GK1504 "Masse, Spektrum, Symmetrie" Symposium - 28.09.2009

Supergravity in the sky

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DESY

HELMHOLTZ
ASSOCIATION

Outline

- Introduction:
  Cosmology & the present Universe
- SUGRA Part I:
  Gravitino Dark Matter
- SUGRA Part II:
  de Sitter/inflation in String-inspired SUGRA
- Outlook

Introduction

Einstein's legacy: Energy is Geometry

$$\mathcal{R}_\mu^\nu - \frac{1}{2}\delta_\mu^\nu \mathcal{R} \; = \; 8\pi G_N \, T_\mu^\nu + \Lambda \delta_\mu^\nu$$

Einstein's Tensor:
Geometry of Space-time

Energy-momentum Tensor:
ALL the Physics content

The birth of Cosmology as a science:
the Universe's dynamics and fate is determined
by its Energy (Particle) content,
both the known and the unknown....!

Standard Cosmology

Cosmological Principle (nowadays also experimental result...):

The Universe is homogeneous and isotropic
on large scales (i.e. larger than ~100 Mpc)

It is described by the Friedmann-Robertson-Walker Metric:

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega\right)$$

- Only one dynamical variable: the scale factor $a(t)$
- One constant parameter: the spatial curvature κ

Friedmann equation:

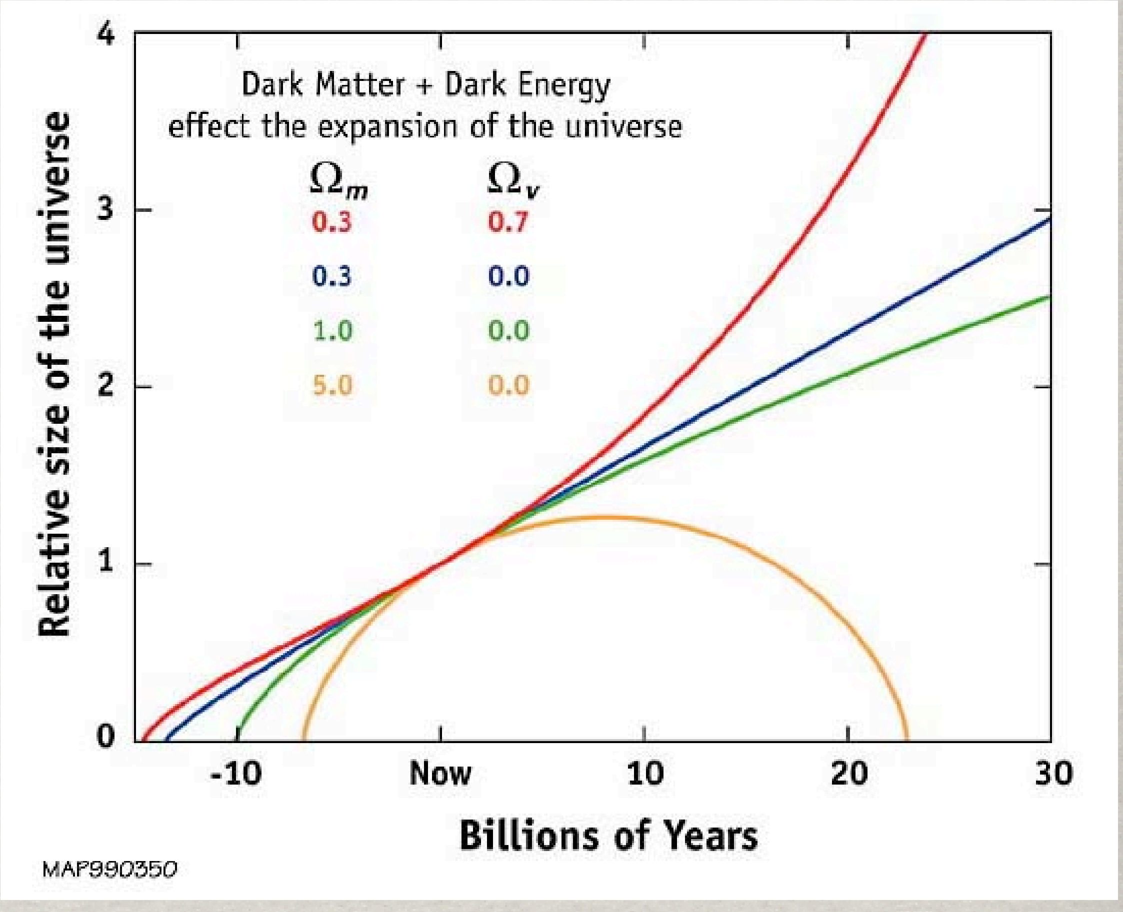
$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho + \Lambda - \frac{\kappa}{a^2}$$

- The energy density & curvature decree the time evolution of the scale factor
- Key parameter is the critical density:

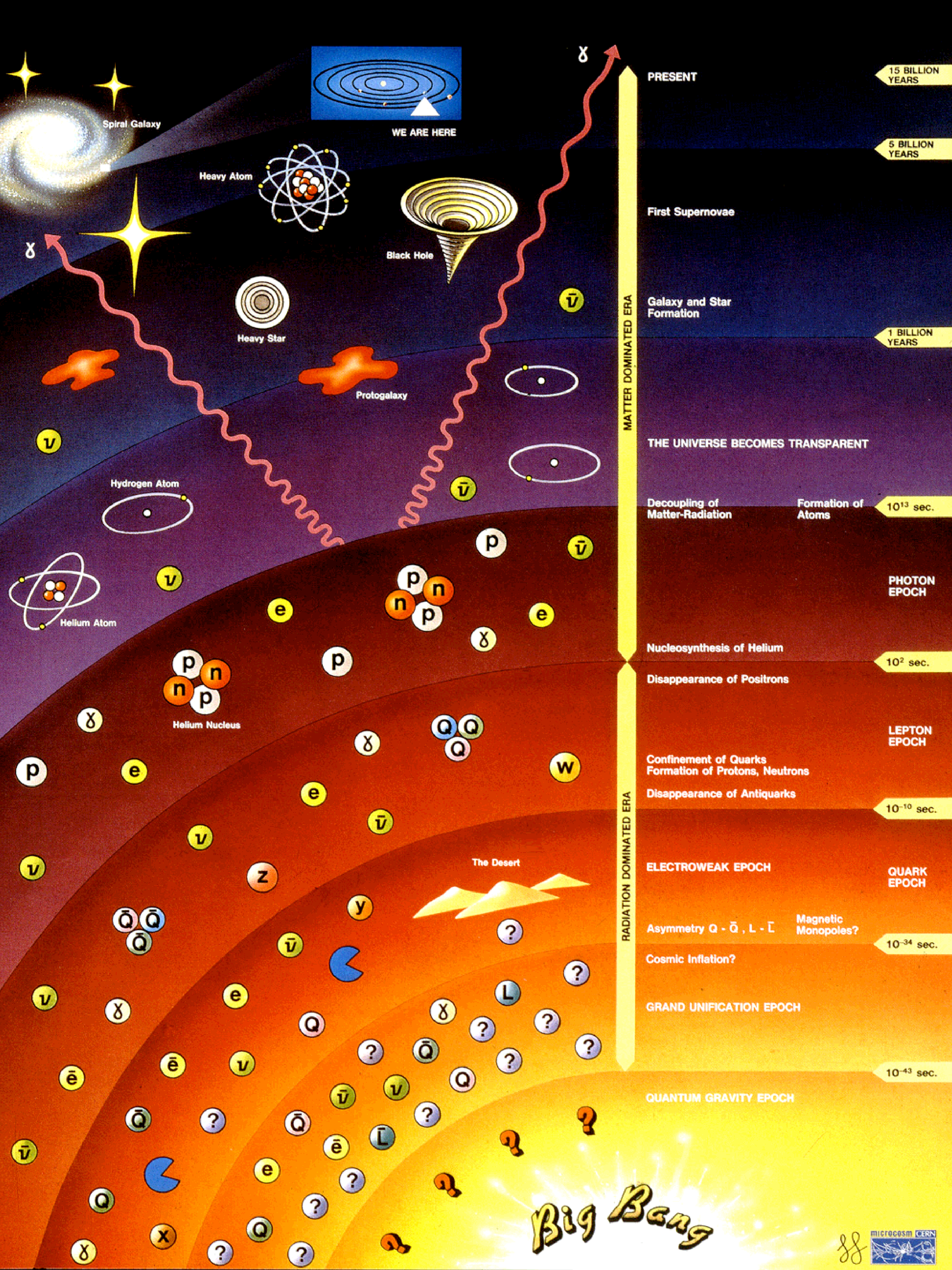
$$\rho_c = \frac{3H^2}{8\pi G_N} \qquad \Omega_i = \frac{\rho_i}{\rho_c}$$

Ω_i :density in $\sim 10^4 \mathrm{eV/cm}^3$

(~10 protons/m3)

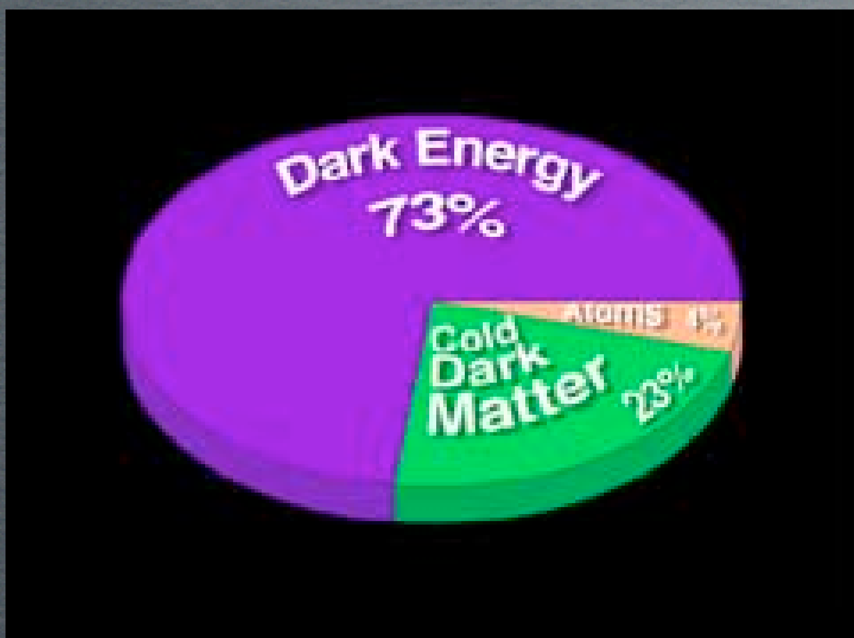


http:/www.wmap.gsfc.nasa.gov



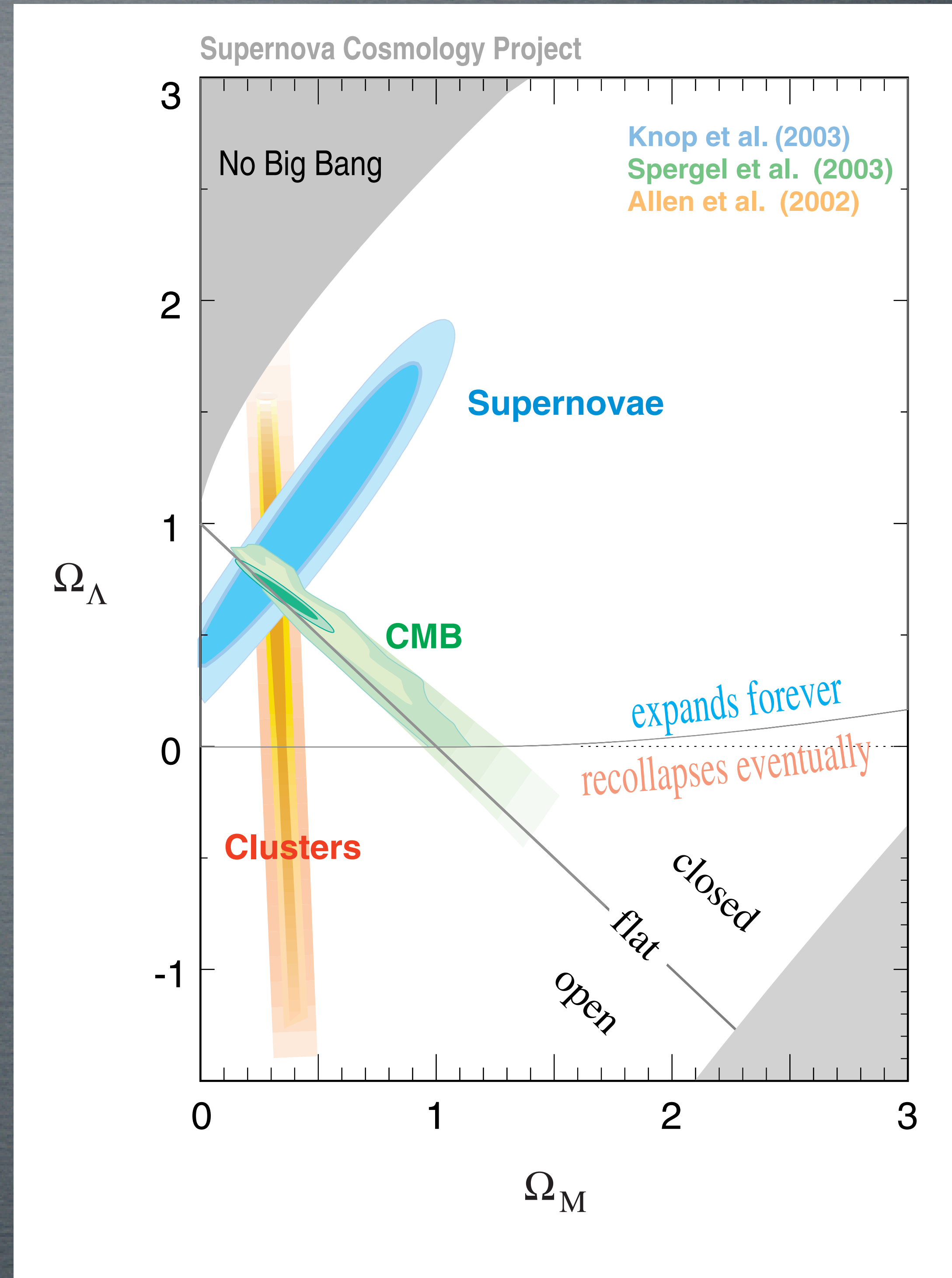
Spiral Galaxy
WE ARE HERE
Heavy Atom
Black Hole
Heavy Star
Protogalaxy
Hydrogen Atom
Helium Atom
Helium Nucleus
The Desert
Big Bang
MATTER DOMINATED ERA
RADIATION DOMINATED ERA
PRESENT
15 BILLION YEARS
5 BILLION YEARS
First Supernovae
Galaxy and Star Formation
1 BILLION YEARS
THE UNIVERSE BECOMES TRANSPARENT
Decoupling of Matter-Radiation
Formation of Atoms
10^{13} sec.
PHOTON EPOCH
Nucleosynthesis of Helium
10^{2} sec.
Disappearance of Positrons
LEPTON EPOCH
Confinement of Quarks
Formation of Protons, Neutrons
Disappearance of Antiquarks
10^{-10} sec.
ELECTROWEAK EPOCH
QUARK EPOCH
Asymmetry $Q - \bar{Q}$, $L - \bar{L}$
Magnetic Monopoles?
10^{-34} sec.
Cosmic Inflation?
GRAND UNIFICATION EPOCH
10^{-43} sec.
QUANTUM GRAVITY EPOCH
microcosm CERN

PRESENT ENERGY CONTENT

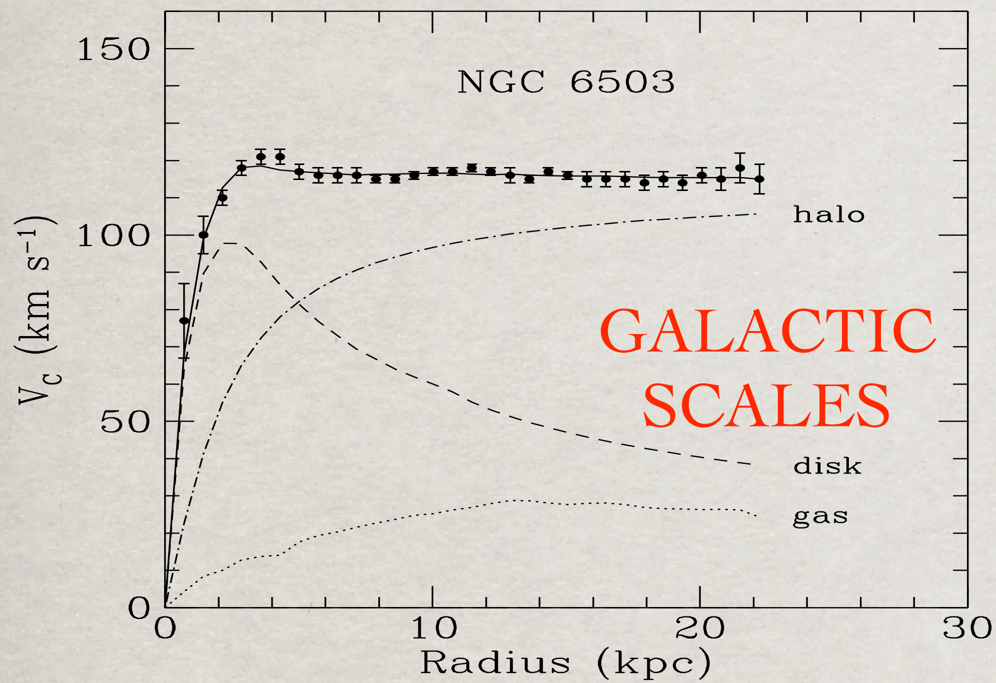


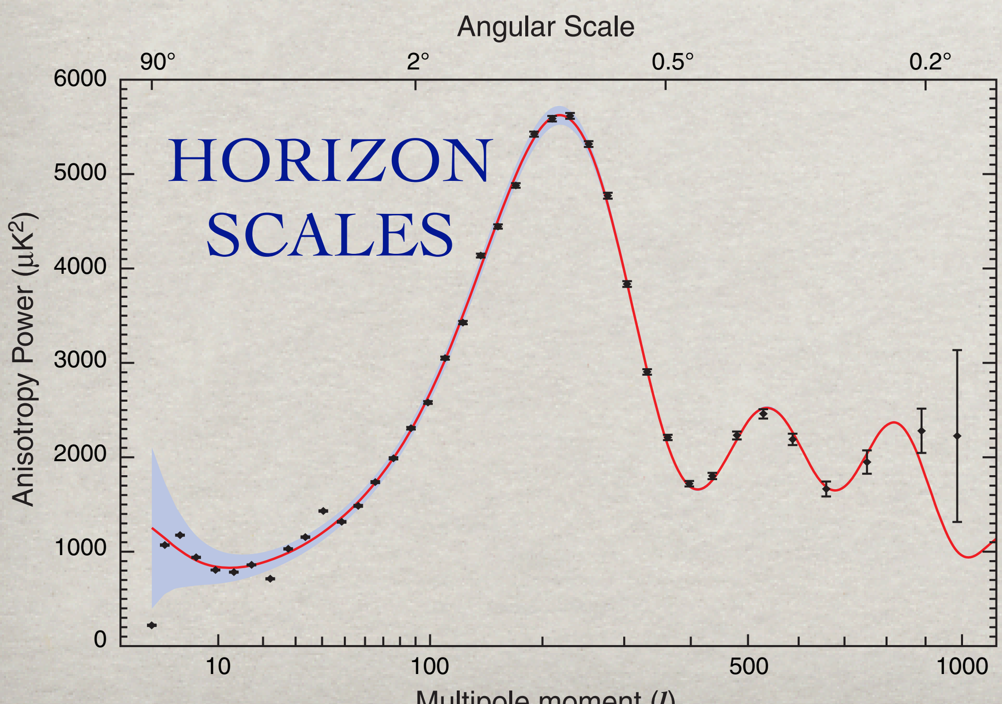
with traces of photons, neutrinos & ... ?

What are DE and DM ???



DARK MATTER EVIDENCE





Particles	Ωh^2	Type
Baryons	0.0224	Cold
Neutrinos	< 0.01	Hot
Dark Matter	0.1-0.13	Cold

DARK MATTER PROPERTIES

- Interacts very weakly, but surely gravitationally (non-baryonic & decoupled from the baryon-photon plasma !)
- It must have the right density profile to “fill in” the galaxy rotation curves.
- No pressure and small free-streaming velocity, it must cluster & cause structure formation.

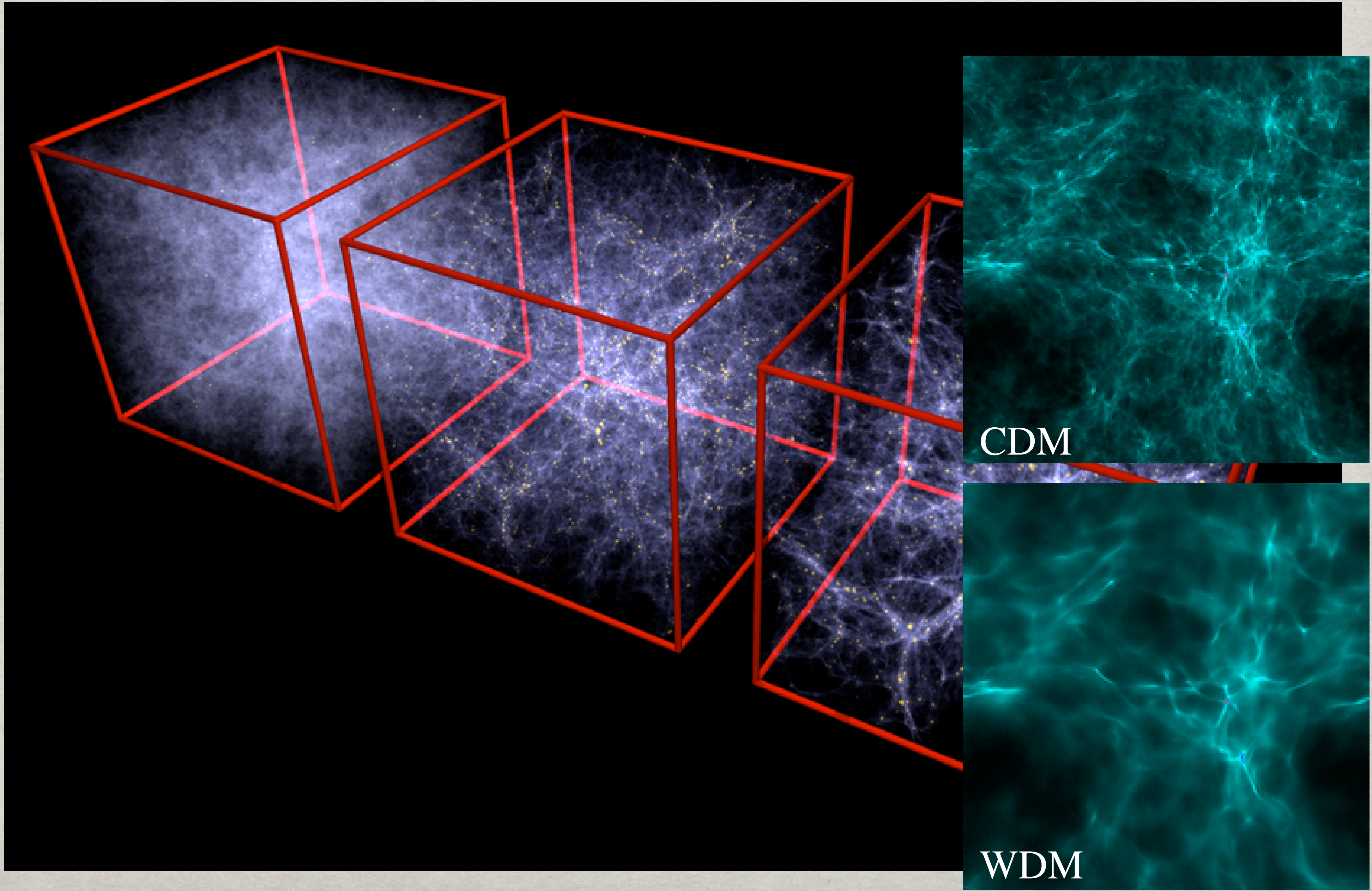
↓

COLD DARK MATTER

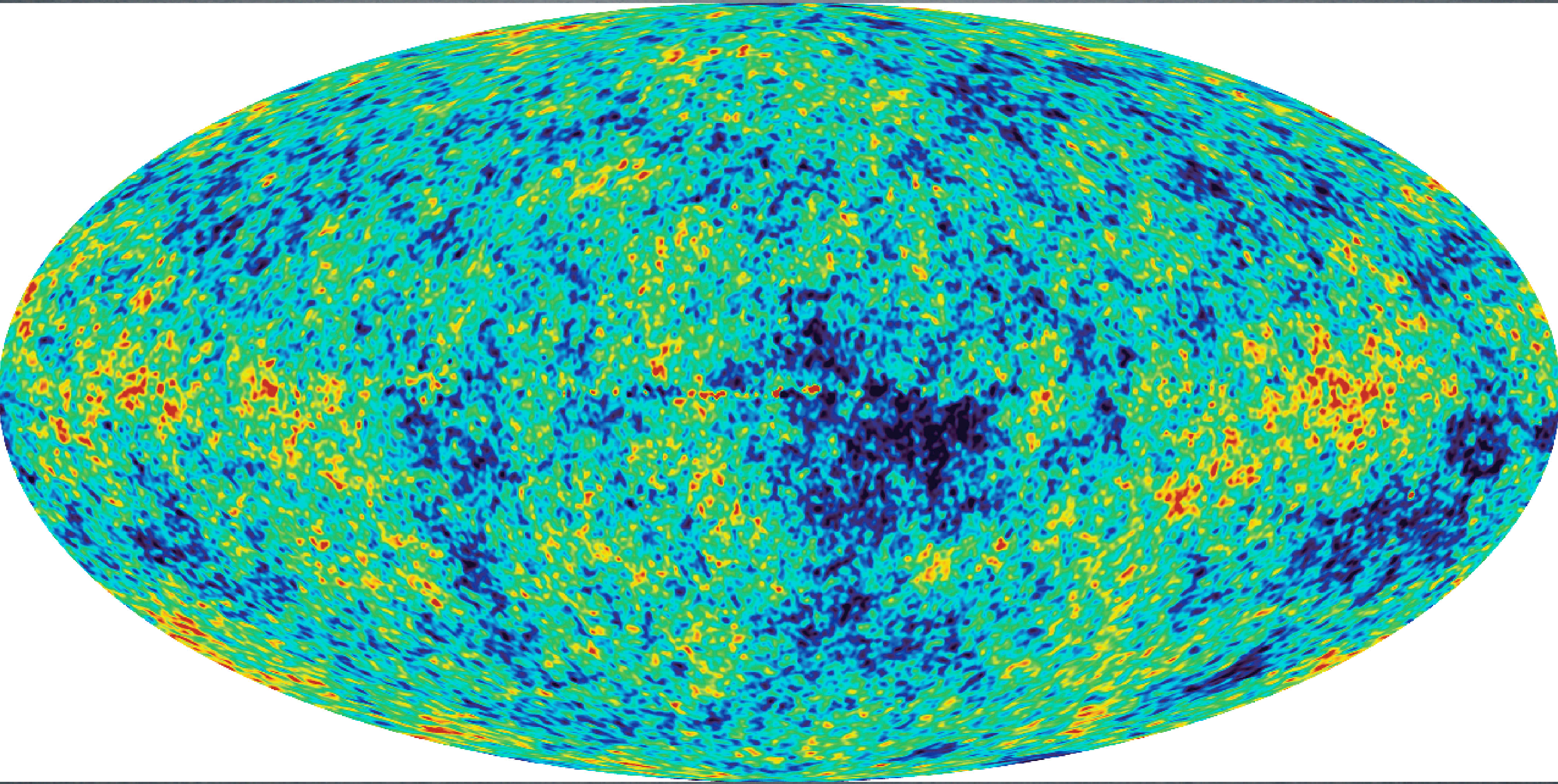
Structure Formation

V. Springel @MPA Munich

Yoshida et al 03



The Universe is NOT perfectly homogeneous !



[WMAP 06]

Tiny ripples on the black body spectrum at level of 0.01%...

WHY IS THE UNIVERSE FLAT, HOMOGENEOUS & ISOTROPIC ?

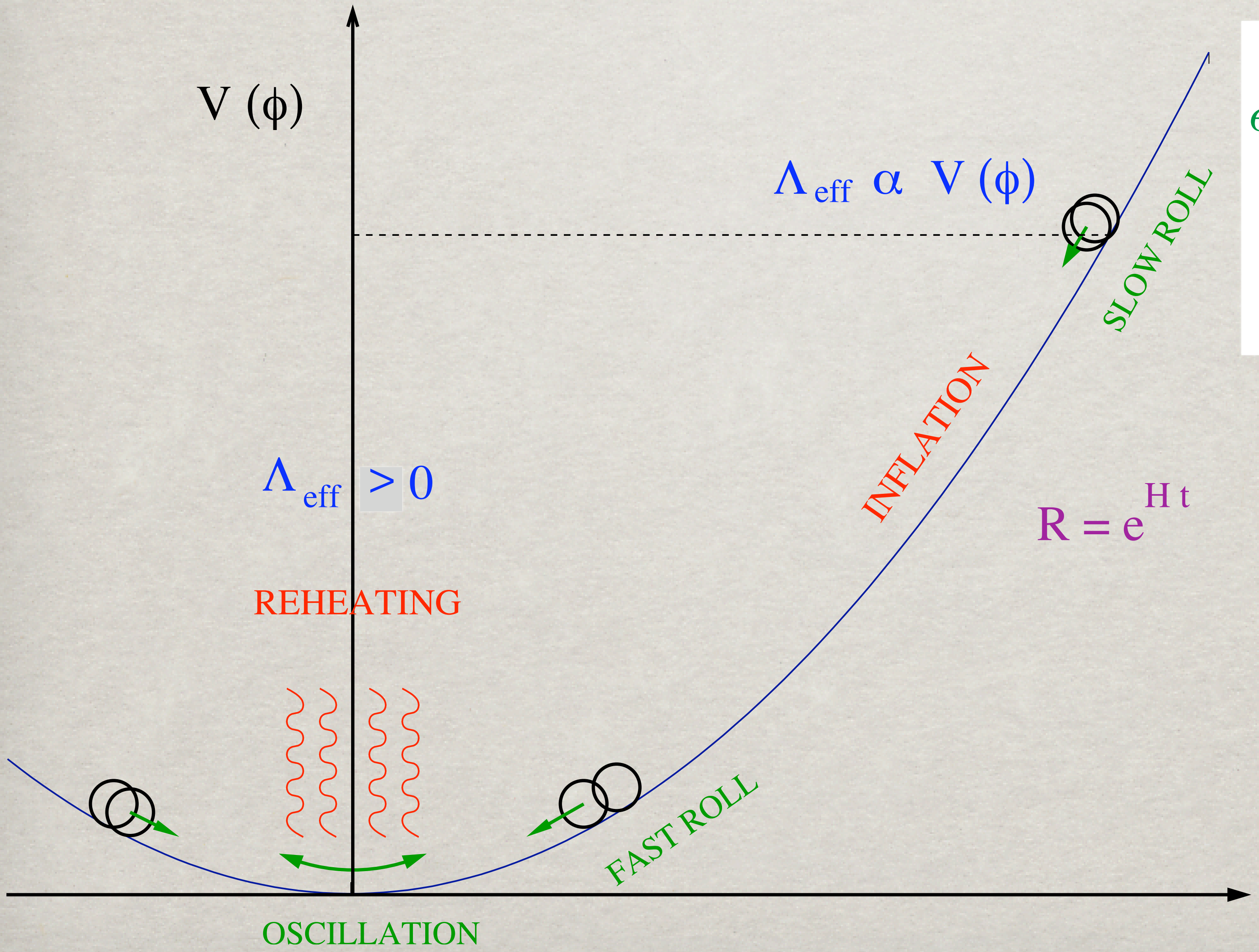
WHAT CAUSED THE TINY RIPPLES, WHICH ARE ORIGIN OF STRUCTURE?

↓

INFLATION

EARLY PHASE OF EXPONENTIAL EXPANSION

Inflation: driven by a scalar field ϕ



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta| = \left|\frac{V''}{V}\right| \ll 1$$

$R = e^{Ht}$ Quasi - de Sitter

de Sitter ?

INFLATON: A QUANTUM FIELD !

Apart for the classical motion, there are fluctuations:

$$\phi = \varphi_c + \delta\varphi$$

In an inflationary (~ de Sitter) phase these are given by

$$\delta\varphi = \frac{H}{2\pi}$$

They remain imprinted in the metric and are stretched to cosmological scales !!!

Look for a signal there !

Power spectrum of the fluctuations

Testing inflation: Single field inflation $\Longleftrightarrow$ Flat Potential $V(\phi)$

The scalar power spectrum is given by $$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2}\Big|_{k=aH} \propto k^{n-1}$$

and its spectral index is: $$n(k)-1 = \frac{d\log(\mathcal{P}_{\mathcal{R}})}{d\log(k)}\Big|_{k=aH} = 2\eta - 6\epsilon + \ldots$$

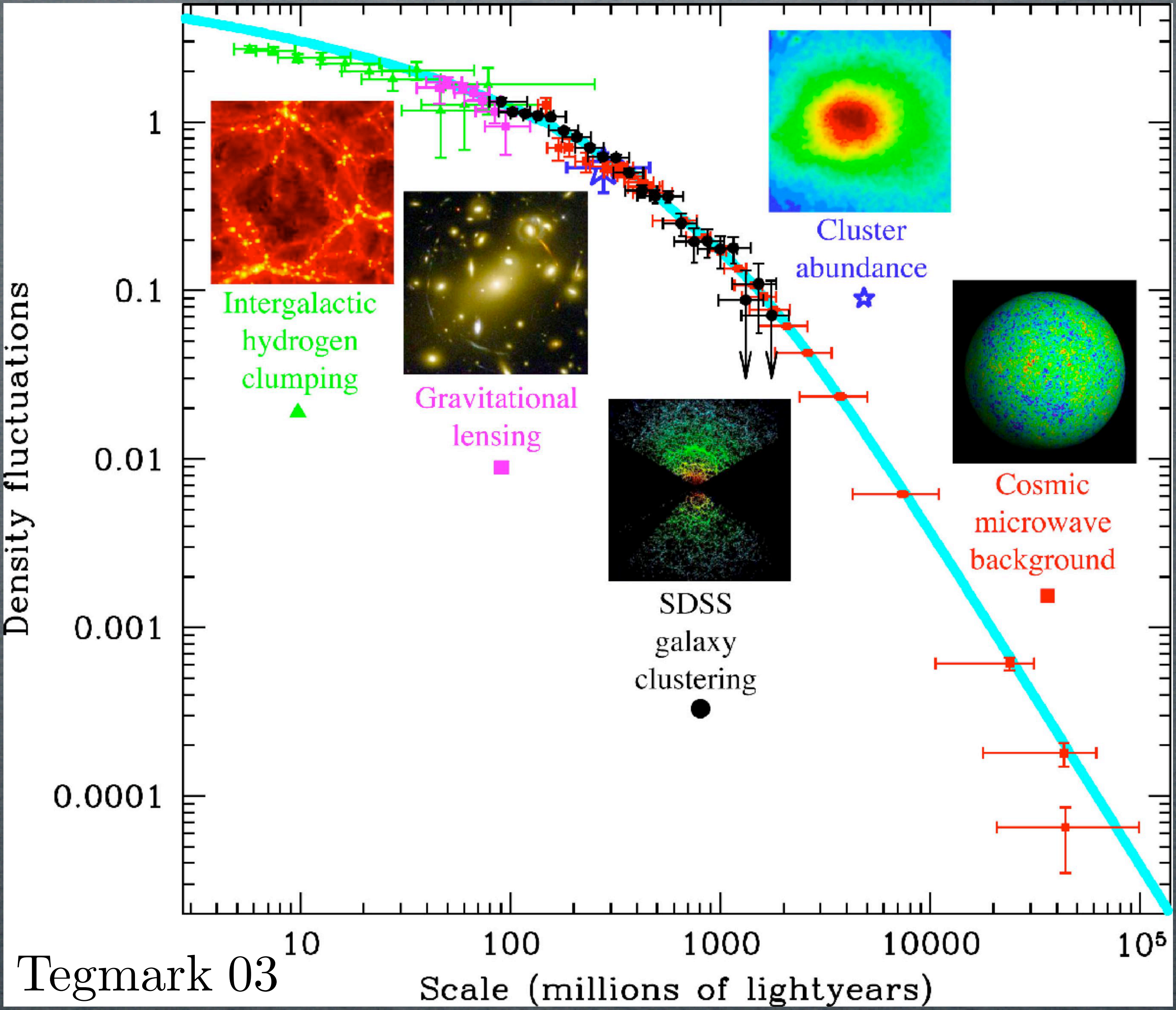
n' arises only at 2^{nd} order in SLOW ROLL: $$n'(k) = \frac{2}{3}\left((n-1)^2 - 4\eta^2\right) + 2\xi$$

where the slow roll parameters are $$\epsilon = \frac{M_P^2}{16\pi}\frac{(V')^2}{V^2} \qquad \eta = \frac{M_P^2}{8\pi}\frac{V''}{V} \qquad \xi = \frac{M_P^4}{64\pi^2}\frac{V'V'''}{V^2}$$

so we expect $n' \propto (n-1)^2 < |n-1|$!!!

The simplest picture seems to fit the data very well, but is it possible to go beyond ???

Density fluctuations on all scales



Part I:
gravitino
Dark Matter

WHY SUPERGRAVITY ?

- Theoretically attractive: supersymmetry gives gauge unification, solves hierarchy problem,etc...

- Provides a coherent framework to study different signal in high energy physics, astrophysics and cosmology.

- It is surely necessary to extend supersymmetry to supergravity to discuss cosmology !

- Allows extension to string theory...:
  the low energy 4D limit of some string theories is a N=1 supergravity of the no-scale type.

What is supergravity ?

- Largest and unique extension of the Poincare` symmetry, includes general coordinate transformations and hence gravity !!!

local SUPERSYMMETRY: boson <-> fermion

Standard Model			
Matter			Forces
e	μ	τ	γ
ν_e	ν_μ	ν_τ	$W^\pm, Z$
u	c	t	g
d	s	b	G

SUSY SM				
SMatter			SForces	
$\tilde{e}$	$\tilde{\mu}$	$\tilde{\tau}$	$\tilde{\gamma}$	$\tilde{\chi}$
$\tilde{\nu}_e$	$\tilde{\nu}_\mu$	$\tilde{\nu}_\tau$	$\tilde{W}^\pm, \tilde{Z}$	
$\tilde{u}$	$\tilde{c}$	$\tilde{t}$	$\tilde{g}$	
$\tilde{d}$	$\tilde{s}$	$\tilde{b}$	$\tilde{G}$	$\psi_{3/2}$

Gravity multiplet

GRAVITINO properties: completely fixed by SUGRA !

Gravitino mass: set by the condition of ”vanishing” cosmological constant

$$m_{3/2} = \langle W e^{K/2} \rangle = \frac{\langle F_X \rangle}{M_P}$$

~~SUSY~~ scale

It is proportional to the SUSY breaking scale and varies depending on the mediation mechanism, e.g. gauge mediation can accomodate very small $\langle F_X \rangle$ giving $m_{3/2} \sim$ keV, while in anomaly mediation we can even have $m_{3/2} \sim$ TeV (but then it is not the LSP...).

Gravitino couplings: determined by masses, especially for a light gravitino since the dominant piece becomes the Goldstino spin 1/2 component: $\psi_\mu \simeq i\sqrt{\frac{2}{3}}\frac{\partial_\mu \psi}{m_{3/2}}$. Then we have:

$$-\frac{1}{4M_P}\bar{\psi}_\mu \sigma^{\nu\rho}\gamma^\mu \lambda^a F^a_{\nu\rho} - \frac{1}{\sqrt{2}M_P}\mathcal{D}_\nu \phi^* \bar{\psi}_\mu \gamma^\nu \gamma^\mu \chi_R - \frac{1}{\sqrt{2}M_P}\mathcal{D}_\nu \phi \bar{\chi}_L \gamma^\mu \gamma^\nu \psi_\mu + h.c.$$

$$\Rightarrow \quad \frac{-m_\lambda}{4\sqrt{6}M_P m_{3/2}}\bar{\psi}\sigma^{\nu\rho}\gamma^\mu \partial_\mu \lambda^a F^a_{\nu\rho} + \frac{i(m^2_\phi - m^2_\chi)}{\sqrt{3}M_P m_{3/2}}\bar{\psi}\chi_R \phi^* + h.c.$$

Couplings proportional to SUSY breaking masses and inversely proportional to $m_{3/2}$.

SUSY breaking mechanism determines which particle is the LSP and the gravitino couplings !

The gravitino gives us direct information on SUSY breaking

THE WIMP MECHANISM

Primordial abundance of stable massive species

[see e.g. Kolb & Turner '90]

The number density of a stable particle X in an expanding Universe is given by the Bolzmann equation

$$\frac{dn_X}{dt} + 3Hn_X = \langle\sigma(X + X \to \text{anything})v\rangle \left(n_{eq}^2 - n_X^2\right)$$

Hubble expansion   Collision integral

The particles stay in thermal equilibrium until the interactions are fast enough, then they freeze-out at $x_f = m_X/T_f$

defined by $n_{eq}\,\langle\sigma_A v\rangle_{x_f} = H(x_f)$ and that gives

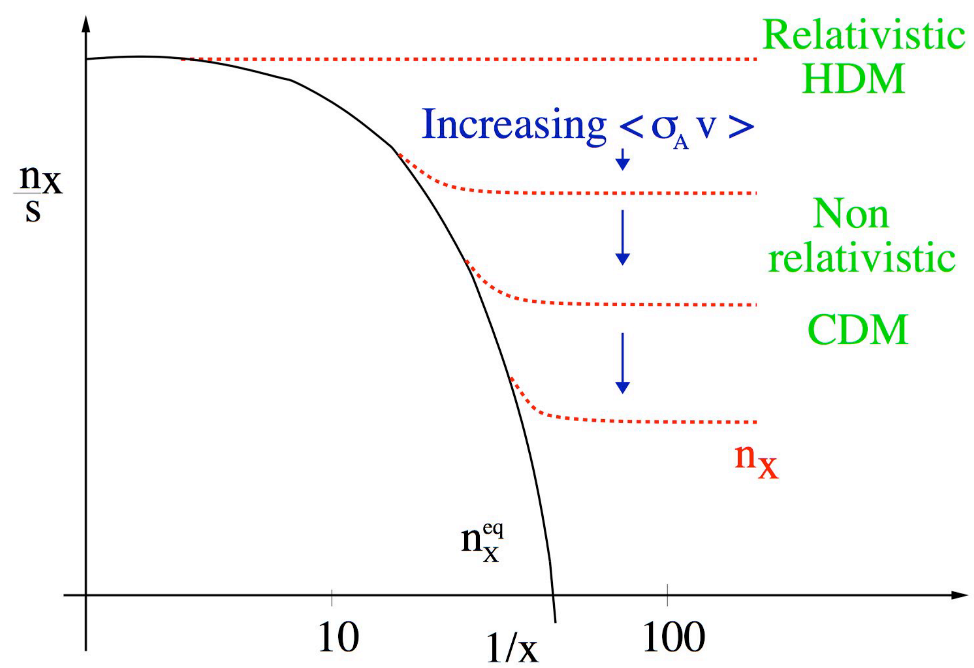
$$\Omega_X = m_X n_X(t_{now}) \quad \propto \quad \frac{1}{\langle\sigma_A v\rangle_{x_f}}$$

Abundance $\Leftrightarrow$ Particle properties

For $m_X \simeq 100$ GeV a WEAK cross-section is needed !

Weakly Interacting Massive Particle

For weaker interactions need lighter masses HOT DM !



CAN THE GRAVITINO BE COLD DARK MATTER ?

YES, if the Universe was never hot enough
for gravitinos to be in thermal equilibrium...

Very weakly interacting particles as the gravitino
are produced even in this case, at least by two mechanisms

PLASMA SCATTERINGS

$$\Omega_{3/2}h^2 \propto \frac{m_{1/2}^2}{m_{3/2}} T_R$$

NLSP DECAY OUT OF EQUILIBRIUM

Ω ^{2} $\propto$ h^2

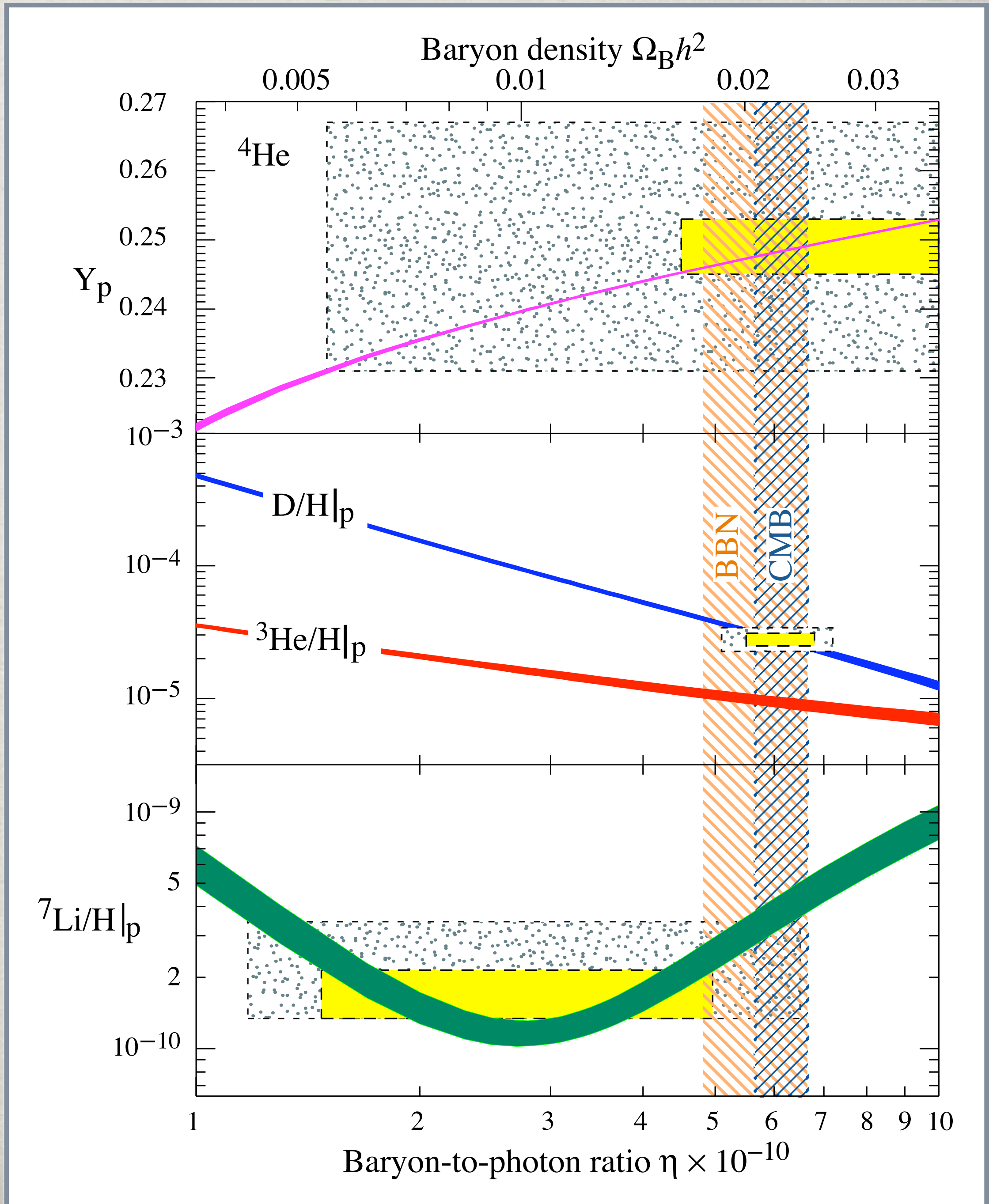
!

DANGER !!!
BBN at risk !

Big Bang Nucleosynthesis

[Fields & Sarkar PDG 07]

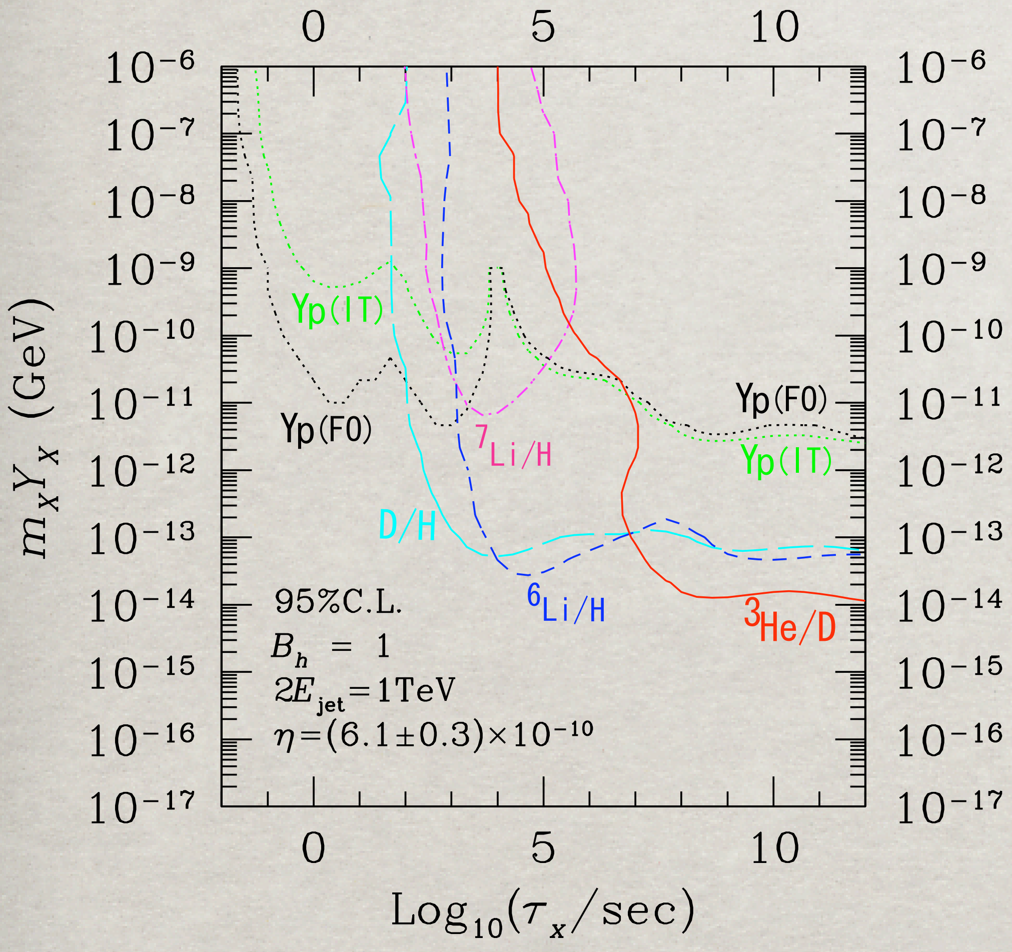
- Light elements abundances obtained as a function of a single parameter $\Omega_B h^2$
- Perfect agreement with WMAP determination
- Some trouble with Lithium 6/7



BBN bounds on NLSP decay

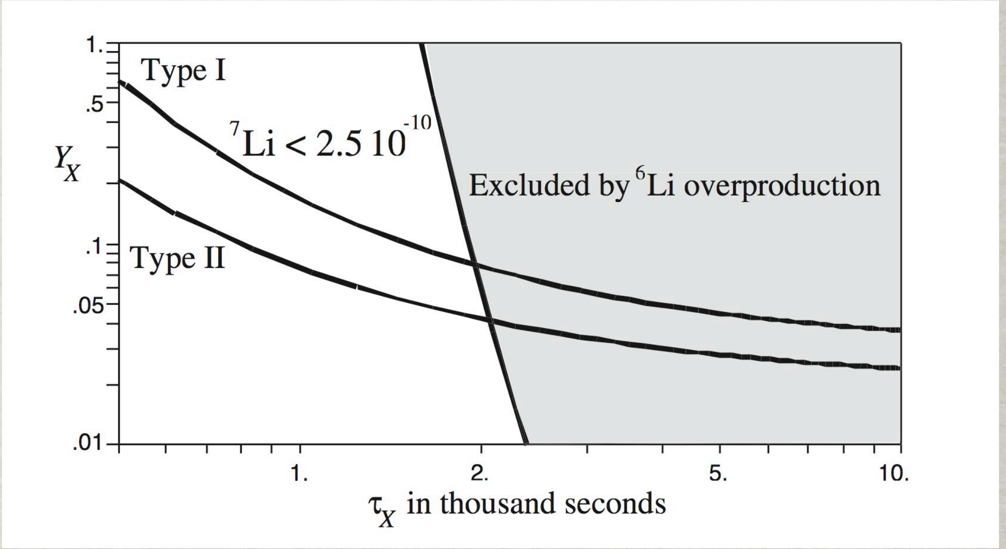
Neutral relics

[...,Kohri, Kawasaki & Moroi 04]



Charged relics

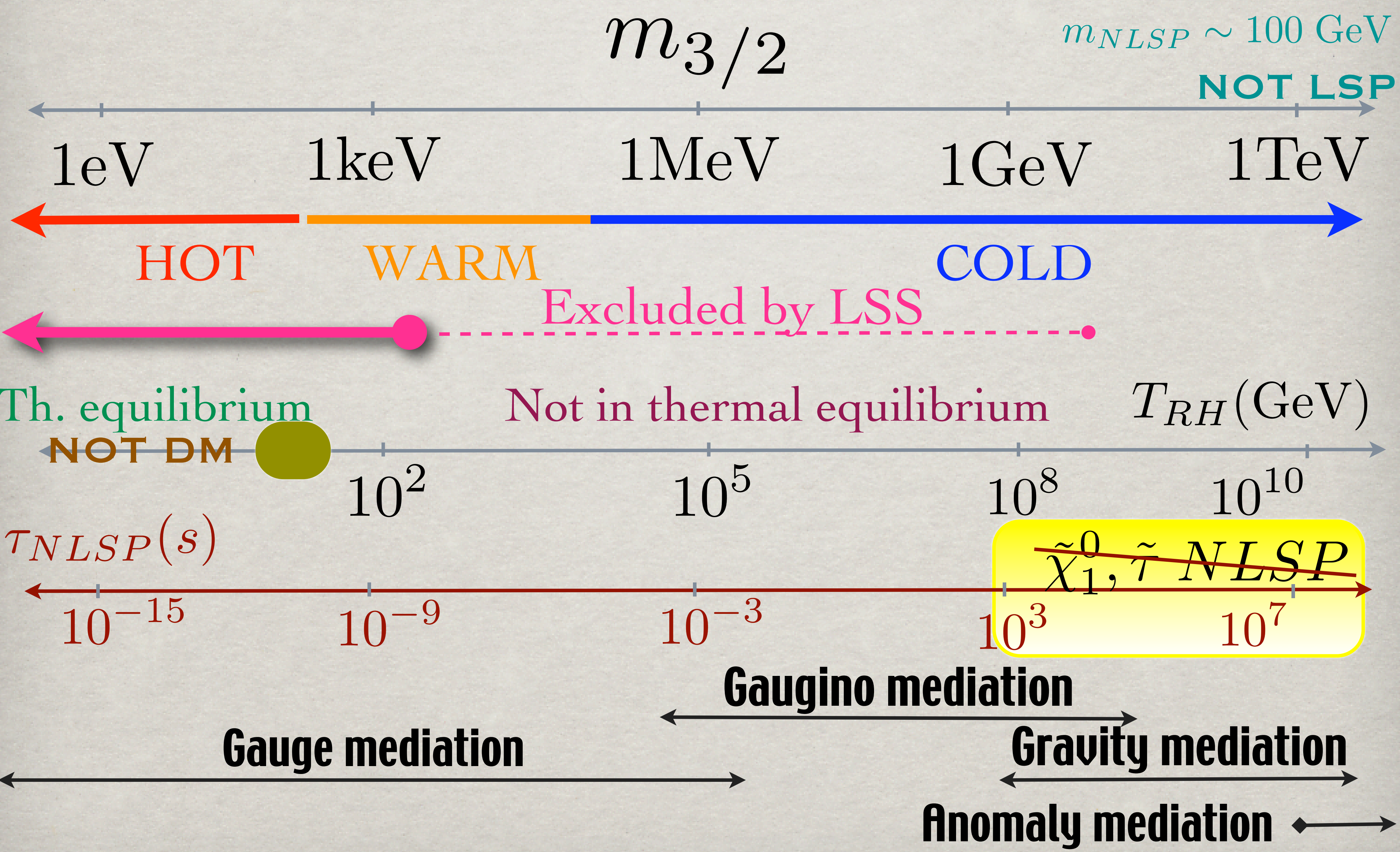
[Pospelov 05, Kohri & Takayama 06, Cyburt at al 06, Jedamzik 07,...]



Need short lifetime & low abundance for NLSP

Big problem for gravitino LSP with 10-100 GeV mass...

Gravitino DM summary



HOW TO EVADE BOUNDS

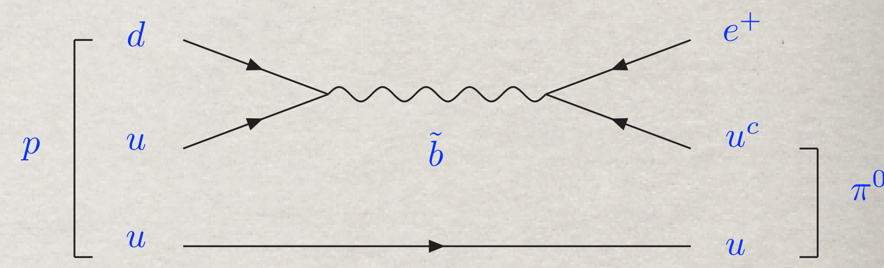
- Make the lifetime shorter:
  heavy(er) NLSP or light(er) gravitino LSP

  $$\tau_{NLSP} \sim 10^5 s \left(\frac{m_{NLSP}}{200GeV}\right)^{-5} \left(\frac{m_{3/2}}{10GeV}\right)^2$$

  violate R-parity
- Choose a harmless NLSP/reduce its density:
  sneutrino LH or RH (weaker bounds...) [LC, S. Kraml 07]
  stop (low abundance and annihilation at QCD transition)
  [Diaz-Cruz, Ellis, Olive & Santoso 07]
  [Berger, LC, Kraml & Palorini 08]
- dilute the NLSP abundance with entropy production
  [Buchmuller et al 05, Hamaguchi et al 07...]

R-parity or not R-parity ?

R-parity is imposed by hand in the MSSM in order to avoid fast proton decay due to renormalizable couplings explicitly violating B and L:



$$W = \lambda L L E^c + \lambda' L Q D^c + \lambda'' U^c D^c D^c + \mu_i L_i H_2$$

$\Rightarrow$ Dimension 4 proton decay operators $\propto \frac{\lambda' \lambda''}{m_{\tilde{q}}^2}$

R-parity = $(-1)^{3B+L+2s}$ forbids these terms $\Rightarrow$ No dimension 4 proton decay (and LSP is stable)!

Proton decay can be avoided also if only B violating couplings λ'' are forbidden. So do we really need R-parity to have gravitino DM ? NO: the decay rate of the gravitino is doubly suppressed by M_P and the R-parity breaking couplings:

$$\tau_{3/2} \simeq 10^{26} s \left(\frac{\lambda^{(')}}{10^{-7}} \right)^2 \left(\frac{m_{3/2}}{10\text{GeV}} \right)^{-3}$$

It is sufficient to have $\lambda, \lambda' < 10^{-7}$ for the gravitinos to live long enough. Such small value also gives sufficient suppression to L violating wash out processes and allows for leptogenesis. On the other hand, requiring the NLSP to decay before BBN just gives $\lambda, \lambda' > 10^{-14}$.

ANY NLSP is allowed if R-parity is broken and still we can have supersymmetric DM !

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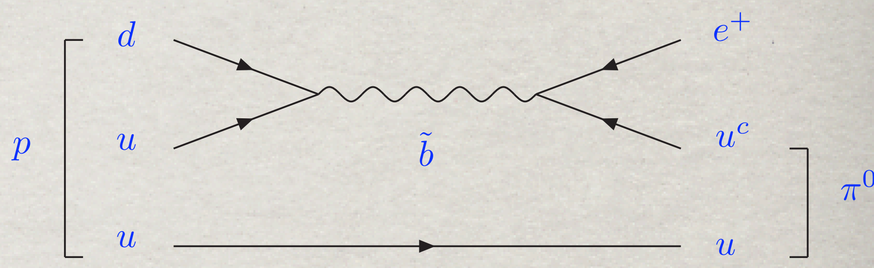

$$W = \lambda LLE^c + \lambda' LQD^c + \lambda'' U^c D^c D^c + \mu_i L_i H_2$$

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$$\tau_{3/2} \simeq 10^{26} s \left(\frac{\lambda^{(\prime)}}{10^{-7}}\right)^2 \left(\frac{m_{3/2}}{10\text{GeV}}\right)^{-3} \gg H_0^{-1} \sim 10^{17}\text{s}$$

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➡ GRAVITINO CDM WITH R-parity VIOLATION

A SIMPLE MODEL with (suppressed) BROKEN R-PARITY

[Buchmüller, LC, Hamaguchi, Ibarra & Yanagida 07]

R-parity is usually not a fundamental symmetry of the MSSM completion. Our idea is to tie the R-parity breaking to the $B-L$ breaking: the v.e.v. of a single field Φ generates both the Majorana mass for RH neutrinos and bilinear R-parity breaking $\mu_i L_i H_u$:

$$W_{B-L} = X(NN^c - \Phi^2) + \frac{NNN_i^c N_j^c}{M_P} \quad \Rightarrow \quad \langle N \rangle = \langle N^c \rangle = \langle \Phi \rangle = v_{B-L}$$

$$\delta K_1 = \left[\frac{(a_i Z + a_i' Z^\dagger)\Phi^\dagger N^c}{M_P^3} + \frac{(c_i Z + c_i' Z^\dagger)\Phi N^\dagger}{M_P^3} \right] H_u L_i \quad \Rightarrow \quad \delta W_1 = \mu_i H_u L_i$$

Then we have

$$M_3 = \frac{v_{B-L}^2}{M_P} \qquad \mu_i \propto m_{3/2} \frac{v_{B-L}^2}{M_P^2}$$

The charge of Φ is such that the other R-parity breaking terms are generated only with higher powers of $\left(\frac{v_{B-L}}{M_P}\right)^{4+n}$ and are harmless.

	$\mathbf{16}_i$	H_u	H_d	N	N^c	Φ	X	Z
R	1	0	0	0	-2	-1	4	0

Effectively a model with bilinear R-parity violation, but with a coupling smaller than those usually discussed in the literature...

$$\epsilon_i = \frac{\mu_i}{\mu} \leq 10^{-7}$$

Gravitino LSP decay

[Takayama & Yamaguchi 00, Buchmuller et al 07]

If R-parity is broken, the gravitino can decay into photon and neutrino via neutralino-neutrino mixing or via a one-loop diagram or into 3 SM fermions via the trilinear couplings.

$$\tilde{G} \to \gamma\nu, Z\nu, W^{\pm}\ell^{\mp} \qquad \tilde{G} \to \ell_L \bar{\ell}_L e_R \quad \tilde{G} \to \ell_L \bar{q}_L d_R$$

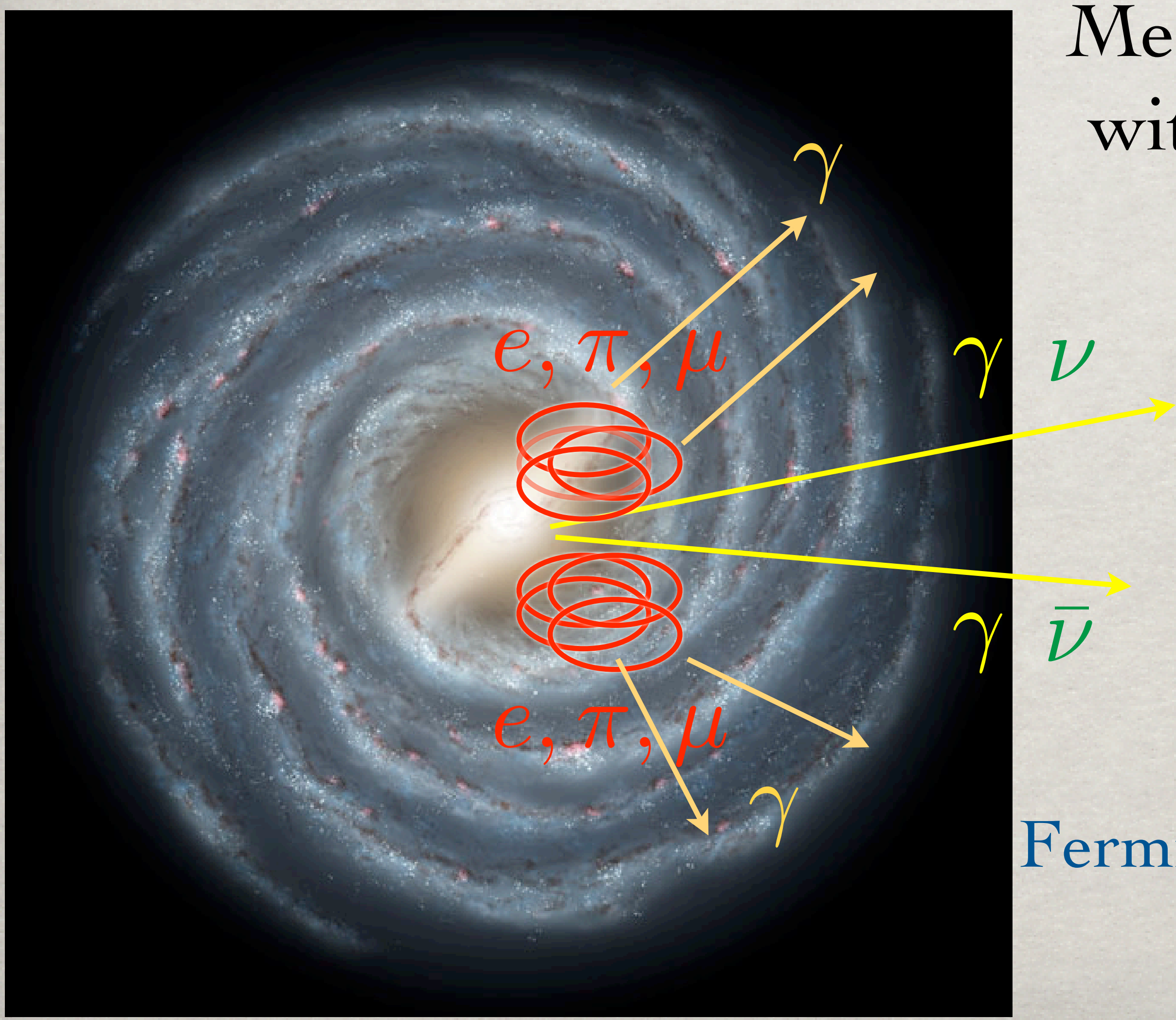
For bilinear R-parity breaking the 2-body channel dominates:

$$\tau_{\tilde{G}} = 4 \times 10^{27} s \left(\frac{U_{\tilde{\gamma}\nu}}{10^{-8}} \right)^2 \left(\frac{m_{\tilde{G}}}{10\mathrm{GeV}} \right)^{-3}$$

[Lola, Osland & Raklev 07] computed also the 2-body one-loop decay and found it also important for most parameter space. For heavy gravitino the decays prefers to go into EW gauge boson final states. [Ibarra & Tran 07]

THE HOPE: DETECT DM !

- Look for decay signal from the Milky Way, other galaxies, clumps of DM, etc...



Measure the decay products with balloons or satellites !

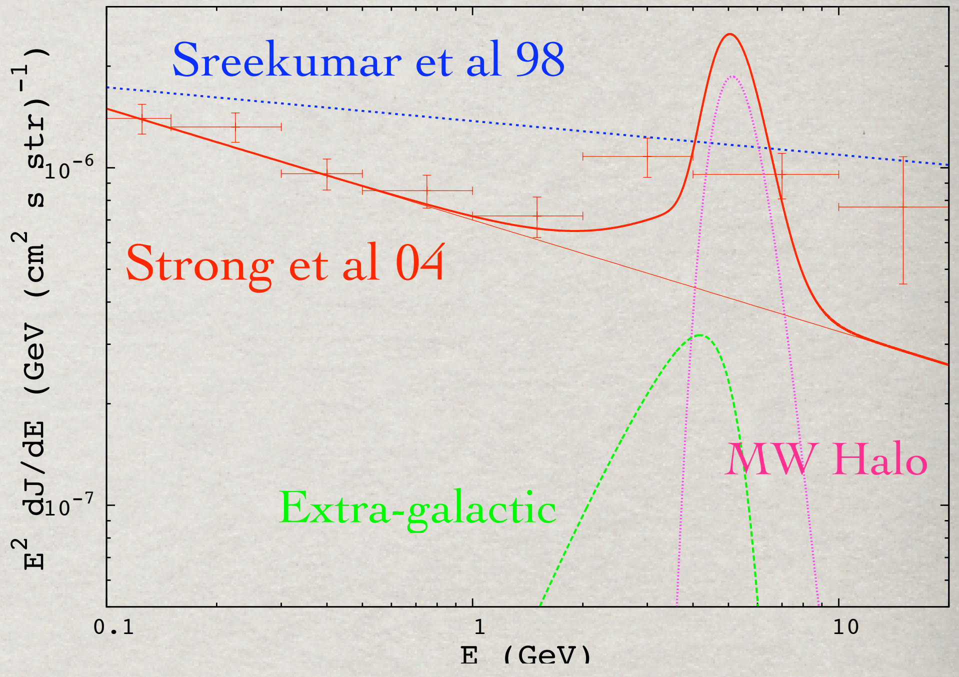
Fermi Gamma-Ray Space Telescope

PAMELA

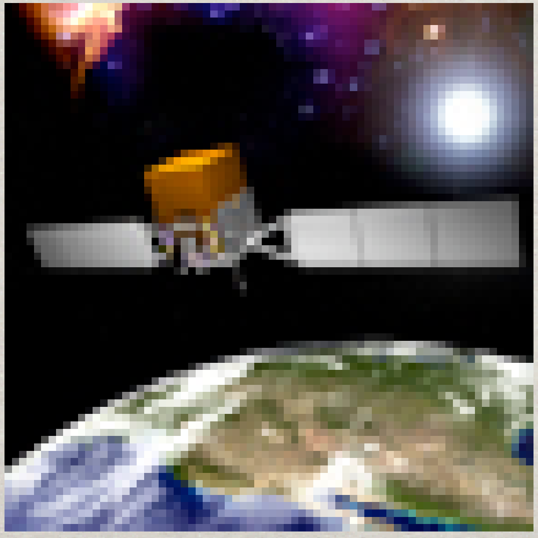
How to see the gravitino

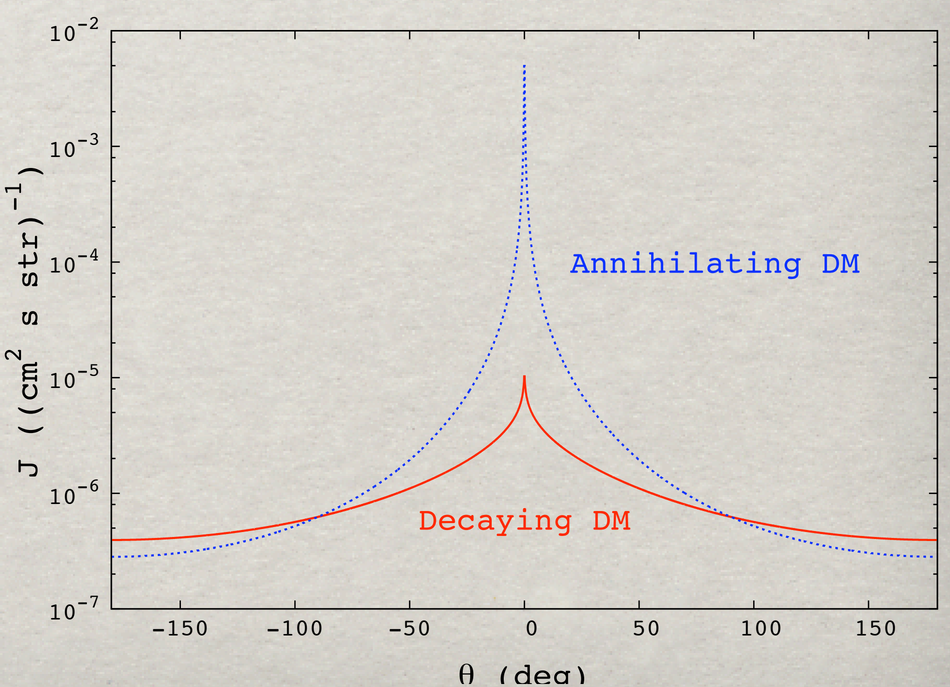
For bilinear R-parity breaking, the gravitino decays into photon and neutrino with flux:

$$J \sim 10^{-7}(\mathrm{cm}^2\mathrm{s\ str})^{-1}\left(\frac{\tau_{DM}}{10^{27}s}\right)^{-1}\left(\frac{m_{DM}}{10\mathrm{GeV}}\right)^{-1}$$



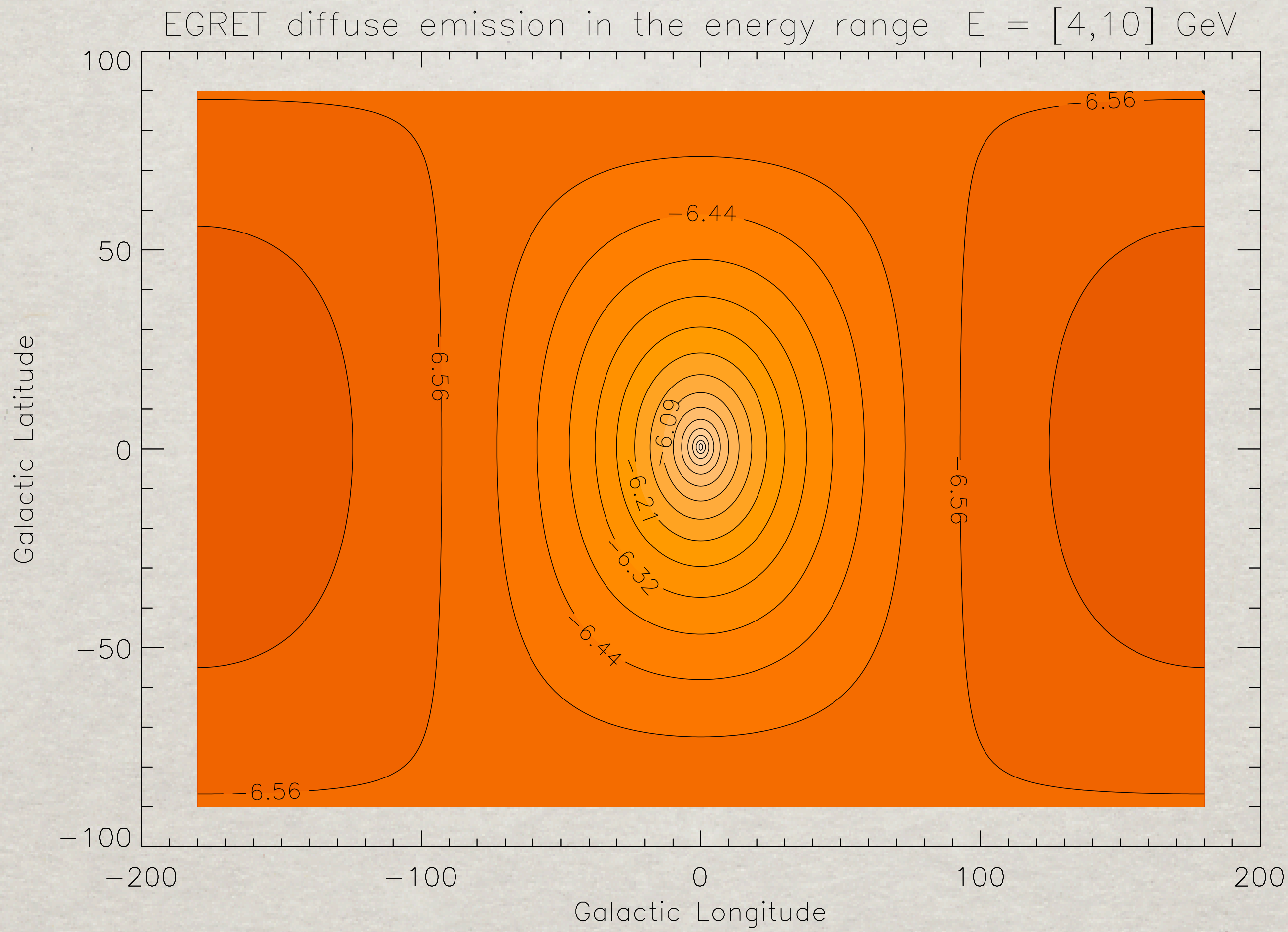
Look at the photons with FGST !





Bertone, Buchmuller, LC, Ibarra '07

The Milky Way signal in gamma-ray

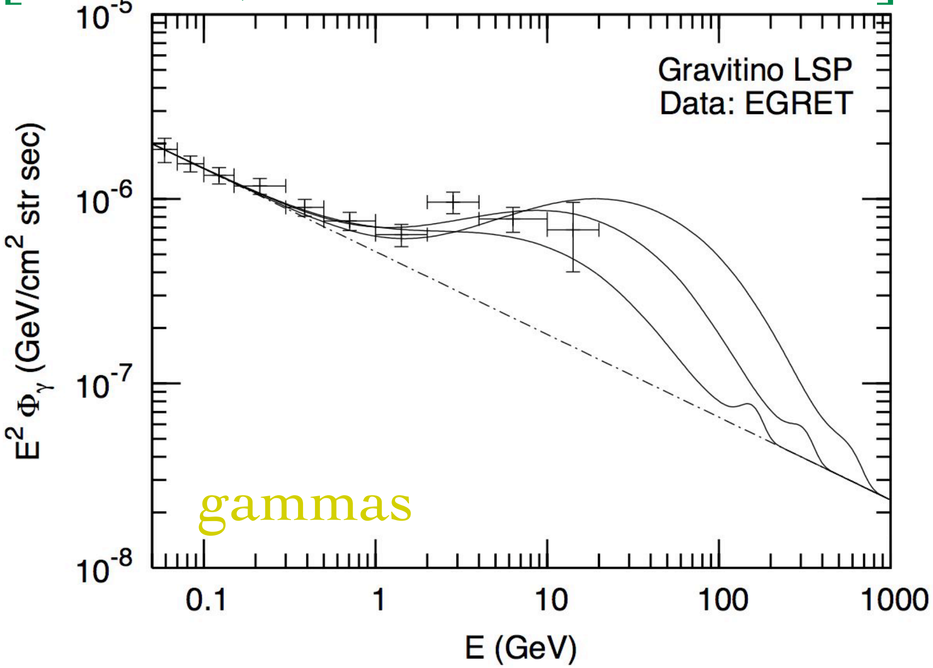


[Bertone, Buchmuller, LC & Ibarra 07]

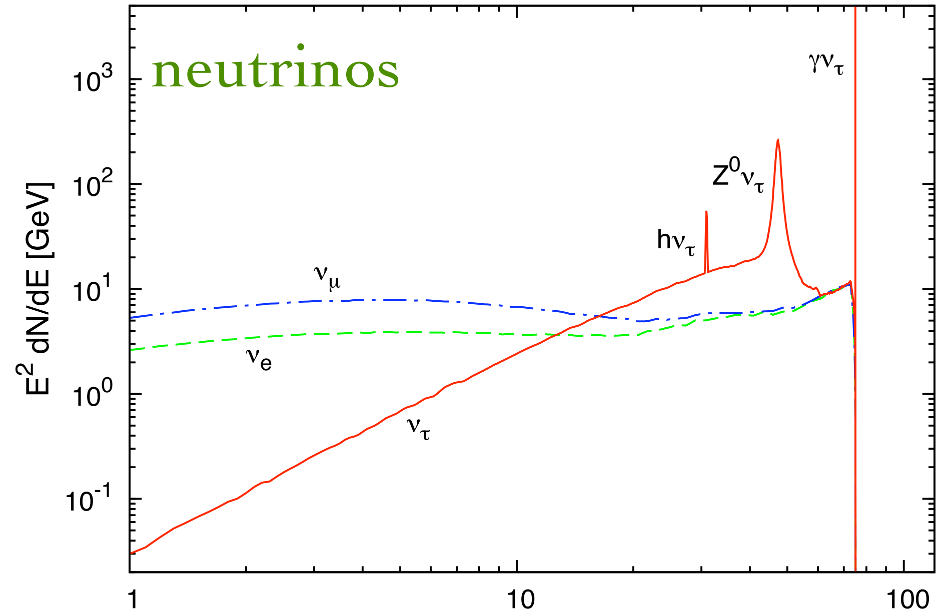
Hopefully the FERMI telescope will be able to see it !

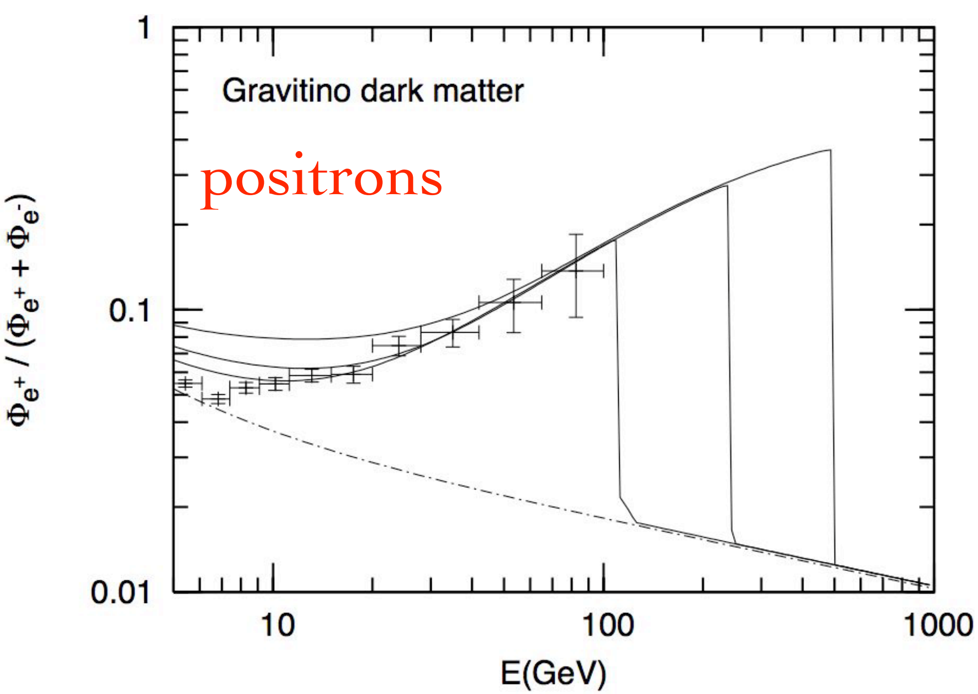
Gravitino DM without R_p

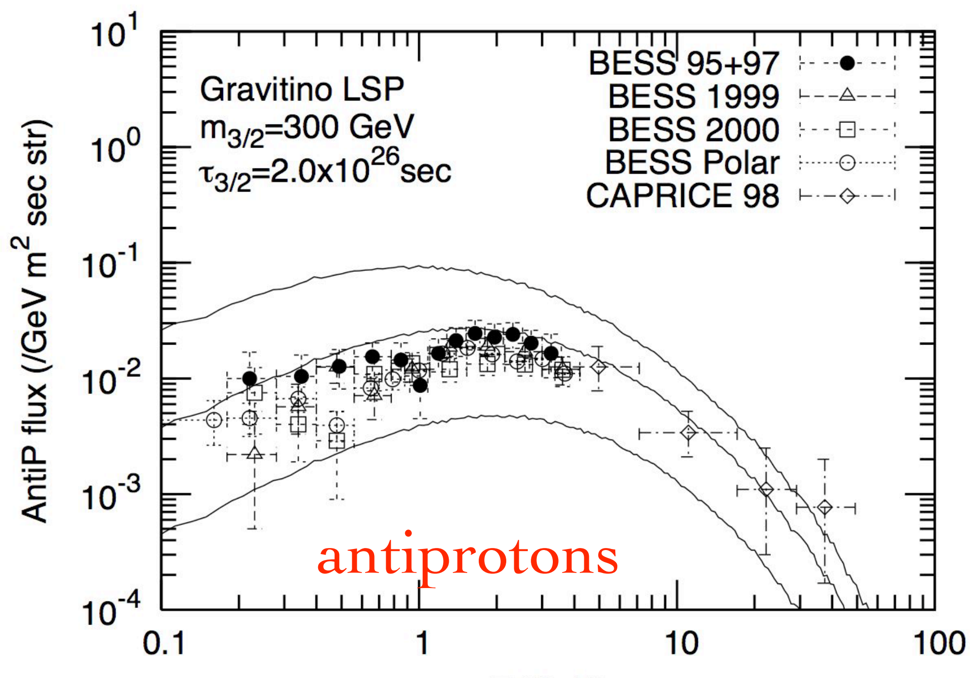
[Ishiwata, Matsumoto & Moroi 09]



[LC, Grefe, Ibarra & Tran 08]



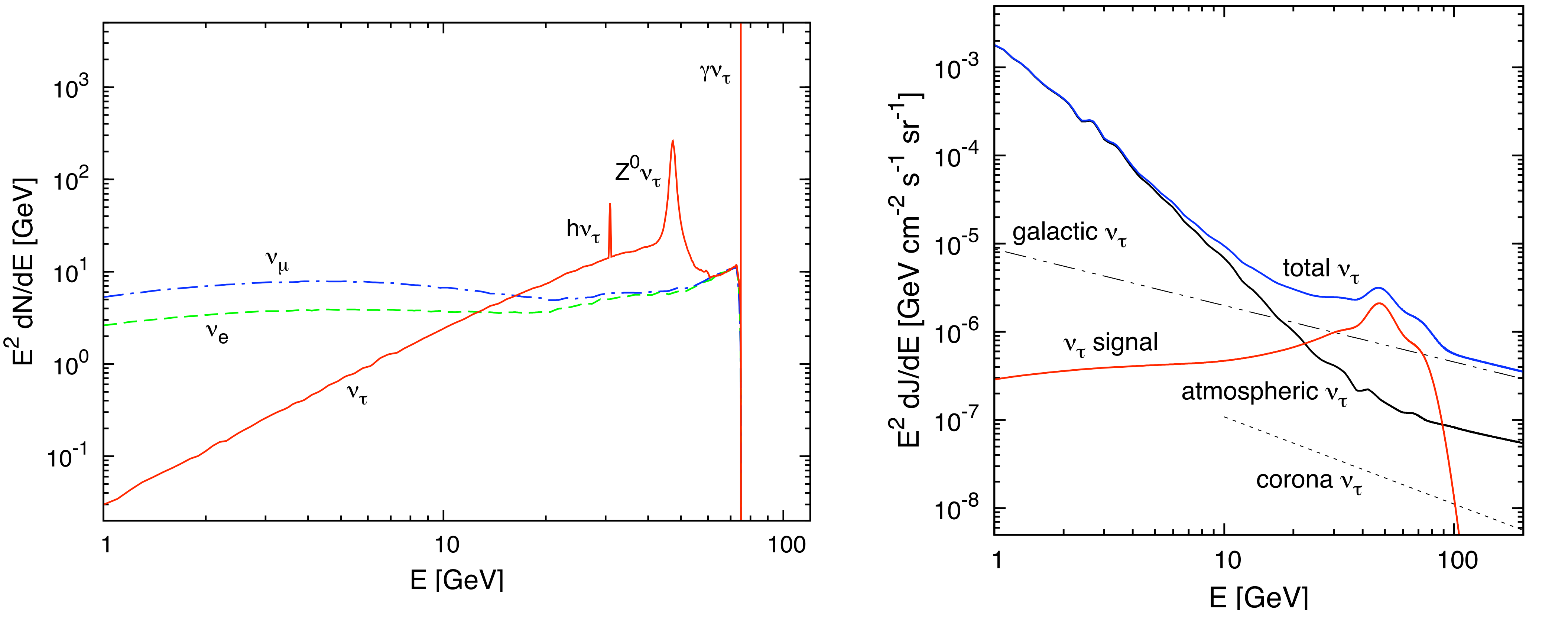




Neutrino Signal...

Neutrino detector's resolution not sufficient to see the peaks

[LC, Grefe, Ibarra & Tran 08]

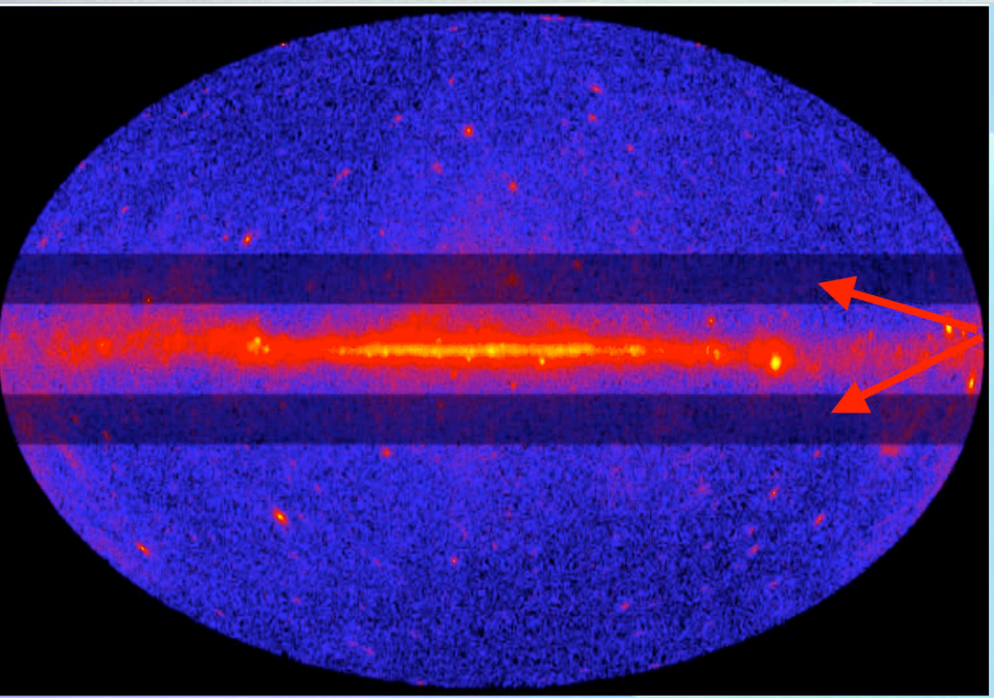


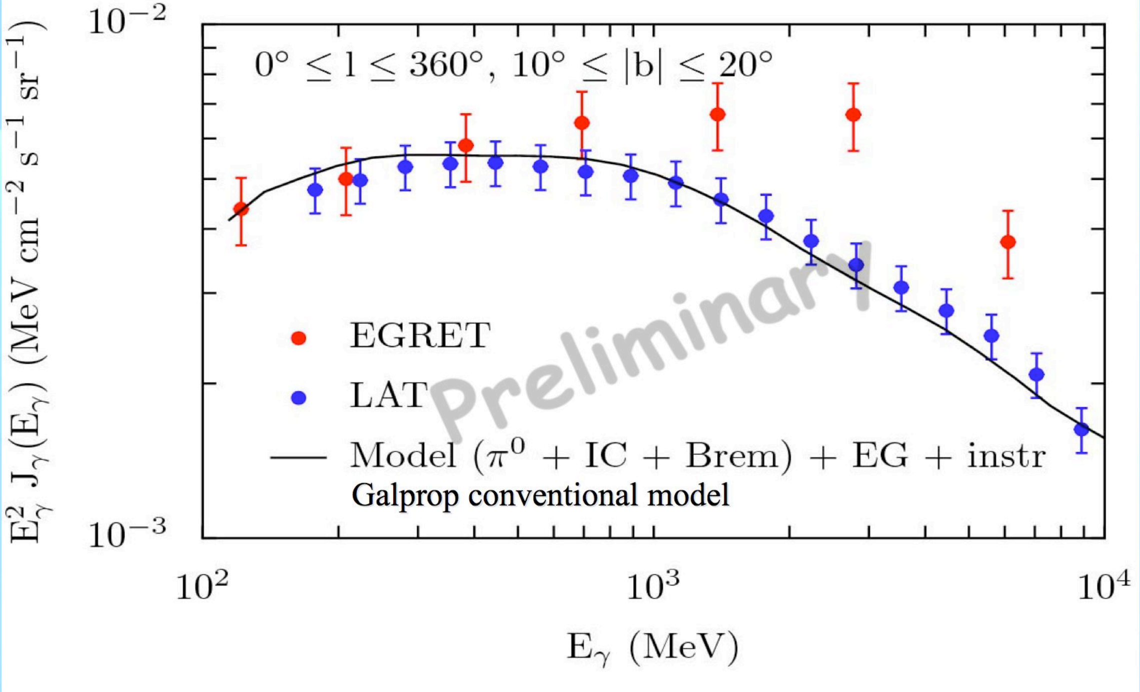
Best signal to background ratio for a tau neutrino looking up...

News from the sky

The FERMI satellite has preliminary results on the gamma-ray emissions around the galactic centre in the strip |b|=10-20

[from a talk by A. Morselli at GGI 09]

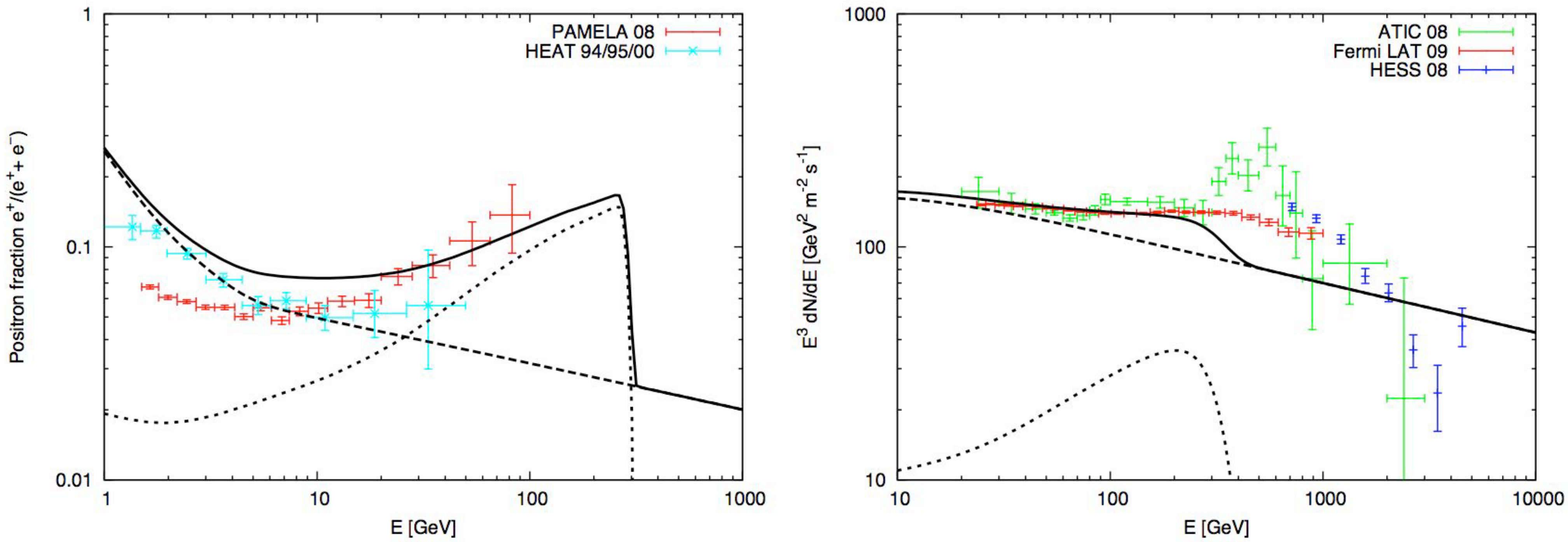




The spectrum seems perfectly consistent with "non-optimized" model of the background, no need of any DM signal there...

Decaying Gravitino vs ID

[Buchmueller, Ibarra, Shindou, Takayama & Tran 09]



Difficult to explain both spectra purely by gravitino decay with bilinear R-parity violation (also without overproducing antiprotons) for reasonable gravitino masses < 600 GeV...

Still gravitino could be part of the signal

Different signals @ LHC depending on the NLSP...

- NLSP decaying within the detector... Need

$$\tau_{NLSP} \leq 10^{-7}\ \mathrm{s} \quad \Rightarrow \quad m_{3/2} \leq 10\ \mathrm{keV}$$

or R-parity breaking at the level larger than 10^{-7}

- Charged meta-stable NLSP: $\tilde{\tau}_R$

- Colored meta-stable NLSP: $\tilde{t}_R$

- Neutral meta-stable NLSP: χ_1^0 vs $\tilde{\nu}_L$

(N)LSP decay at colliders

Same signals as in classical gauge mediation/R-parity breaking scenarios, the main decay channels for neutralino or stau are

R-parity conserved

$$\chi^0 \to \psi_{3/2}\, \gamma$$
$$\tilde{\tau} \to \psi_{3/2}\, \tau$$

R-parity violated

$$\chi^0 \to \tau W, \nu Z, b\bar{b}\nu$$
$$\tilde{\tau} \to \tau\nu_\mu, \mu\nu_\tau, \bar{b}bW$$

but with longer lifetimes than expected if gravitino is DM...

$$m_{3/2} > 4 \text{ keV}$$

$$\tau_{NLSP} > 10^{-13} \text{ s} \left(\frac{m_{NLSP}}{2\text{TeV}}\right)^{-5}$$

$$\tau_{3/2} > 10^{27} \text{ s}$$

$$\tau_{NLSP} > 10^{-9} \text{ s}$$

DISPLACED VERTICES... perhaps even too much !

Part II:
de Sitter in no-scale SUGRA

(quasi)de Sitter in SUGRA

- A de Sitter or quasi-de Sitter phase is needed to account for the present cosmological constant and for inflation

- But in SUGRA the absolute minima are either anti-de Sitter or Minkowski... and do not break SUSY !

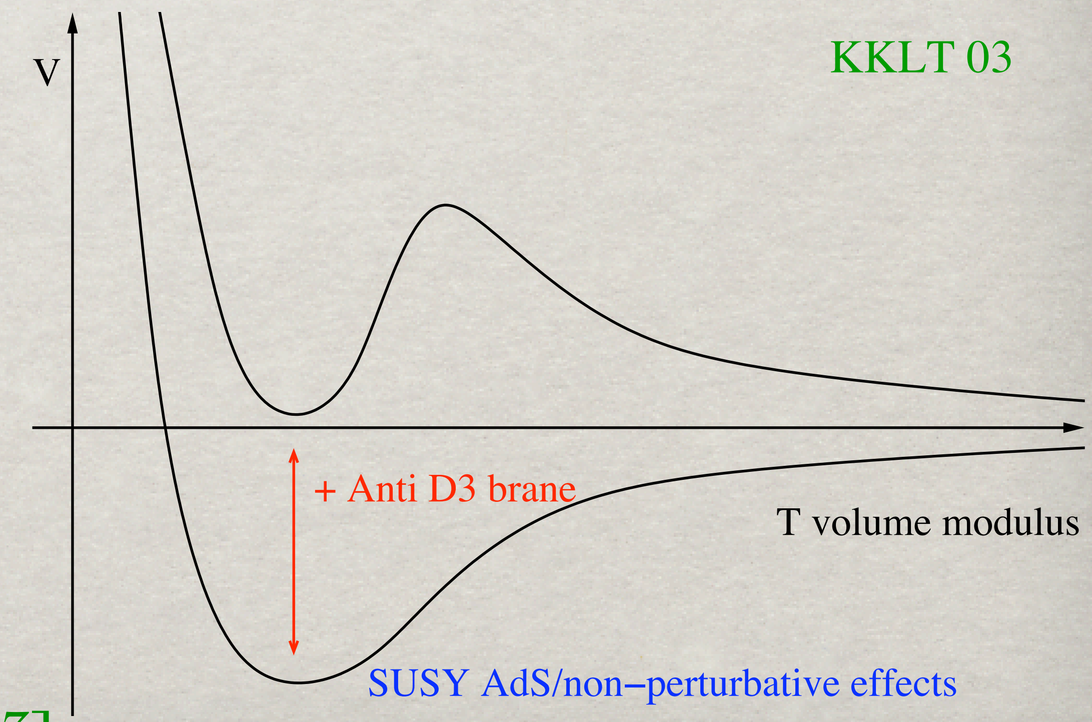
$$V = e^K (K^{i\bar{j}} (W_i + K_i W)(\bar{W}_{\bar{j}} + K_{\bar{j}} \bar{W}) - 3|W|^2$$

- Also inflation is difficult → η problem
the SUGRA potential is usually steep with $V'' \sim V$
as long as one does not resort to some tuning...
... SLOW ROLL inflation not easy to realise !

[Copeland et al 94; Guth, Randall & Thomas 94, ...]

de Sitter vacua and moduli stabilisation

- One of the historical problems of string theory is to stabilise all the moduli fields.
- Progress in the last years: possible to stabilise most moduli using flux compactifications !
- But in these models one has to rely to explicit SUSY breaking terms to stabilise all the moduli and up-lift the vacuum (e.g. KKLT...)



[Kachru, Kallosh, Linde & Trivedi 03]

SUGRA and scalar fields

Thanks to the Kaehler symmetry the scalar potential can be written very simply in terms of a single function

$$G(\Phi,\bar{\Phi}) = K(\Phi,\bar{\Phi}) + \ln\left[W(\Phi)\right] + \ln\left[\bar{W}(\bar{\Phi})\right]$$

i.e. the potential is $V(\Phi,\bar{\Phi}) = e^{G(\Phi,\bar{\Phi})}(G_i G^i - 3)$

where $G_i = \partial_{\Phi_i} G(\Phi,\bar{\Phi})$ is the derivative w.r.t. fields

and indices are lowered and raised by the metric and its inverse

$$g_{i\bar{j}} = \partial_{\Phi_i}\partial_{\Phi_{\bar{j}}} G(\Phi,\bar{\Phi}) \qquad g^{\bar{j}i} g_{i\bar{k}} = \delta_{\bar{j}\bar{k}}$$

Supersymmetry is broken if $\langle G_i \rangle \neq 0$

and the Goldstino field is given by $\eta = G_i \Psi^i$

Scalar mass matrix

- Project the scalar mass matrix along the Goldstino direction for any V and obtain

$$\lambda = e^{-G} V_{i\bar{j}} G^i G^{\bar{j}} = -\frac{2}{3} e^{-G} V (e^{-G} V + 3) + \sigma$$

where $$\sigma = \frac{2}{3}(g_{i\bar{j}} G^i G^{\bar{j}})^2 - R_{i\bar{j}n\bar{m}} G^i G^{\bar{j}} G^n G^{\bar{m}}$$

- A necessary condition for metastability is that λ is positive, then if $V > 0$ we need $\sigma > 0$

- Note: the curvature tensor depends only on the Kaehler potential, while the Goldstino direction on the whole G, including W

No-Scale Kaehler

[Cremmer, Ferrara, Kounas & Nanoupoulos 83, ...]

- The no-scale property requires $K_i K^i = 3$ so that the cosmological constant is zero at tree level since the potential vanishes if $W_i = 0$

$$V = e^{K(\Phi,\bar{\Phi})} \left[|W_i + K_i W|^2 - 3|W|^2 \right]$$
$$= e^{K(\Phi,\bar{\Phi})} \left[|W_i|^2 + 2Re[K_i W \bar{W}_i] \right]$$

- For a single field the no-scale Kaehler is simply

$$K = -3 \ln[T + \bar{T}]$$

The trouble of no-scale

- The problem is the logarithmic Kaehler potential...

$$K = -3\ln(T+\bar{T}) \qquad G = K + \ln(|W|^2)$$

- For a single modulus in de Sitter one mass is always negative for any superpotential W [Brustein & de Alwis 04]
- In general Minkowski metastable vacua with broken SUSY need the holomorphic sectional curvature for the metric $K_{i\bar{j}}$ to be bounded: $R_{i\bar{j}n\bar{m}}G^iG^{\bar{j}}G^nG^{\bar{m}} < 6$

[Gomez Reino & Scrucca 04]

- This result can be generalised to de Sitter into:

$$\sigma = \frac{2}{3}(g_{i\bar{j}}G^iG^{\bar{j}})^2 - R_{i\bar{j}n\bar{m}}G^iG^{\bar{j}}G^nG^{\bar{m}} > 0$$

- $\sigma = 0$ for $G_i \propto K_i$: NO GO for a single field !

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca I 08]

Two moduli in strings

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca I 08]

Heterotic Calabi-Yau

$$K = -\log(\mathcal{V})$$

$$\mathcal{V} = \frac{4}{3} d_{ijk} v^i v^j v^k$$

$$\Re(T^i) = v^i$$

Type II b orientifolds

$$K = -2\log(\mathcal{V})$$

$$\mathcal{V} = \frac{1}{48} d^{ijk} v_i v_j v_k$$

$$\Re(T^i) = \frac{1}{16} d^{ijk} v_j v_k$$

Then we have simply

$$\sigma \sim -\frac{3}{8} e^{4K} \frac{\Delta}{detg} |C|^4$$

$$\sigma \sim \frac{3}{8} e^{4K} \Delta \, detg \, |C|^4$$

Where Δ is the discriminant of the cubic polynomial

Explicit Model(s)

[LC, Gomez-Reino, Gross, Palma, Scrucca 09]

Expand the superpotential around the minimum as

$$W = W_0 + W_i(T_i - T_i^0) + W_{ij}(T_i - T_i^0)(T_j - T_j^0)$$
$$+W_{ijk}(T_i - T_i^0)(T_j - T_j^0)(T_k - T_k^0) + \dots$$

heterotic: $\Delta < 0$

T_0^1	0.405666
T_0^2	0.749277

W_0	1.00000
W_1	1.64415
W_2	2.60392
W_{11}	−17.4400
W_{22}	3.82418
W_{111}	616.732
W_{222}	2.31275

orientifold: $\Delta > 0$

T_0^1	0.412741
T_0^2	0.714888

W_0	1.000000
W_1	2.021311
W_2	0.931223
W_{11}	0.999657
W_{22}	−0.797685
W_{111}	−0.827204
W_{222}	3.308820

Stringy Model(s)

[LC, Gomez-Reino, Gross, Palma, Scrucca 09]

Match to a string-inspired superpotential like

$$W = \Lambda + A_1 e^{a_1 T_1} + B_1 e^{b_1 T_1} + A_2 e^{a_2 T_2} + B_2 e^{b_2 T_2}$$

heteroticu: $\Delta < 0$

Λ	-5.97604×10^{-1}
A_1	-3.62358×10^{5}
B_1	-1.46692×10^{0}
A_2	7.98841×10^{-1}
B_2	7.49672×10^{-1}

a_1	4.36876×10^{1}
b_1	2.66924×10^{0}
a_2	-1.28225×10^{0}
b_2	5.33848×10^{0}

orientifold: $\Delta > 0$

Λ	2.63036×10^{1}
A_1	7.37726×10^{1}
B_1	-9.77287×10^{1}
A_2	-1.50213×10^{0}
B_2	-2.80545×10^{0}

a_1	3.49830×10^{-1}
b_1	2.79764×10^{-1}
a_2	7.30908×10^{0}
b_2	4.19646×10^{-1}

in units of

$m_{3/2}\mathcal{V}_H^{1/2}$ $\quad$ $\mathcal{V}_H^{-1/3}$ $\quad$ $m_{3/2}\mathcal{V}_0$ $\quad$ $\mathcal{V}_0^{-2/3}$

What about inflation ?
A New η problem !

[LC, Gomez Reino, Gross, Luis, Palma & Scrucca II 08]

- In modular inflation eta is constrained:

$$\eta \leq -\frac{2}{3} + \frac{\sigma}{9\gamma\,(1+\gamma)} + \mathcal{O}(\sqrt{\epsilon})$$

where $\gamma = \frac{H_I^2}{m_{3/2}^2}$ for $m_{3/2}^2 = e^G = e^K |W|^2$

- To realise slow roll inflation, i.e. $\epsilon, |\eta| \sim 0$, we need

$$\sigma \geq 6\gamma(1+\gamma)$$

For $\gamma \ll 1$ this reduces to $\sigma > 0$ as for pure de Sitter, while for $\gamma \geq 1$ it is more stringent !

INFLATION at HIGH SCALE is more difficult !

What can we say then ?

- We need more than one field contributing to modular inflation..., possibly one which has a Kaehler potential with zero curvature, e.g.

$$K = -3\ln(T+\bar{T}) + \bar{X}X$$

- We can rely on quantum corrections to modify the curvature and allow de Sitter or inflation, but with some tuning...

- An early inflationary phase, makes present (at least metastable) de Sitter possible...

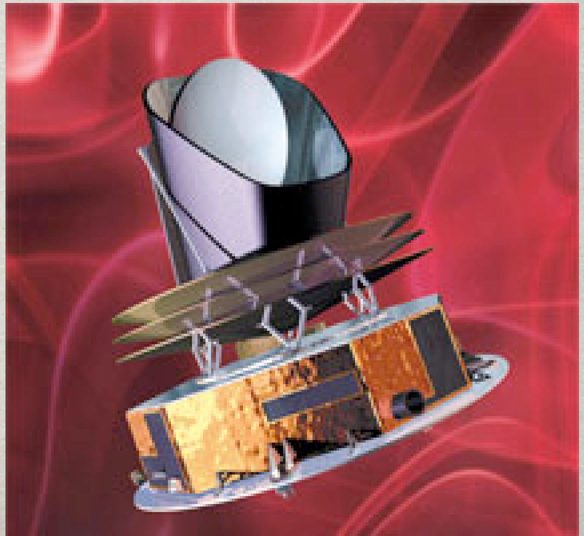
- Explicit model building still ongoing work !

General predictions:

- With more than one modular field needed for inflation, we may expect deviation from the single field predictions, i.e. isocurvature perturbations and non-gaussianities

- Low scale inflation is preferred ! Probably no detectable gravity waves for modular inflation... apart if the gravitino mass was very large during inflation.

The Planck satellite was launched
on the 14th May this year
and will measure the CMB
with better precision !



Outlook

Outlook

The next decade will bring us some answers:

- Cosmic Microwave Background & Large Scale Structure measurements will be able to tell us more about dark matter and inflation...

- Accelerator and DM experiments together with astrophysical observations could soon find out if Dark Matter is made of gravitinos.

- Perhaps we will be able to know soon one key parameter: the gravitino mass !
  This would open up a unique window on Supergravity and improve model building.

At the end,
the good news
for our graduate
students:

We still have 95% of the Universe to discover !