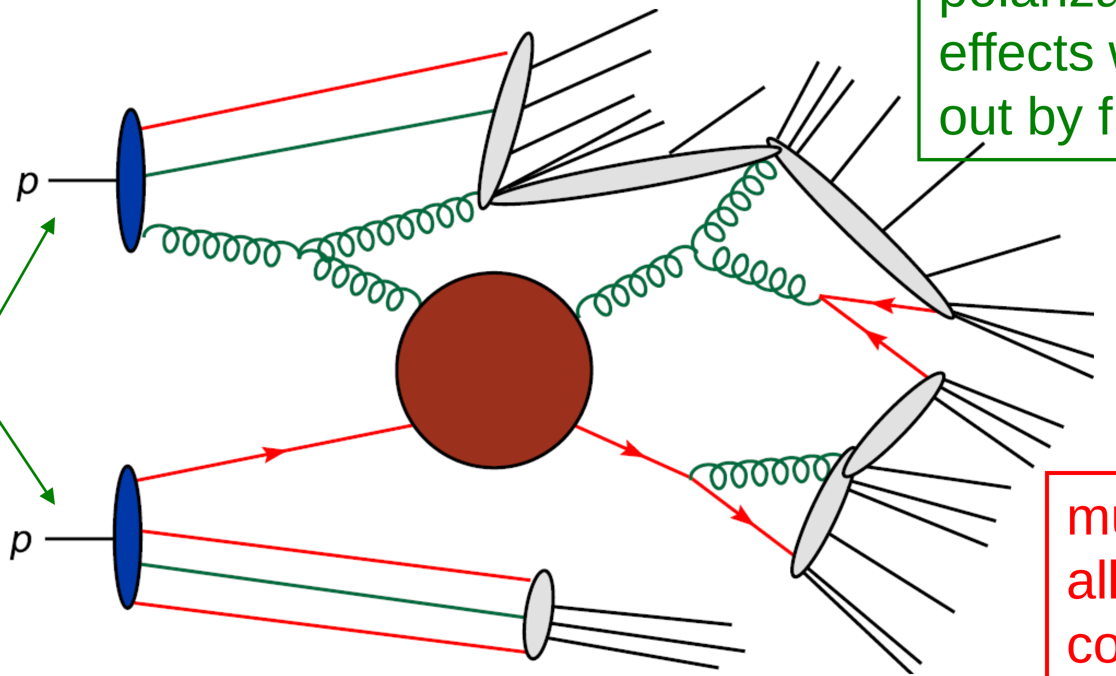
What the Biologists Didn't Know

Particle theorists have also evolved capability to communicate results via helicity formalism

LHC experimentalists are blind to it

any final-state polarization effects washed out by fragmentation

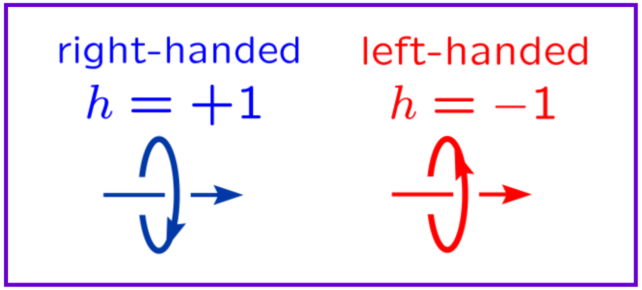
unpolarized

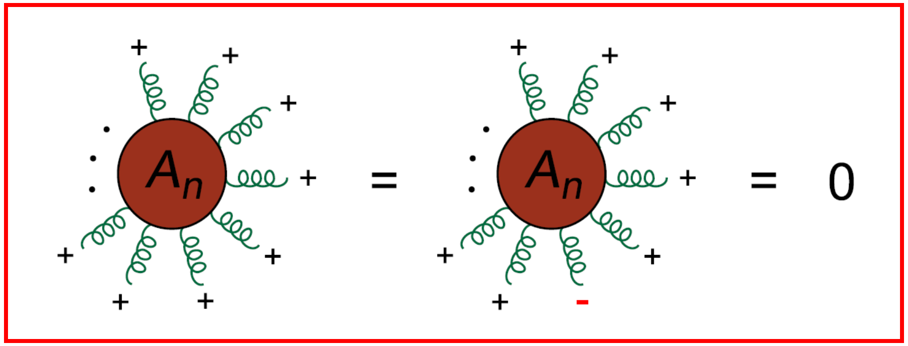


must sum over all helicity configurations

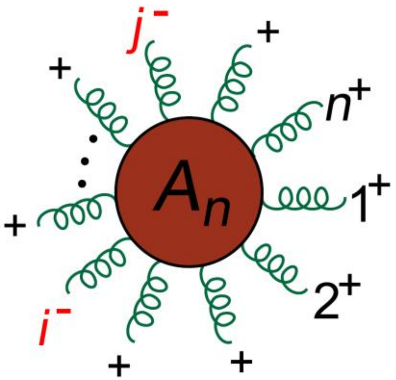
Helicity Formalism
→ Tree-Level Simplicity in QCD

Many helicity amplitudes either vanish or are very short





Analyticity makes it possible to recycle this simplicity into loop amplitudes

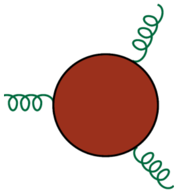


$$= \frac{\langle i\, j \rangle^4}{\langle 1\, 2 \rangle \langle 2\, 3 \rangle \cdots \langle n\, 1 \rangle}$$

Parke-Taylor formula (1986)

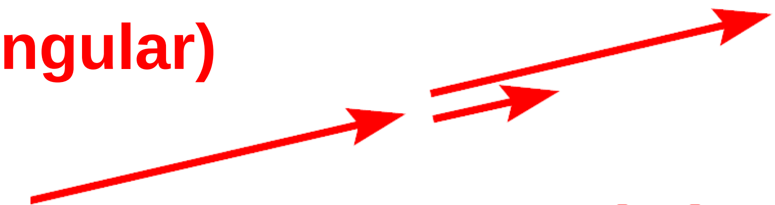
Special Complex Momenta

- Makes sense of most basic process with all 3 particles massless



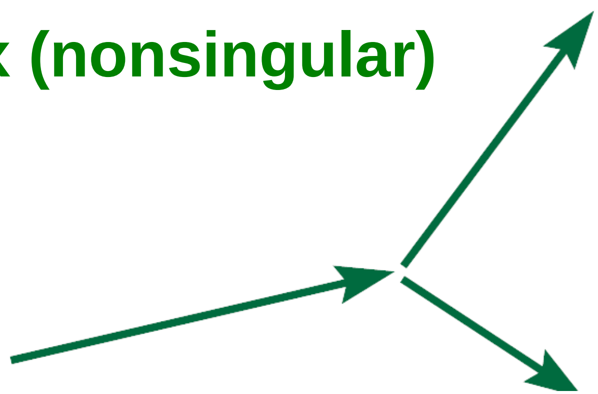
$$s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2 = 0 \quad \forall\ i,j$$

real (singular)



$$\langle i\, j \rangle = [i\, j] = s_{ij} = 0 \qquad \forall\ i,j$$

complex (nonsingular)



$$[i\, j] = 0 \quad \text{but} \quad \langle i\, j \rangle \neq 0$$

$$\frac{\langle i\, j \rangle^4}{\langle 1\, 2 \rangle \langle 2\, 3 \rangle \langle 3\, 1 \rangle}$$

makes sense

For Efficient Computation

Reduce

the number of “diagrams”

Reuse

building blocks over & over

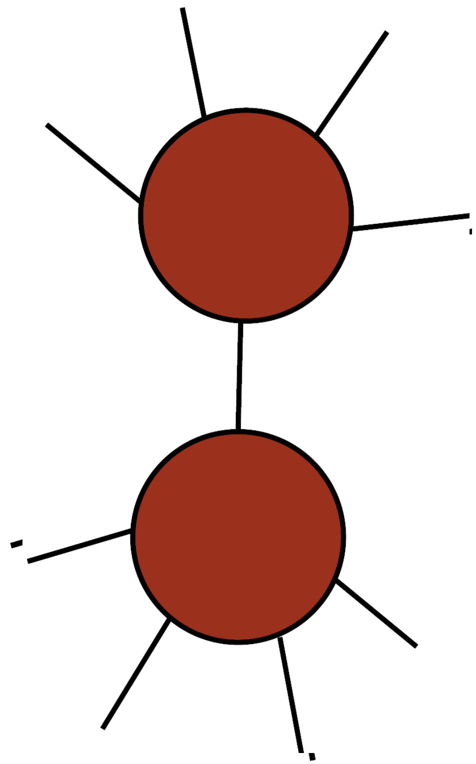
Recycle

lower-point (1-loop) & lower-loop (tree) on-shell amplitudes

Recurse

Amplitudes Are "Plastic"

They fall apart – factorize – into simpler ones in special limits

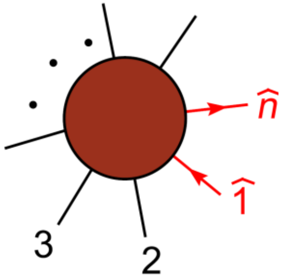


Explore Limits in Complex Plane

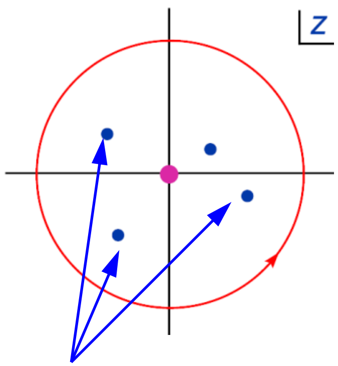
Britto, Cachazo, Feng, Witten, hep-th/0501052

Inject complex momentum at leg 1, remove it at leg *n.*

$$k_1(z) + k_n(z) = k_1 + k_n \qquad \Rightarrow \qquad A(0) \to A(z)$$
$$k_1^2(z) = k_n^2(z) = 0$$



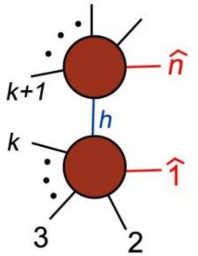
special limits $\Leftrightarrow$ poles in z



Cauchy: If $A(\infty) = 0$ then

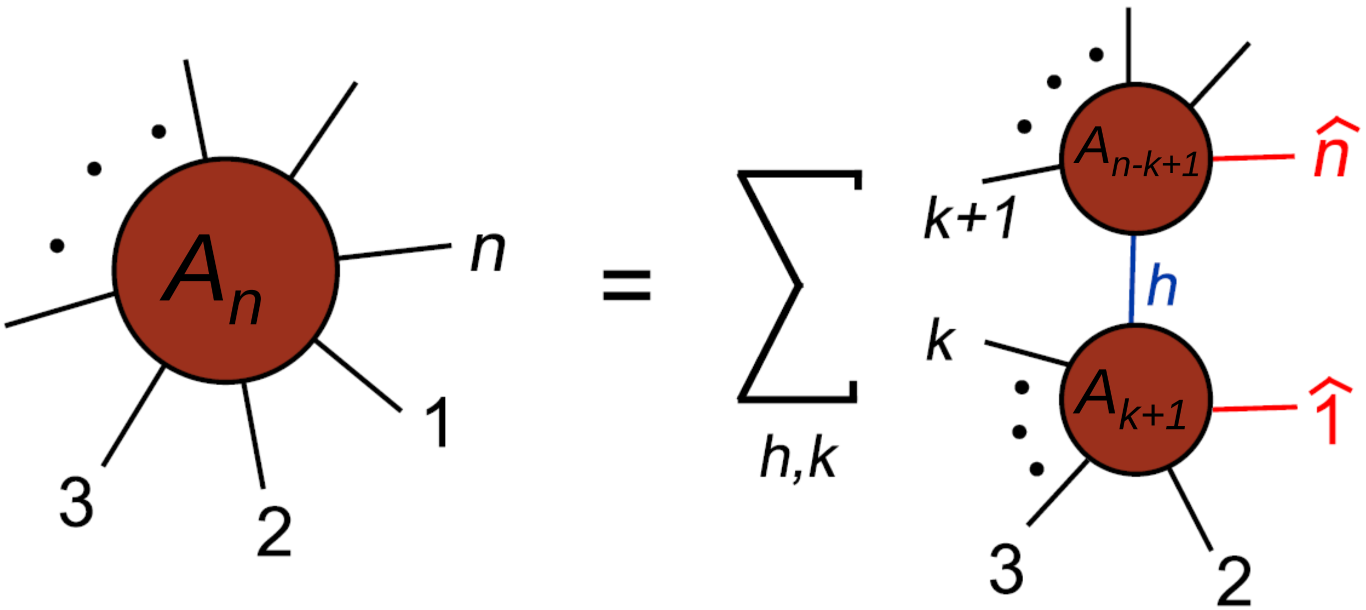
$$0 = \frac{1}{2\pi i} \oint dz \frac{A(z)}{z} = A(0) + \sum_k \text{Res}\left[\frac{A(z)}{z}\right]\Big|_{z=z_k}$$

residue at z_k = [k^{th} factorization limit] =



→ BCFW (On-shell) Recursion Relations

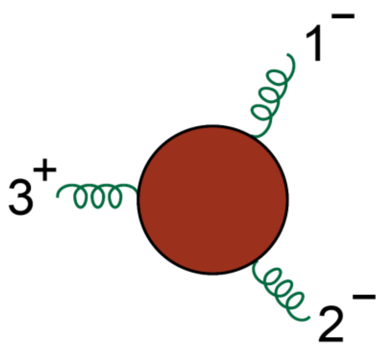
Britto, Cachazo, Feng, hep-th/0412308



A_{k+1} and A_{n-k+1} are **on-shell** tree amplitudes with **fewer** legs, and with momenta shifted by a **complex** amount

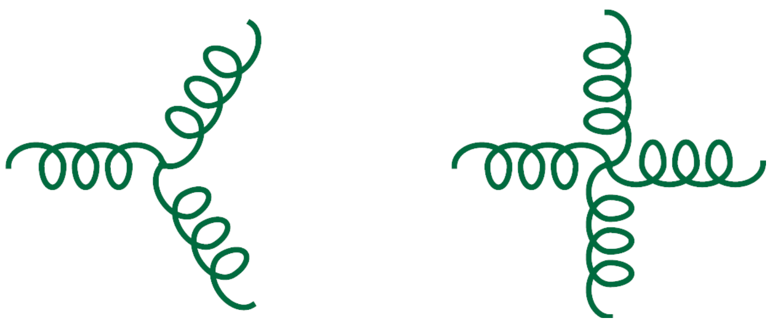
Trees recycled into trees

All Gluon Tree Amplitudes Built From:



$$= \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}$$

(In contrast to Feynman vertices, it is on-shell, gauge invariant.)

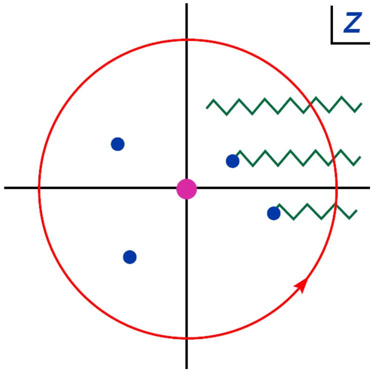


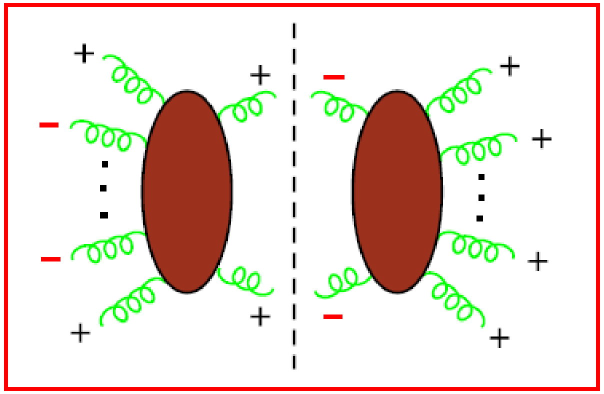
On-Shell Recursion at One Loop

Bern, LD, Kosower, hep-th/0501240, hep-ph/0505055, hep-ph/0507005;
Berger, et al., hep-ph/0604195, hep-ph/0607014, 0803.4180

- **Same techniques** work for one-loop QCD amplitudes

- **New features** compared with tree case, especially **branch cuts**
- Determine cut terms efficiently using (generalized) unitarity



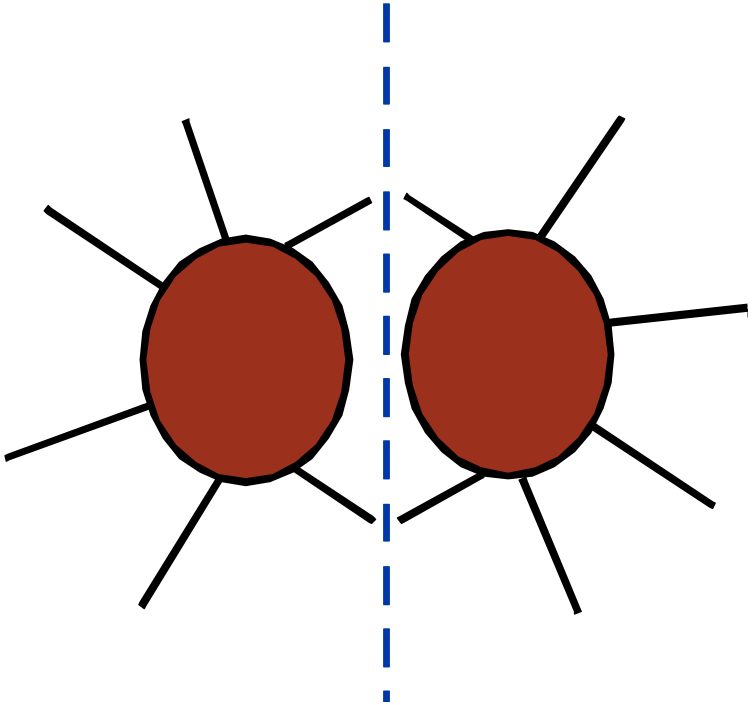


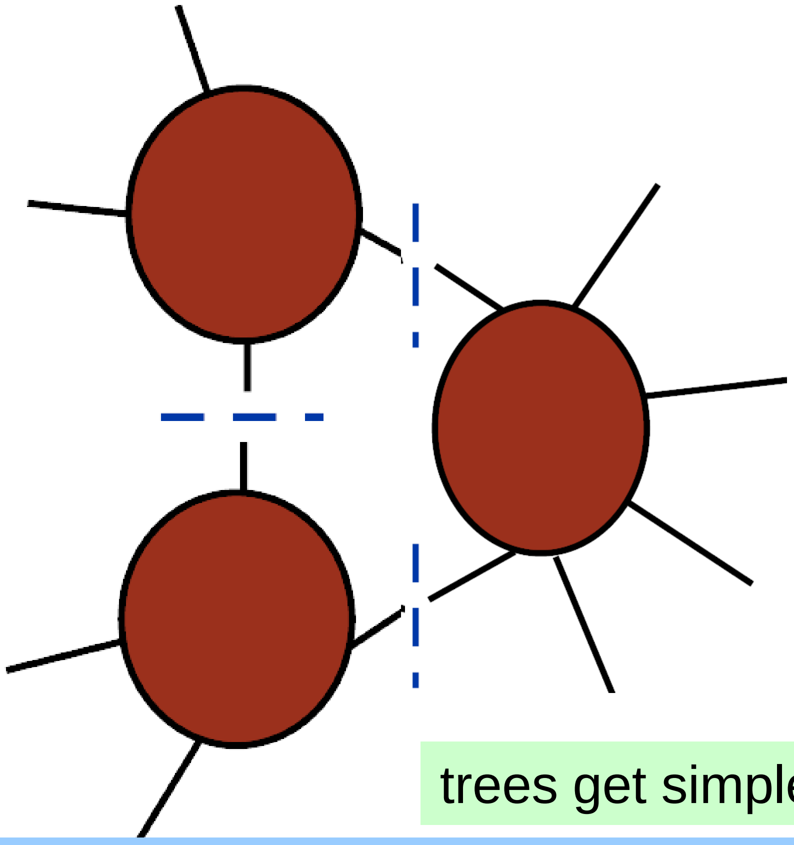
Trees recycled into loops!

Generalized Unitarity

Ordinary unitarity:
put 2 particles on shell

Generalized unitarity:
put 3 or 4 particles on shell





trees get simpler

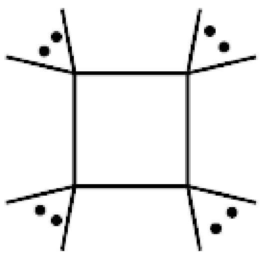
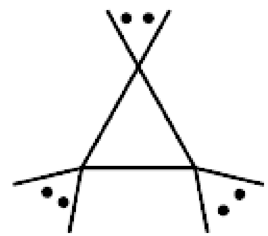
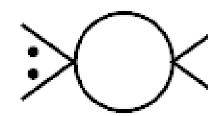
cut conditions require complex loop momenta

One-Loop Amplitude Decomposition

Missing from the old, nonpertubative analytic *S*-matrix

When all external momenta are in *D=4*, loop momenta in *D=4-2ε* (dimensional regularization), one can write: BDDK (1994)

coefficients are all rational functions – determine algebraiclally from products of trees using (generalized) unitarity

$A^{\text{1-loop}} = \sum_i d_i$  $+ \sum_i c_i$  $+ \sum_i b_i$ 

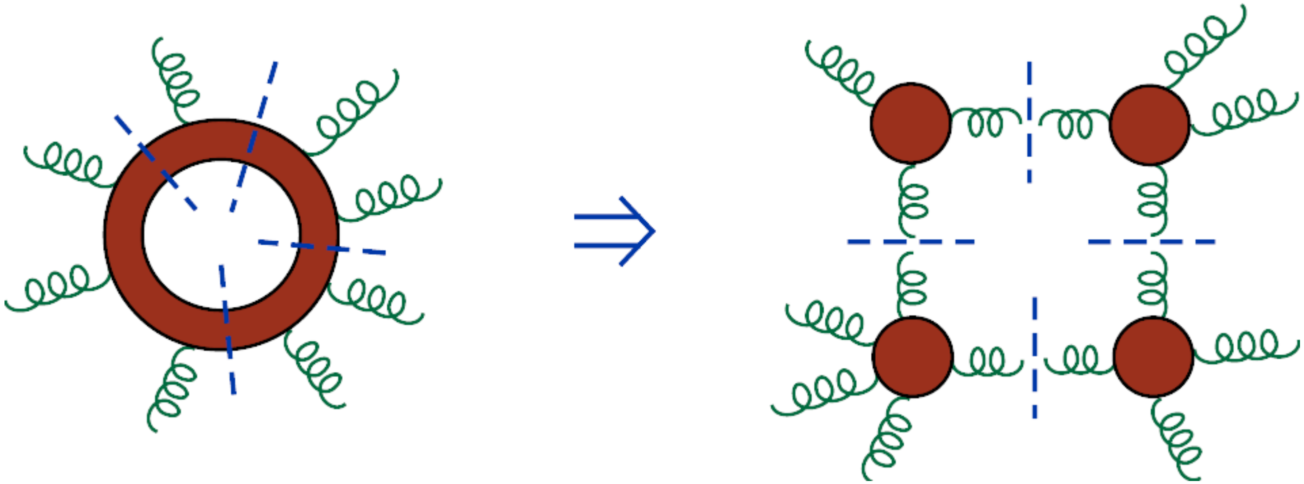
$+ R + \mathcal{O}(\epsilon)$

rational part

known **scalar** one-loop integrals, same for all amplitudes

Generalized Unitarity for Box Coefficients d_i

Britto, Cachazo, Feng, hep-th/0412308



$$\int d^4\ell \; \delta(\ell_1^2 - m_1^2)\, \delta(\ell_2^2 - m_2^2) \times \delta(\ell_3^2 - m_3^2)\, \delta(\ell_4^2 - m_4^2) \times A^{1-\text{loop}}(\ell_i)$$
$$= A_1^{\text{tree}}(\ell_0) A_2^{\text{tree}}(\ell_0) A_3^{\text{tree}}(\ell_0) A_4^{\text{tree}}(\ell_0)$$
$$= d_i$$

no. of dimensions = 4 = no. of constraints → discrete solutions (2)

Ideas Now Implemented Numerically and Automatically

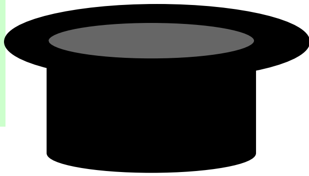
CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO production of *WWW* Binoth+OPP, 0804.0350

Rocket: Giele, Zanderighi, 0805.2152
One-loop n-gluon amplitudes for n up to 20;
W + 3 jets amplitudes EGKMZ, 0810.2762

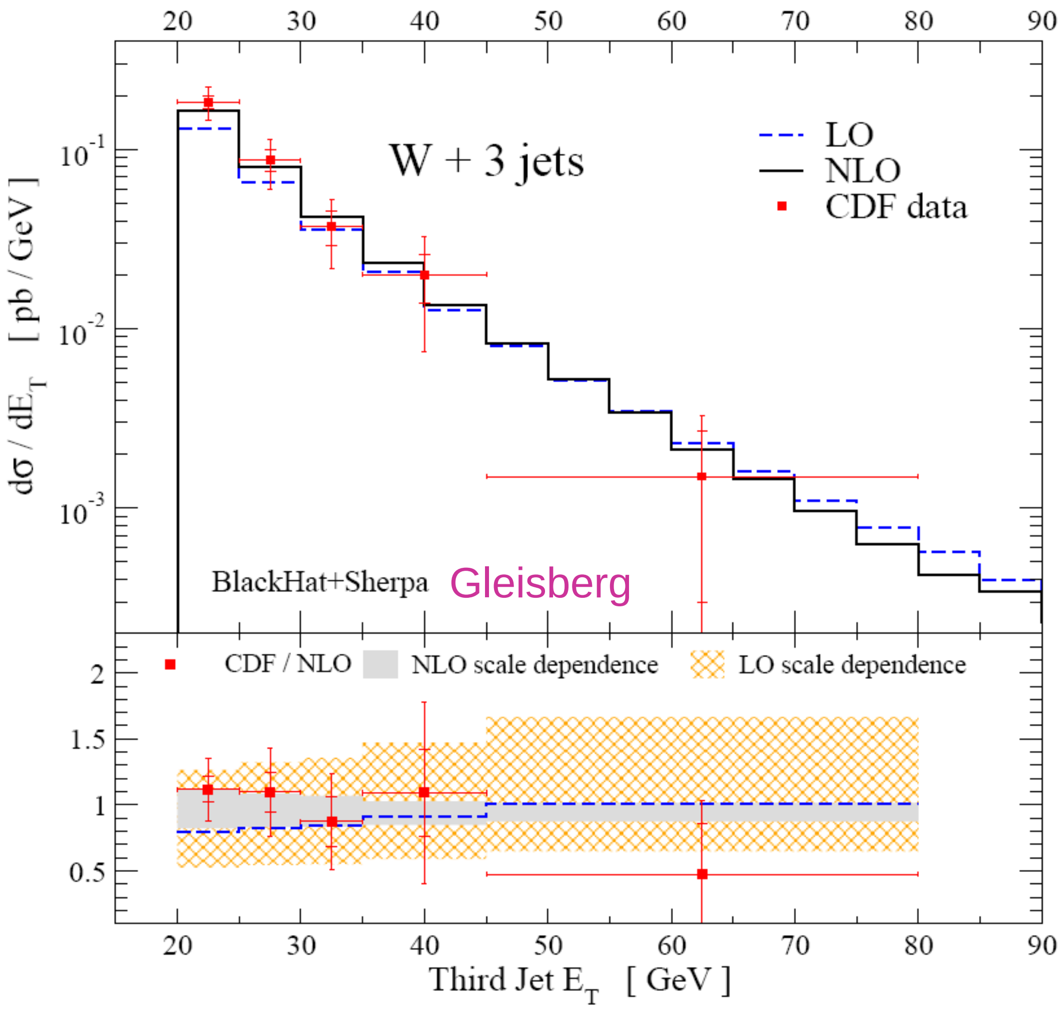
} D-dim'l unitarity

+ on-shell recursion

Blackhat: Berger, Bern, LD, Febres Cordero, Forde, H. Ita, D. Kosower, D. Maître, 0803.4180, 0808.0941
One-loop n-gluon amplitudes for n up to 7,…;
amplitudes needed for NLO production of *W,Z* + 3 jets



W + 3 jets at Tevatron at NLO



Phys. Rev. Lett. 102: 222001, 2009

and

0907.1984 [hep-ph]

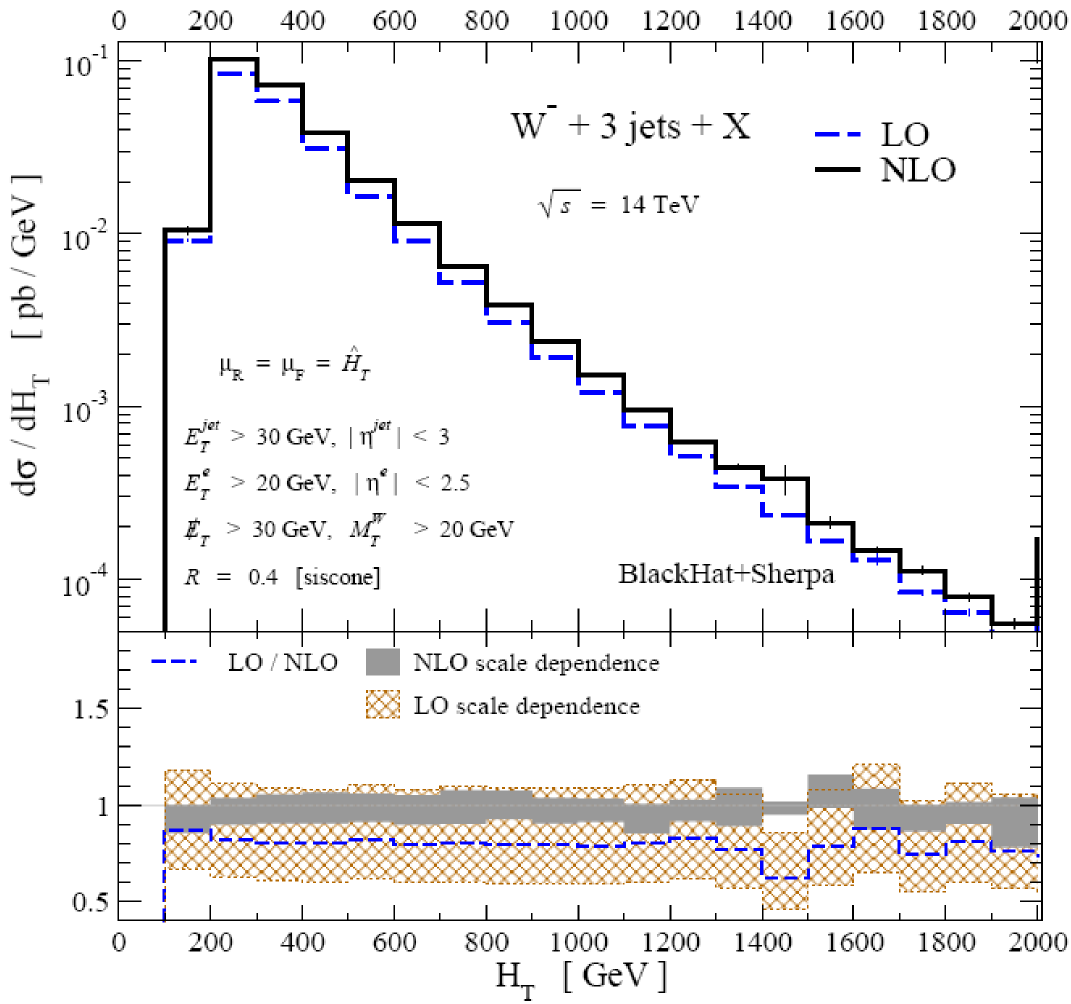
same cuts as CDF

- Much smaller uncertainties than at LO.
- Agrees well with data; more data coming soon.

Total Transverse Energy H_T at LHC

$$H_T = \sum_j E_{T,j}^{\text{jet}} + E_T^e + E_T^\nu$$

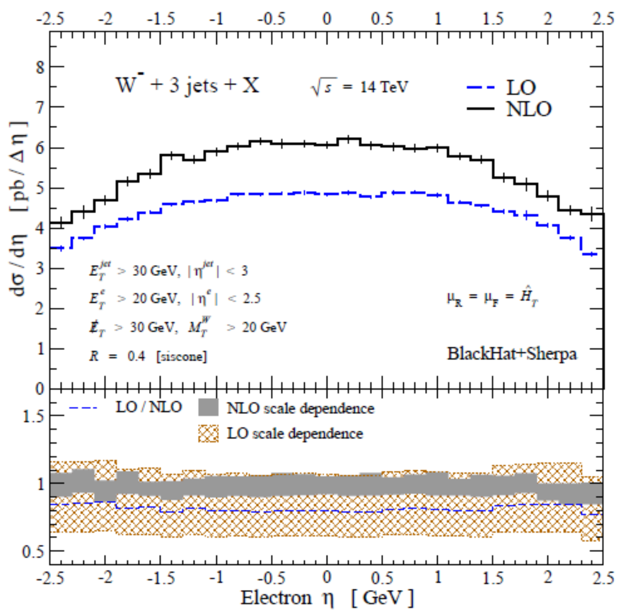
often used in supersymmetry searches

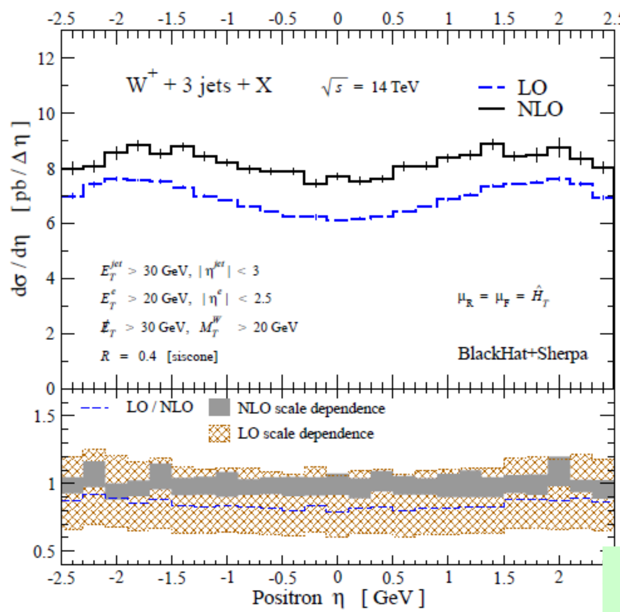


0907.1984

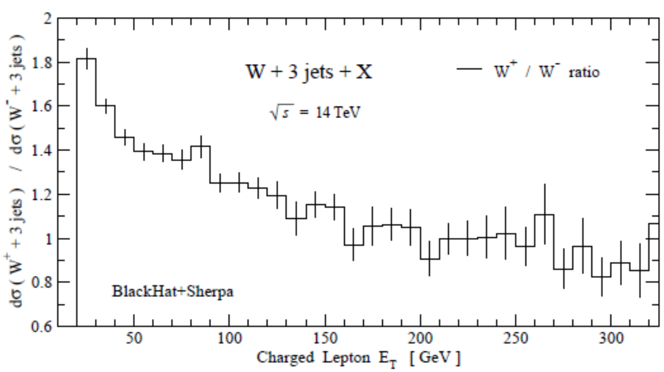
flat LO/NLO ratio due to good choice of scale $\mu = H_T$

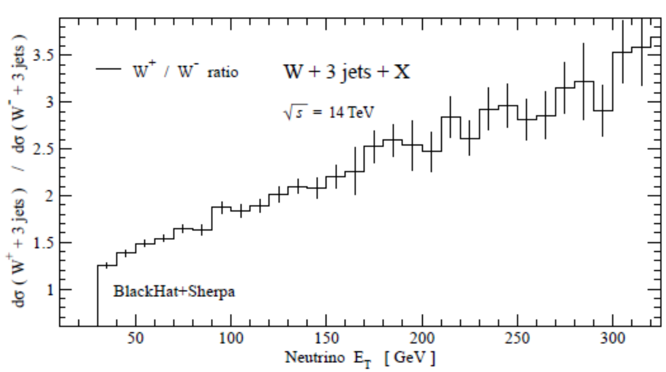
Leptonic Variables in W + 3 jets at LHC





rapidity distributions remember $u(x)/d(x)$ large as $x \rightarrow 1$





W^+W^- transverse ratios trace a remarkably large left-handed W polarization – may be useful to separate it from top, new physics

NLO $pp \to t\bar{t}\, b\bar{b}$ at LHC

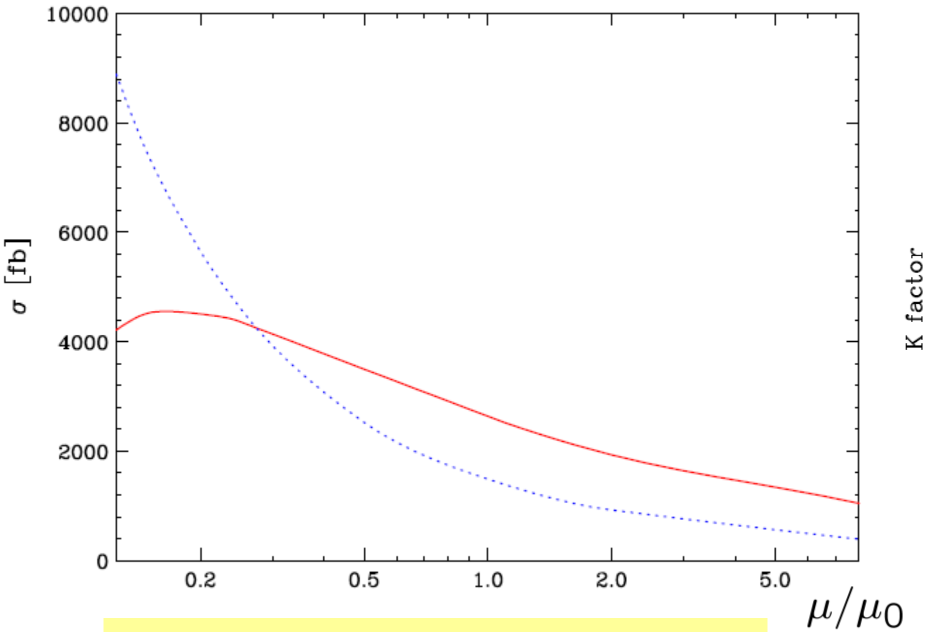
Higgs (+ tt̄) background at LHC

First done using Feynman diagrams

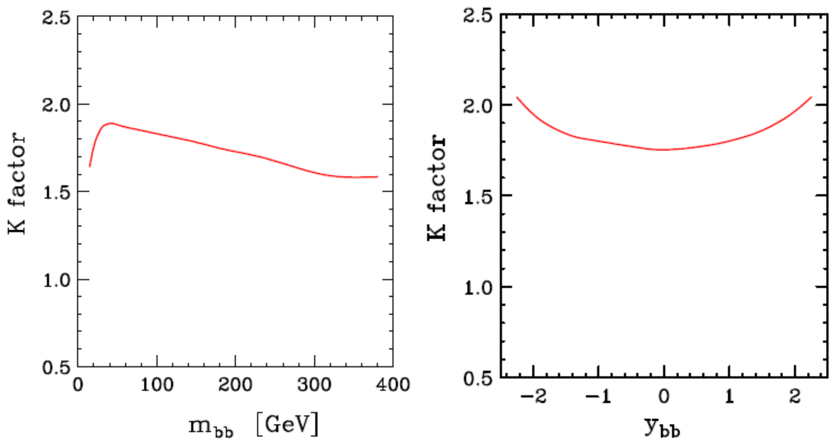
Recently recomputed in **CutTools** framework

Bredenstein et al., 0807.1248, 0905.0110

Bevilacqua et al., 0907.4723



much improved scale uncertainties at NLO



shape changes in bb distributions from LO to NLO (K=NLO/LO)

Conclusions

- New and efficient computational approaches to one-loop QCD amplitudes needed for important Tevatron and LHC backgrounds:
  - exploit **analyticity**: build loop amplitudes up out of trees
  - implemented numerically in C++ program **BlackHat**, as well as **CutTools** and **Rocket**
- Validated at Tevatron and now producing useful new NLO results for the LHC
- *W* + 3 jets completed; *Z* + 3 jets in process
- *W/Z* + 4 jets also now feasible
- Other groups have produced NLO results for several other processes using similar methods (*VVV*, *ttbb*, …)
- Success here an essential ingredient for optimal exploitation of LHC data!

Extra slides

Spinor products

Instead of Lorentz products:

$$s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$$

Use spinor products:

$$\varepsilon^{\alpha\beta}(\lambda_i)_\alpha(\lambda_j)_\beta = \langle i\, j\rangle$$

$$\varepsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\alpha}(\tilde\lambda_j)_{\dot\beta} = [i\, j]$$

Which always obey:

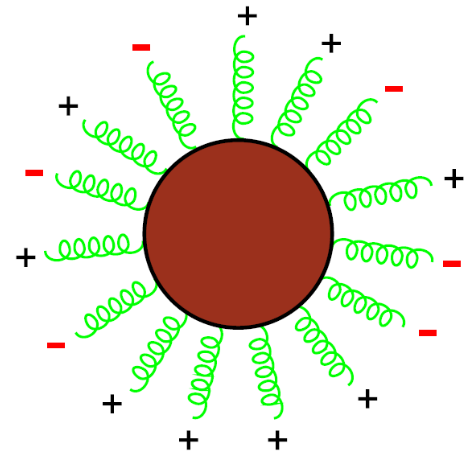
$$\langle i\, j\rangle [j\, i] = s_{ij}$$

If the momenta k_i are real, they are **complex square roots** of the Lorentz products:

$$\langle i\, j\rangle = \sqrt{s_{ij}}\, e^{i\phi_{ij}} \qquad [j\, i] = \sqrt{s_{ij}}\, e^{-i\phi_{ij}}$$

Spinor variables

Scattering amplitudes for massless plane waves of definite 4-momentum:
Lorentz vectors k_i^μ $\qquad k_i^2 = 0$



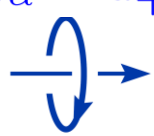
Textbook: use Lorentz-invariant products (invariant masses): $s_{ij} = 2k_i \cdot k_j = (k_i + k_j)^2$

But for particles with **spin**
there are better variables

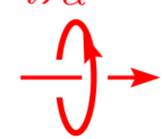
massless q,g,γ
all have 2 helicities

Take "square root" of 4-vectors k_i^μ (spin 1)
use 2-component Dirac (Weyl) spinors $u_\alpha(k_i)$ (spin ½)

right-handed: $(\lambda_i)_\alpha = u_+(k_i)$
$h = +1/2$

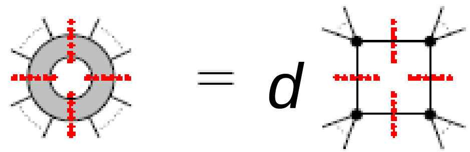


left-handed: $(\tilde{\lambda}_i)_{\dot{\alpha}} = u_-(k_i)$
$h = -1/2$

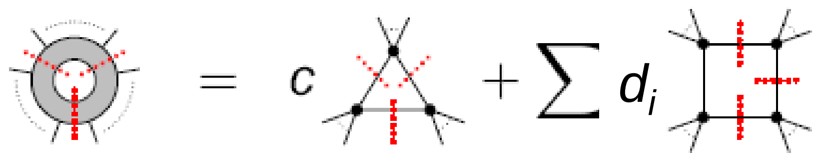


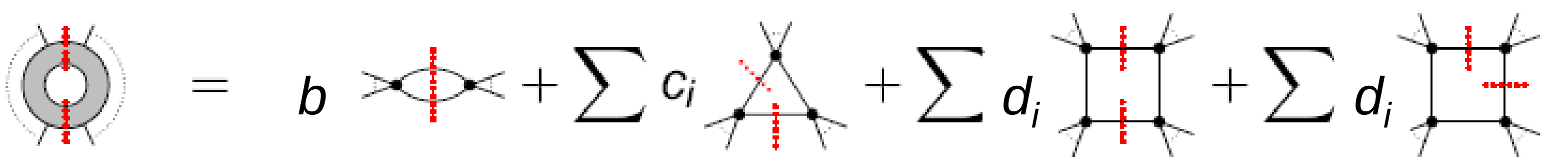
Other integral coefficients

With a 4-ple cut we select one coefficient



Triangle and bubble coefficients are more complicated since a double or triple cut does not isolate a single coefficient.





Also, solutions to cut constraints are now continuous, so there are multiple ways to solve and eliminate d_i , etc.

Britto et al. (2005,2006); Ossola, Papadopoulos, Pittau, hep-ph/0609007; Mastrolia hep-th/0611091; Forde, 0704.1835; Ellis, Giele, Kunszt, 0708.2398; …

Rational function *R*

No cuts in D=4 – can't get from D=4 unitarity
However, can get using D=4-2ε unitarity:

$$\int d^{4-2\epsilon}\ell \quad \Rightarrow \quad R(s_{ij}) \;\rightarrow\; R(s_{ij})\,(-s_{12})^{-\epsilon} \;=\; R(s_{ij})\,[1-\epsilon\,\ln(-s_{ij})]$$

Bern, Morgan (1996); Bern, LD, Kosower (1996);
Brandhuber, McNamara, Spence, Travaglini hep-th/0506068;
Anastasiou et al., hep-th/0609191, hep-th/0612277;
Britto, Feng, hep-ph/0612089, 0711.4284;
Giele, Kunszt, Melnikov, 0801.2237;
Britto, Feng, Mastrolia, 0803.1989; Britto, Feng, Yang, 0803.3147;
Ossola, Papadopolous, Pittau, 0802.1876;
Mastrolia, Ossola, Papadopolous, Pittau, 0803.3964;
Giele, Kunszt, Melnikov (2008); Giele, Zanderighi, 0805.2152;
Ellis, Giele, Kunszt, Melnikov, 0806.3467;
Feng, Yang, 0806.4106; Badger, 0806.4600;
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762

OR: Get rational function *R* using on-shell recursion

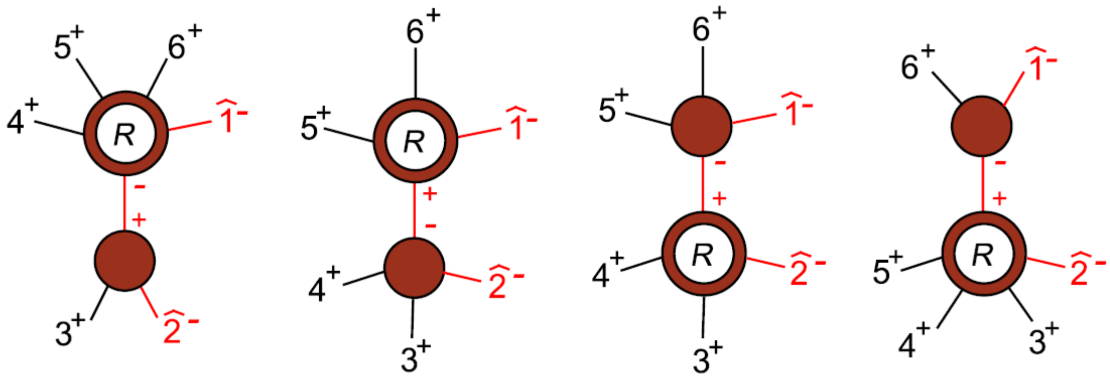
- Used to get infinite series of QCD helicity amplitudes analytically:
  - n-gluon MHV amplitudes at 1-loop $(- + \cdots + - + \cdots +)$
  - n-gluon "split" helicity amplitudes $(- - \cdots - + + \cdots +)$
  - "Higgs" + n-gluon MHV amplitudes $(\phi; - + \cdots + - + \cdots +)$

Forde, Kosower, hep-ph/0509358; Berger, Bern, LD, Forde, Kosower, hep-ph/0604195, hep-ph/0607014; Badger, Glover, Risager, 0704.3194; Glover, Mastrolia, Williams, 0804.4149

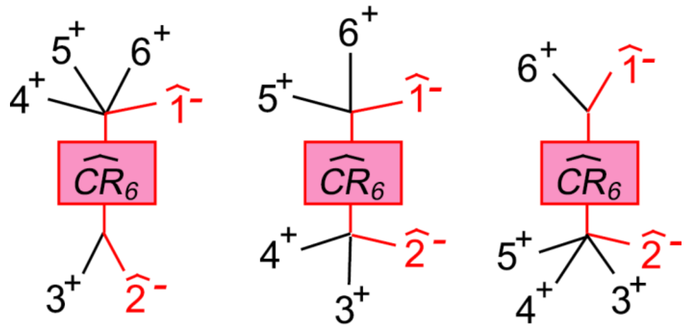
Example of recursive diagrams

For rational part of $A_6^{1-\text{loop}}(1^-, 2^-, 3^+, 4^+, 5^+, 6^+)$

recursive:



overlap:



loops recycled into loops

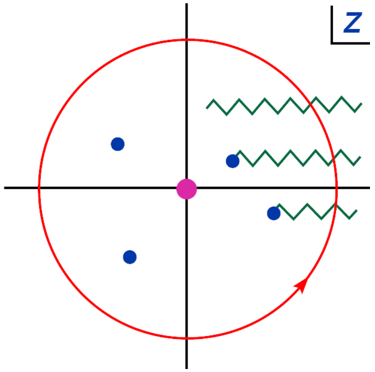
Compared with 10,860 1-loop Feynman diagrams

Loop amplitudes with cuts

Generic analytic properties of shifted 1-loop amplitude, $A_n(z)$

Cuts and poles in z-plane:

$$\ln(s_{23}) \Rightarrow \ln[(\langle 2\,3\rangle + z\langle 1\,3\rangle)[3\,2]]$$



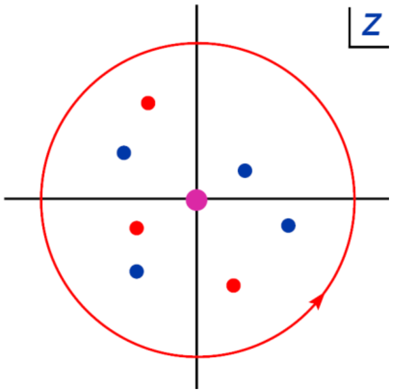
But if we know the cuts (*via* unitarity in D=4), we can subtract them: $R_n \equiv A_n - C_n$

rational part — full amplitude — cut-containing part

Shifted rational function $R_n(z) = A_n(z) - C_n(z)$

has no cuts, but has spurious poles in z because of C_n:

$$C_n \longrightarrow \frac{\ln(r) + 1 - r}{(1-r)^2} \longleftarrow R_n$$



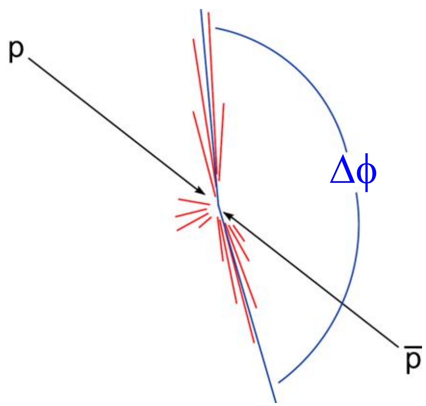
Unmasking a new particle

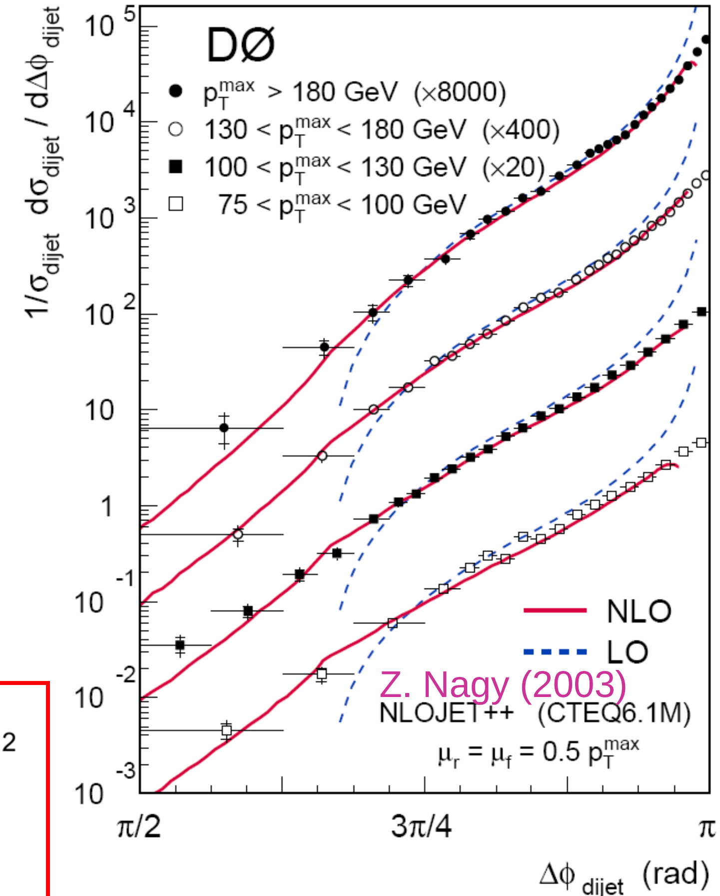
Suppose a new particle is found – how do we know what we have, a Higgs boson or something else?

Particle theorists are really good at proposing alternative explanations...

NLO also improves shapes of distributions

Azimuthal decorrelation of di-jets at D0 at Tevatron, due to additional radiation





NLO

n=3