Laserphysik

WS 2017/2018 — 2. Exercise sheet — 24.10.2017

Lecture: Prof. Dr. Alejandro Saenz, Prof. Dr. Oliver Benson

Deliver your solutions no later than 3pm 01.11.2017 (mail box near room 1'701)

Problem 1: Lensemaker's equation

Use the ABCD formalism (see for example book of Svelto) to derive the lensemaker's equation for a biconvex lense:

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right], \tag{1}$$

which describes the focusing properties of this lense. Here f is the focal length, R_1 and R_2 are the radii of curvatures of the biconvex lense, n is the refractive index of the lense (that is placed in air with $n_{air} \approx 1$), and d is the thickness of the lense.

Problem 2: Slowly varying envelope

Derive the slowly varying envelope (SVE) wave equation for waves propagating in z-direction inside a media with absorption or gain:

$$\Delta_t \mathbf{A}(\mathbf{r},t) + 2ink_0\left[-\frac{\alpha}{2} + \frac{\partial}{\partial z} + \frac{1}{c_{ph}}\frac{\partial}{\partial t}\right]\mathbf{A}(\mathbf{r},t) = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 \mathbf{P}_A}{\partial t^2} exp\left(i(\omega_0 t - nk_0 z)\right) \tag{2}$$

Here Δ_t is the transversal Laplace operator $\begin{bmatrix} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} & 0 \\ 0 & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \end{bmatrix}$, α is the absorption coefficient ($\alpha < 0$: absorber, $\alpha > 0$: gain), k_0 is the wavenumber, c_{ph} is the phase velocity inside the medium, $\mathbf{P}_A$ is the polarization of the medium, A is the amplitude of the wave and n is the real part of the complex refractive index $\tilde{n} = n - i\frac{\alpha}{2k_0}$.
To this end start with the usual wave equation

$$\Delta \mathbf{E} - \frac{\tilde{n}^2}{c_0^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0 c_0^2}\frac{\partial^2 \mathbf{P}_A}{\partial t^2} \tag{3}$$

and insert the test-solution

$$\mathbf{E}(\mathbf{r},t) = \mathbf{A}(\mathbf{r},t) exp\left(-i(\omega_0 t - nk_0 z)\right). \tag{4}$$

Then apply the SVE approximations known from the lecture.

Problem 3: Stability of two mirror cavity

Use the ABCD formalism (see for example book of Svelto) to describe a light ray, that bounces back and forth between two mirrors with radii R_1 and R_2 that are separated by the a distance d (as indicated by the figure). To this end the vector $\begin{bmatrix} y_m \\ \Theta_m \end{bmatrix}$ can be written as the m-fold multiplication of a matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ with the start value $\begin{bmatrix} y_0 \\ \Theta_0 \end{bmatrix}$, where M itself is constructed by the multiplication of four matrixes representing the events: reflection on the first mirror, propagation to the second mirror, reflection on the second mirror and propagation back to the first mirror.

a) Show, that the following recurrence relation holds for such a system:

$$y_{m+2} = (A + D)y_{m+1} + (BC - AD)y_m \tag{5}$$

b) The above relation can be analyzed for stable trapping of the ray, i.e., for the formation of a cavity. Show, that the stability criterion

$$\frac{1}{2}|A + D| \leq 1 \tag{6}$$

is equivalent to

$$0 \leq g_1 g_2 \leq 1 \tag{7}$$

(where $g_i = (1 + d/R_i)$) by explicit evaluation of M.

c) Draw a graph with g_1 being the x-axis and g_2 the y-axis. Mark the general regions of stability as well as three specific points: planar, symmetric concentric and symmetric confocal cavity.