Laserphysik

WS 2017/2018 **3. Exercise sheet** **01.11.2017**

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Deliver your solutions no later than 3pm 08.11.2017 (mail box near room 1'701)

Problem 1: Gaussian Beams 1

Consider the complex amplitude of a Gaussian beam:

$$U(\mathbf{r}) = \frac{W_0}{W(z)} \exp\left[-\frac{r^2}{W(z)^2}\right] \exp\left[(-ikz - ik\frac{r^2}{2R}(z) + i\tan^{-1}(\frac{z}{z_0}))\right] = A(\mathbf{r})e^{-ikz}.$$

With the beam waist radius W_0, the Rayleigh length z_0, the beam waist along the optical axis $W(z)$ and the wave front radius $R(z)$ defined as:

$$\begin{aligned} W_0 &= \sqrt{\frac{\lambda z_0}{\pi}} \\ z_0 &= \frac{\pi W_0^2}{\lambda} \\ W(z) &= W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} \\ R(z) &= z\left[1+\left(\frac{z_0}{z}\right)^2\right] \end{aligned} \tag{1}$$

Show that the complex envelope $A(\mathbf{r})$ of this beam satisfies the paraxial Helmholtz equation:

$$(\nabla_T^2 - 2ik\frac{\partial}{\partial z})A(\mathbf{r}) = 0$$

with ∇_T^2 being the transverse part of the Laplacian operator.

Hint 1: You might show first, that $A(\mathbf{r})$ can be expressed as $A(\mathbf{r}) = \frac{q(0)}{q(z)}e^{-ik\frac{r^2}{2q(z)}}$ with $q(z) = z + iz_0$.

Hint 2: Use an appropriate coordinate system.

Problem 2: Gaussian Beams 2

Consider the same Gauss beam as in problem 1.

a) What is the meaning of z_0 for the beam waist $W(z)$?

b) Show that in the far-field, the beam propagates in the form of a cone of half angle $\tan\Theta_0 = \frac{\lambda}{\pi W_0}$

c) Discuss the radius of curvature $R(z)$ of the wavefronts in the near- and far-field. At which length does maximum wave-front curvature occur?

Problem 3: Gaussian Beams 3

The Gaussian beam described above represents the fundamental mode. Higher order modes, so-called Hermite-Gaussian modes, are expressed as:

$$\begin{aligned} E_{l,m}(x,y,z) =& E_0 \frac{W_0}{W(z)} H_l\left(\frac{\sqrt{2}\,x}{W(z)}\right) H_m\left(\frac{\sqrt{2}\,y}{W(z)}\right) \exp\left(-\frac{r^2}{W^2(z)}\right) \exp\left(-i\frac{kr^2}{2R(z)}\right) \\ &\cdot \exp\left(-ikz + i(1+l+m)\tan^{-1}\left(\frac{z}{z_0}\right)\right) \end{aligned} \tag{2}$$

where the modes are indexed $\mathrm{TEM}_{l,m}$. Plot the intensity of the following modes: $\mathrm{TEM}_{0,0}$, $\mathrm{TEM}_{1,0}$, $\mathrm{TEM}_{0,1}$, and $\mathrm{TEM}_{1,1}$. Use a computer.