Bose-Einstein condensate of dilute atomic gases

In a perfect Bose-Einstein condensate (BEC) all ideal bosons occupy the same lowest energy state (in mean field):

$$\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N) \;=\; \prod_{i=1}^{N} \phi_0(\boldsymbol{R}_i)$$

- In contrast to, e. g., superfluid He interactions in dilute atomic gases are (usually) small $\longrightarrow$ practically 100 % BEC: quantum statistics determines behaviour.

Bose-Einstein condensate of dilute atomic gases

In a perfect Bose-Einstein condensate (BEC) all ideal bosons occupy the same lowest energy state (in mean field):

$$\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N) = \prod_{i=1}^{N} \phi_0(\boldsymbol{R}_i)$$

- In contrast to, e. g., superfluid He interactions in dilute atomic gases are (usually) small $\longrightarrow$ practically 100 % BEC: quantum statistics determines behaviour.
- Lowest known temperatures (nanokelvin), atoms are practically at rest: high-precision spectroscopy, quantum-state resolved cross-sections.

Bose-Einstein condensate of dilute atomic gases

In a perfect Bose-Einstein condensate (BEC) all ideal bosons occupy the same lowest energy state (in mean field):

$$\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N) \;=\; \prod_{i=1}^{N} \phi_0(\boldsymbol{R}_i)$$

- In contrast to, e. g., superfluid He interactions in dilute atomic gases are (usually) small $\longrightarrow$ practically 100 % BEC: quantum statistics determines behaviour.
- Lowest known temperatures (nanokelvin), atoms are practically at rest:
  high-precision spectroscopy, quantum-state resolved cross-sections.
- "Superchemistry" (?): chemical reaction between two coherent molecular clouds.

Bose-Einstein condensate of dilute atomic gases

In a perfect Bose-Einstein condensate (BEC) all ideal bosons occupy the same lowest energy state (in mean field):

$$\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N) = \prod_{i=1}^{N} \phi_0(\boldsymbol{R}_i)$$

- In contrast to, e. g., superfluid He interactions in dilute atomic gases are (usually) small $\longrightarrow$ practically 100 % BEC: quantum statistics determines behaviour.
- Lowest known temperatures (nanokelvin), atoms are practically at rest: high-precision spectroscopy, quantum-state resolved cross-sections.
- "Superchemistry" (?): chemical reaction between two coherent molecular clouds.
- Neutral atoms $\longrightarrow$ variable particle number.
- Different atomic species $\longrightarrow$ different interparticle interactions.

Bose-Einstein condensate of dilute atomic gases

In a perfect Bose-Einstein condensate (BEC) all ideal bosons occupy the same lowest energy state (in mean field):

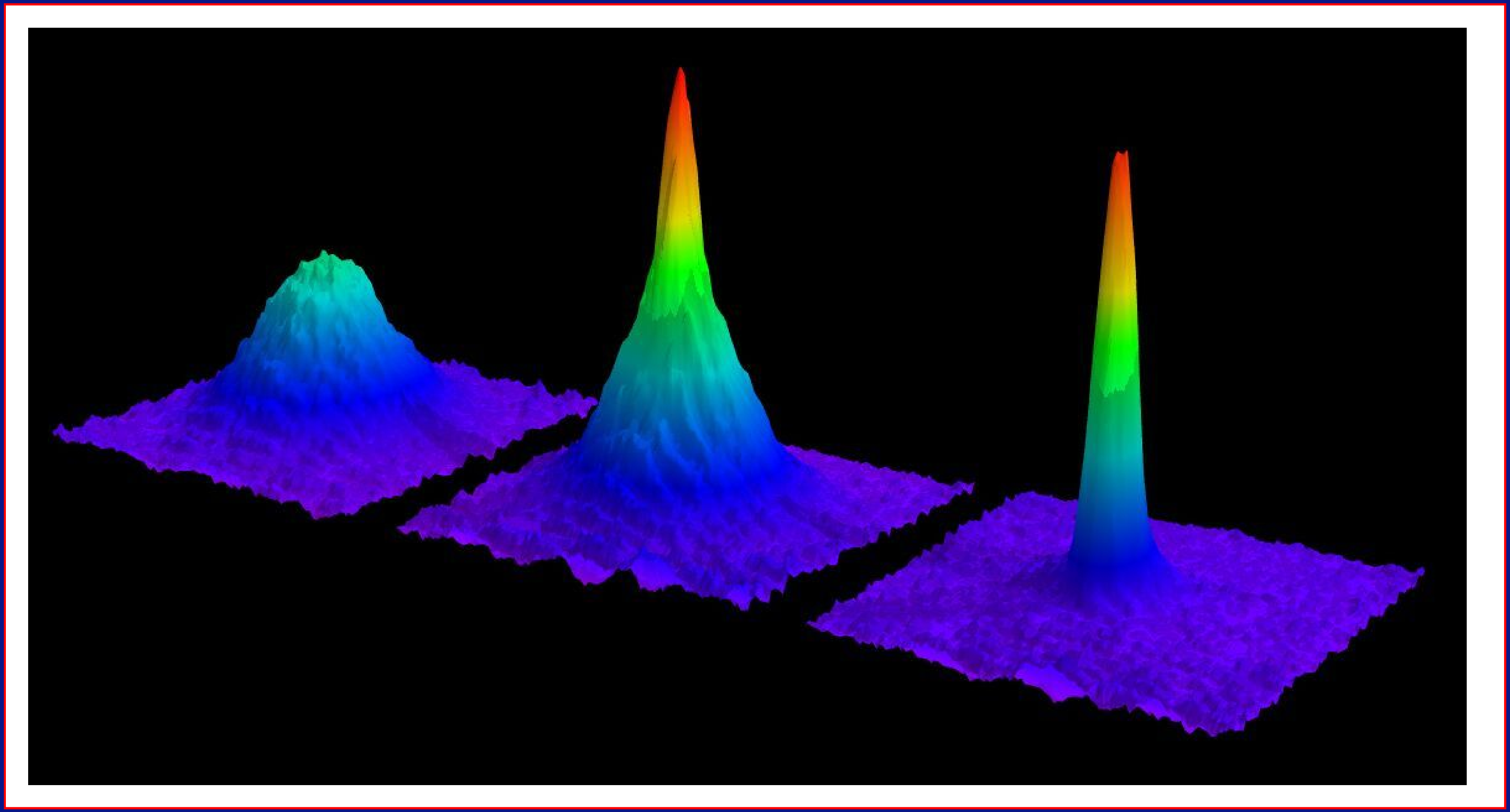
$$\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N) = \prod_{i=1}^{N} \phi_0(\boldsymbol{R}_i)$$

- In contrast to, e. g., superfluid He interactions in dilute atomic gases are (usually) small $\longrightarrow$ practically 100 % BEC: quantum statistics determines behaviour.
- Lowest known temperatures (nanokelvin), atoms are practically at rest:
  high-precision spectroscopy, quantum-state resolved cross-sections.
- "Superchemistry" (?): chemical reaction between two coherent molecular clouds.
- Neutral atoms $\longrightarrow$ variable particle number.
- Different atomic species $\longrightarrow$ different interparticle interactions.

Macroscopic coherent quantum systems

with variable interparticle interactions.

Thermal cloud vs. Bose-Einstein Condensate (BEC)



[from: http://cua.mit.edu/ketterle_group/]

Interactions in ultracold atomic gases

Interactions:

- Ultracold collisions $\longrightarrow$ s-wave scatering:

$$\sigma = 4\,\pi\,a_{l=0}^2 + O(k^2) \approx 4\,\pi\,a_{\rm sc}^2$$

(“hard-spheres”). (Note: p-wave scattering for identical Fermions.)

Interactions in ultracold atomic gases

Interactions:

- Ultracold collisions $\longrightarrow$ s-wave scatering:

$$\sigma = 4\,\pi\,a_{l=0}^2 + O(k^2) \approx 4\,\pi\,a_{\mathrm{sc}}^2$$

("hard-spheres"). (Note: p-wave scattering for identical Fermions.)

- Effective interaction potential:

$$V_{\mathrm{pseudo}}(R) \;=\; \frac{4\pi\,\hbar^2}{\mu\,R^2}\,a_{\mathrm{sc}}\,\delta(R)$$

Extremely simple short-range potential (though strongly dependent on long-range part of true interatomic potential).

Interactions in ultracold atomic gases

Interactions:

- Ultracold collisions $\longrightarrow$ s-wave scatering:

$$\sigma = 4\,\pi\, a_{l=0}^2 + O(k^2) \approx 4\,\pi\, a_{\rm sc}^2$$

(“hard-spheres”). (Note: p-wave scattering for identical Fermions.)

- Effective interaction potential:

$$V_{\rm pseudo}(R) \;=\; \frac{4\pi\,\hbar^2}{\mu\, R^2}\, a_{\rm sc}\, \delta(R)$$

Extremely simple short-range potential (though strongly dependent on long-range part of true interatomic potential).

- Magnetic Feshbach resonances $\longrightarrow$ extreme tunability: $-\infty \le a_{\rm sc} \le +\infty$ (“strong attraction, no interaction, strong repulsion”).

Interactions in ultracold atomic gases

Interactions:

- Ultracold collisions $\longrightarrow$ s-wave scatering:

$$\sigma = 4\,\pi\,a_{l=0}^2 + O(k^2) \approx 4\,\pi\,a_{\rm sc}^2$$

(“hard-spheres”). (Note: p-wave scattering for identical Fermions.)

- Effective interaction potential:

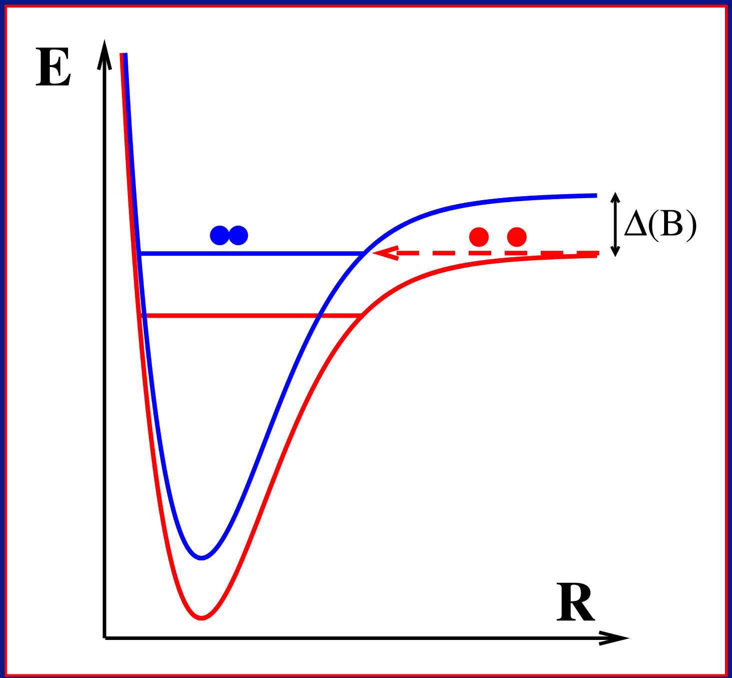
$$V_{\rm pseudo}(R) \;=\; \frac{4\pi\,\hbar^2}{\mu\,R^2}\,a_{\rm sc}\,\delta(R)$$

Extremely simple short-range potential (though strongly dependent on long-range part of true interatomic potential).

- Magnetic Feshbach resonances $\longrightarrow$ extreme tunability: $-\infty \leq a_{\rm sc} \leq +\infty$ (“strong attraction, no interaction, strong repulsion”).

- Dipolar gases (Cr, diatomics, . . .) $\longrightarrow$ non-isotropic interaction (“spins”).

Magnetic Feshbach resonances



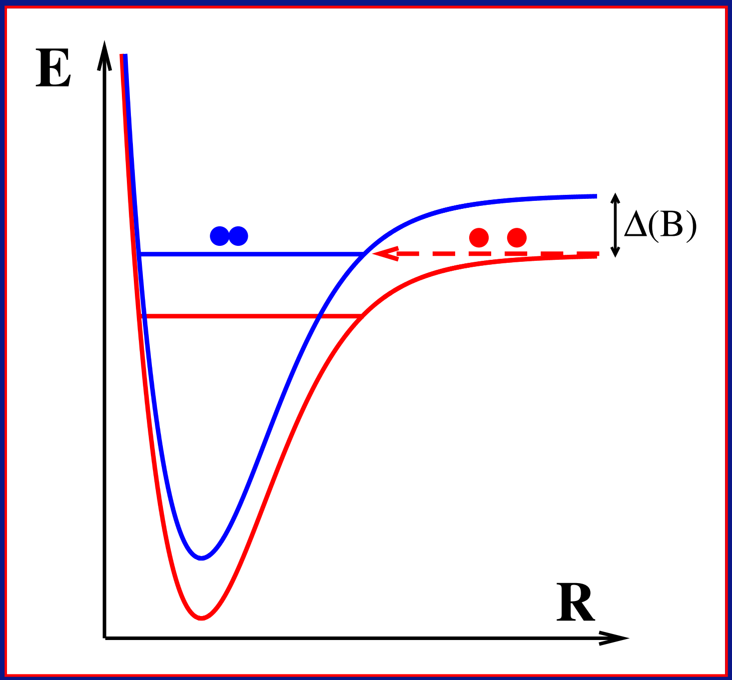
Simple picture:

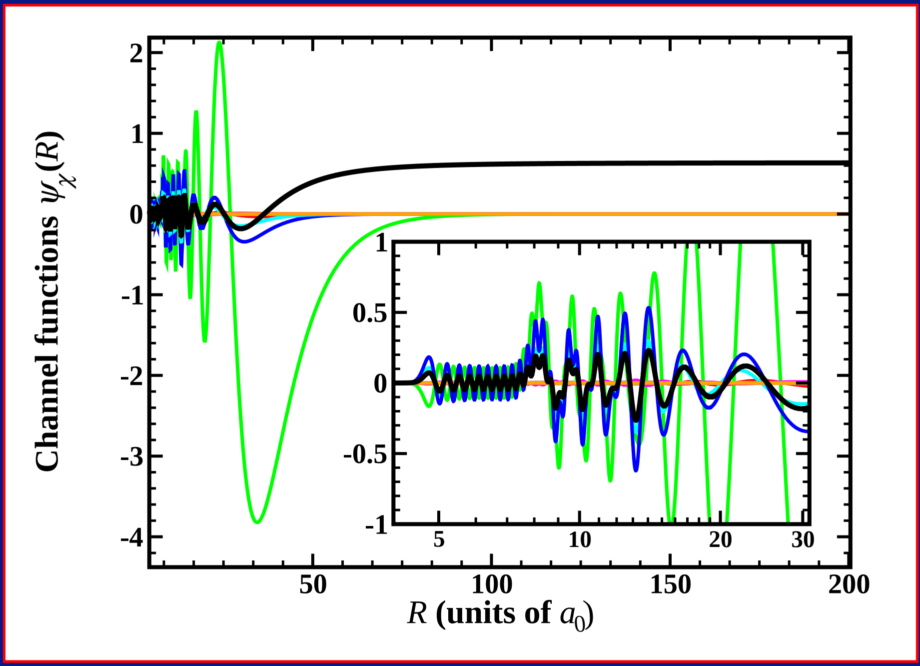
Only **2 channels**:

– open (continuum) channel,

– closed (bound) channel.

Magnetic Feshbach resonances





Simple picture:

Only **2 channels**:

— open (continuum) channel,

— closed (bound) channel.

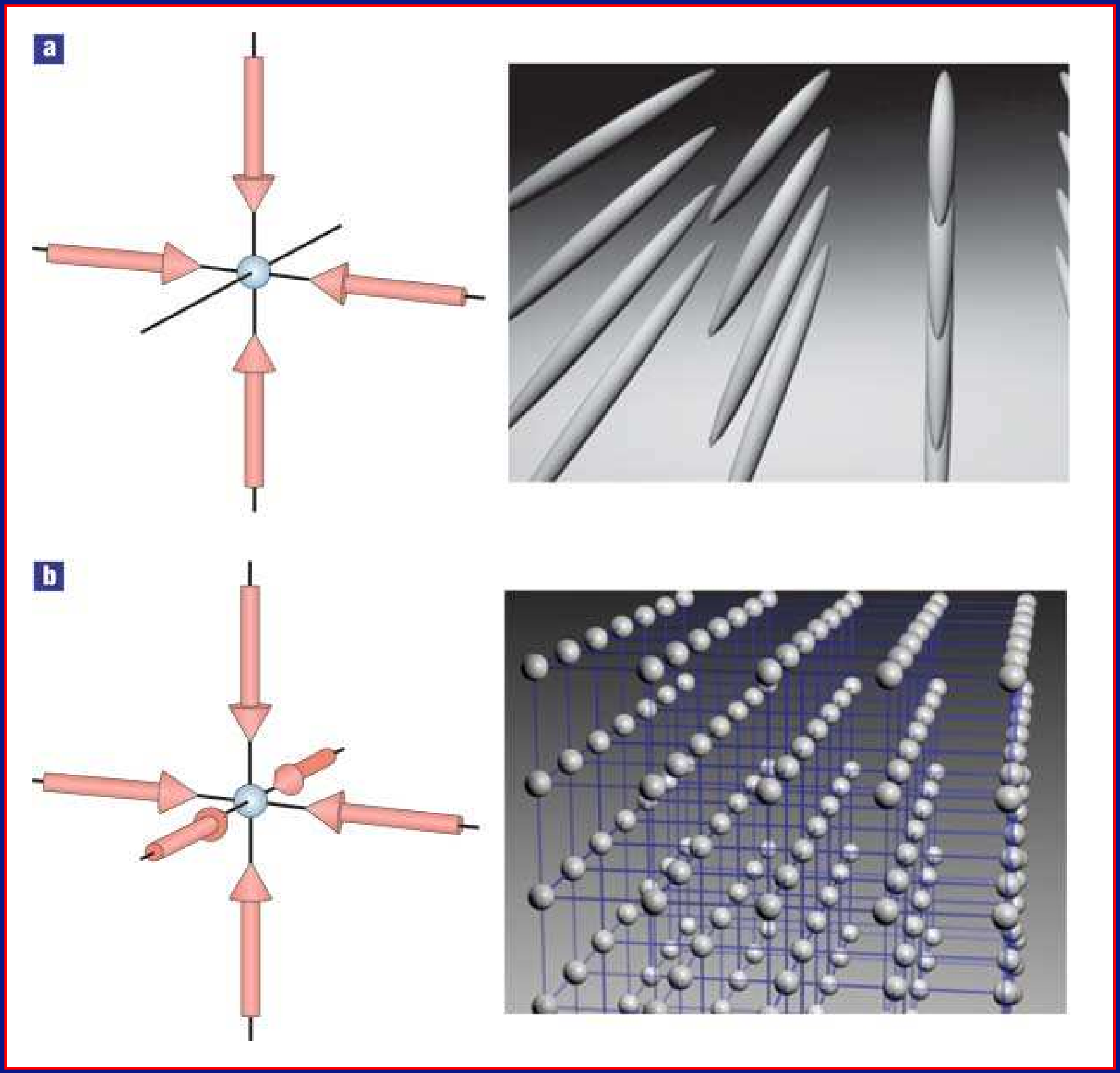
Multichannel reality:

Example ^{6}Li-^{87}Rb : **8 coupled channels**,

— very different length scales involved,

— high quality molecular potential curves required.

Optical lattices: physics on a lattice



[reproduced from I. Bloch, *Nature Physics* **1**, 23 (2005)]

Counterpropagating lasers:

$\longrightarrow$ standing light field.

Trap potential varies as

$$U_{\text{lat}} \sin^2(\vec{k}\vec{r})$$

with

$k = \frac{2\pi}{\lambda}$

λ: laser wavelength.

$$U_{\text{lat}} \propto I\,\alpha(\lambda)$$

with

laser intensity I and

atomic polarizability α.

Applications in quantum information

Optical lattices:

- Perfect periodic structure (phonon free) with variable filling.

Applications in quantum information

Optical lattices:

- Perfect periodic structure (phonon free) with variable filling.
- Lattice shape, depth, geometry tunable (in real time).
  Strong anisotropies possible: physics in reduced dimensionality.

Applications in quantum information

Optical lattices:

- Perfect periodic structure (phonon free) with variable filling.
- Lattice shape, depth, geometry tunable (in real time).
  Strong anisotropies possible: physics in reduced dimensionality.
- Additional vector potential may break Abelian symmetry.

Applications in quantum information

Optical lattices:

- Perfect periodic structure (phonon free) with variable filling.
- Lattice shape, depth, geometry tunable (in real time).
  Strong anisotropies possible: physics in reduced dimensionality.
- Additional vector potential may break Abelian symmetry.

$+$ variability and tunability of the interactions:

$\longrightarrow$ **Quantum emulators for solid-state Hamiltonians and (possibly) lattice gauge theory!**

Applications in quantum information

Optical lattices:

- Perfect periodic structure (phonon free) with variable filling.
- Lattice shape, depth, geometry tunable (in real time).
  Strong anisotropies possible: physics in reduced dimensionality.
- Additional vector potential may break Abelian symmetry.

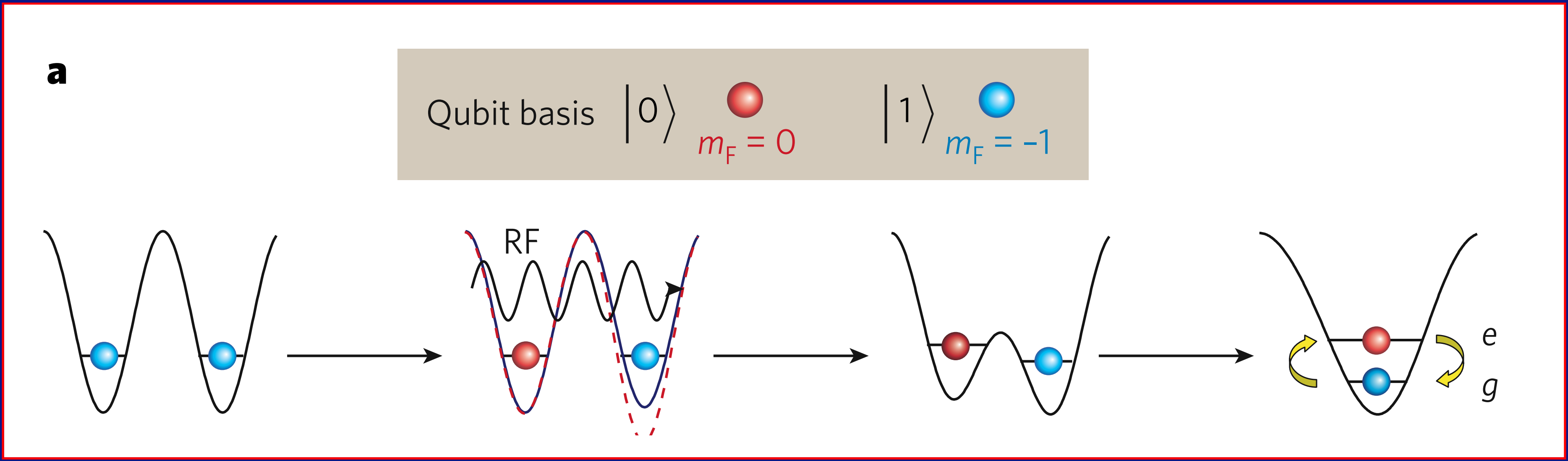
$+$ variability and tunability of the interactions:

$\longrightarrow$ **Quantum emulators for solid-state Hamiltonians and (possibly) lattice gauge theory!**

$+$ single-site addressability:

$\longrightarrow$ **Quantum computers could become possible!**

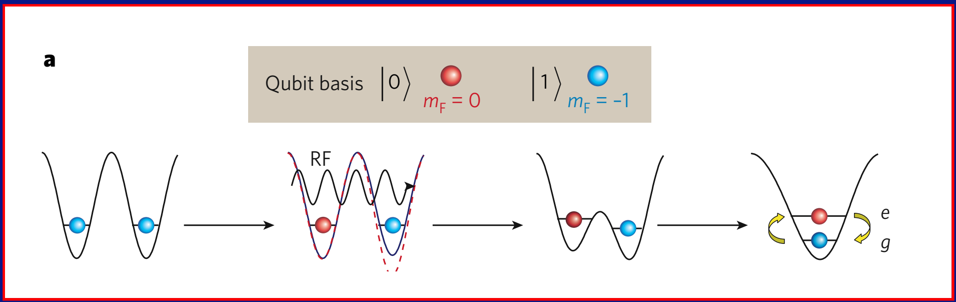
Example of a two-qubit gate (SWAP) (I)

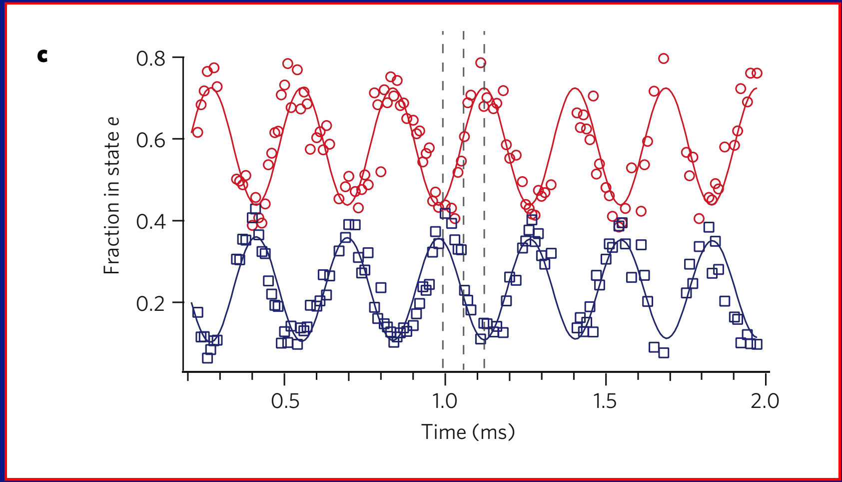


1. Radio-frequency pulse (RF) changes spin state of one atom.
2. Merge the two atoms into a single well.
3. Exchange interaction induces oscillation of spin population in lower and upper vibrational state.

[Anderlini et al., *Nature* **448** 452 (2007)]

Example of a two-qubit gate (SWAP) (II)





Left (Fig. a):

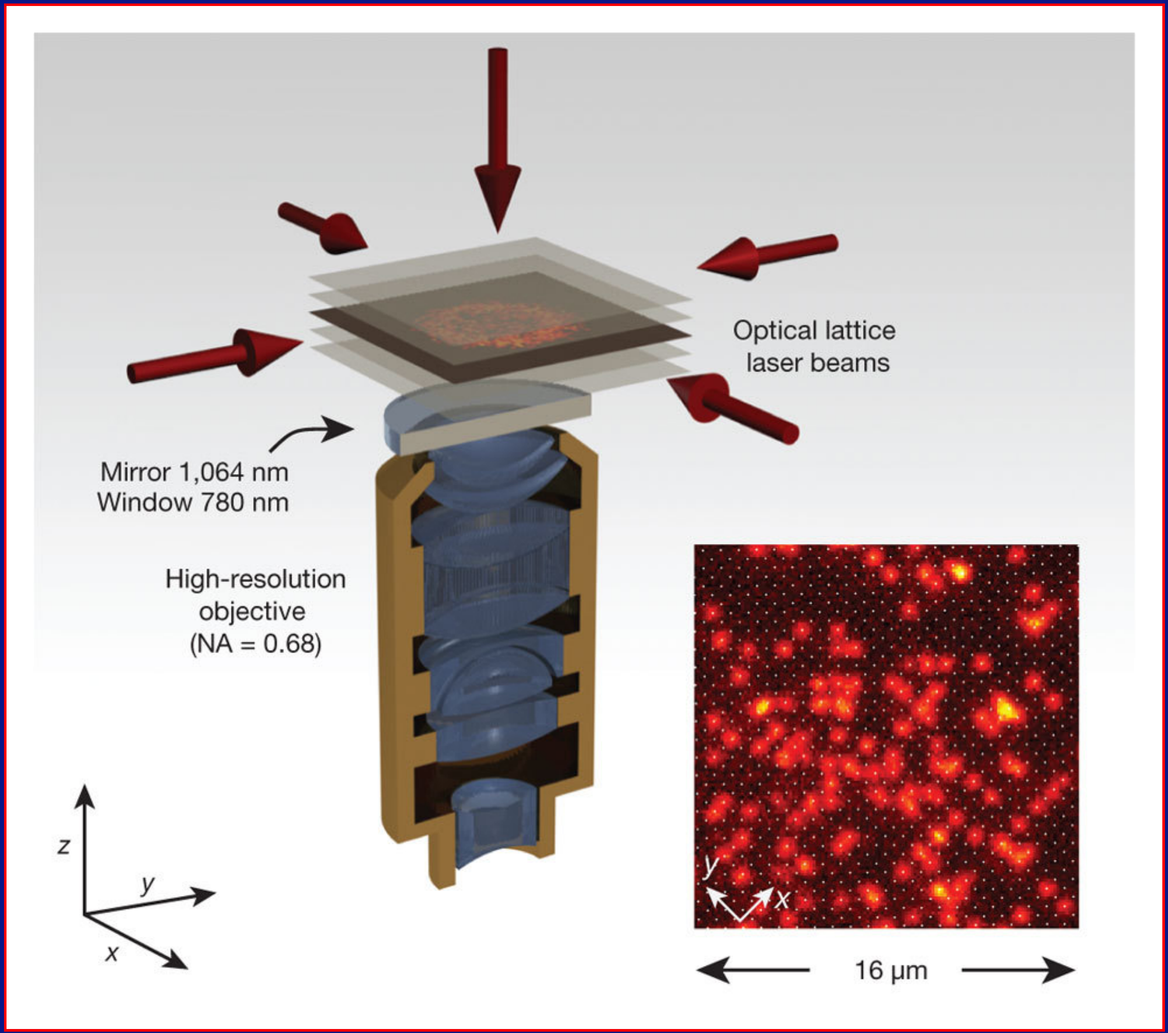
1. Radio-frequency pulse (RF) changes spin state of one atom.
2. Merge the two atoms into a single well.
3. Exchange interaction induces oscillation of spin population in lower and upper vibrational state.

Right (Fig. c):

Measured spin populations in upper vibrational state as a function of interaction time.

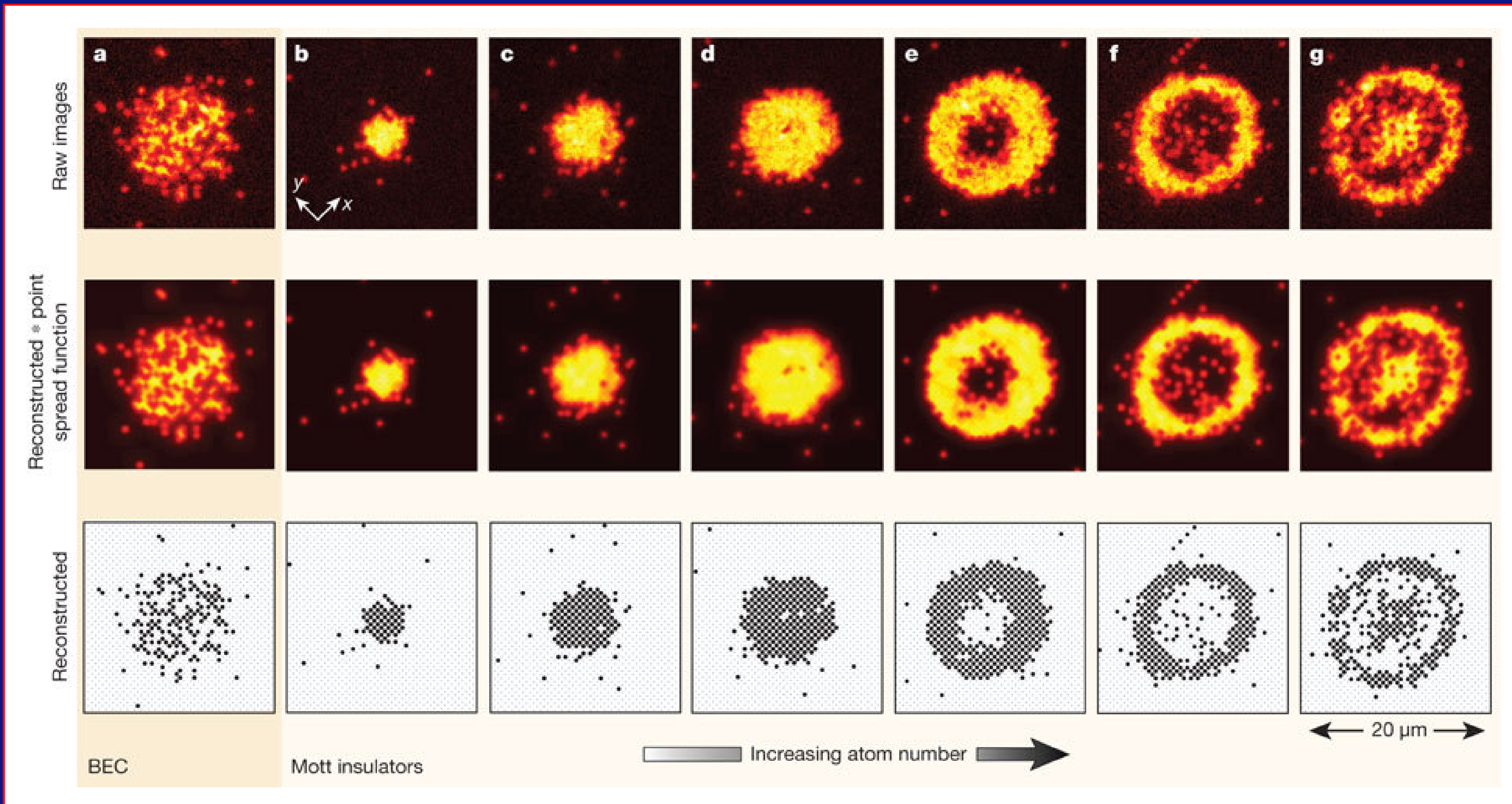
[Anderlini et al., *Nature* **448** 452 (2007)]

Single-site resolution (I)



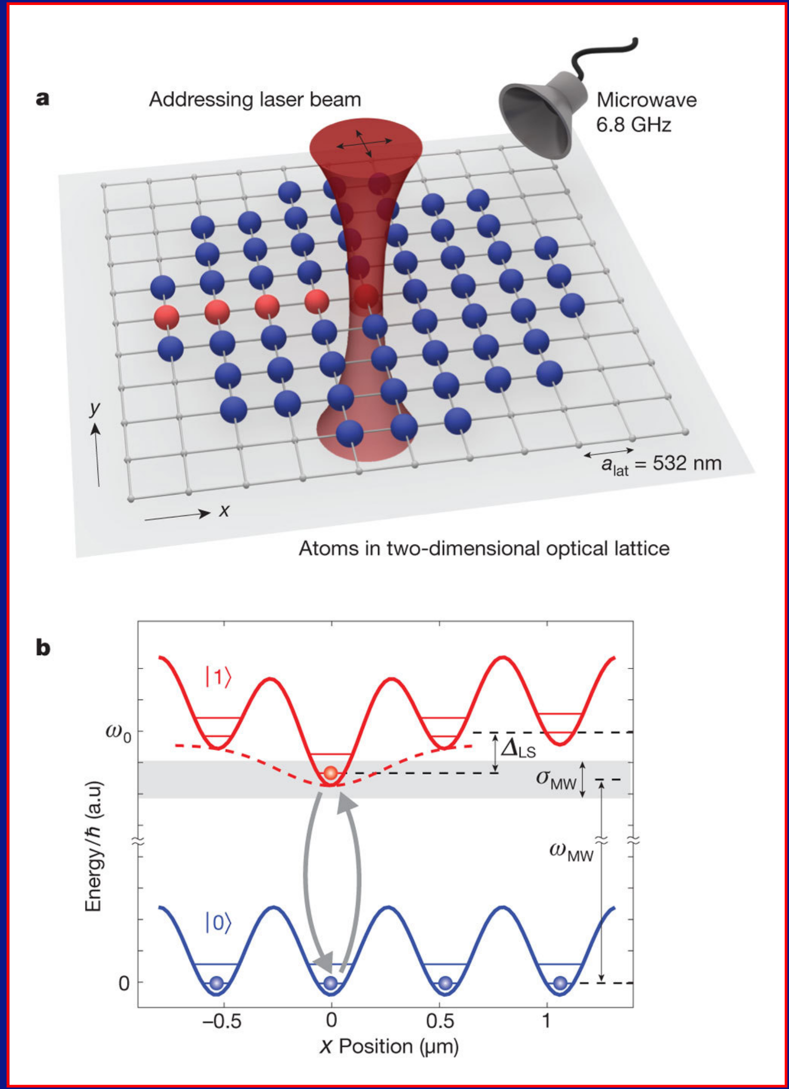
Sherson *et al.*, *Nature* **467**, 68 (2010)

Single-site resolution (II)



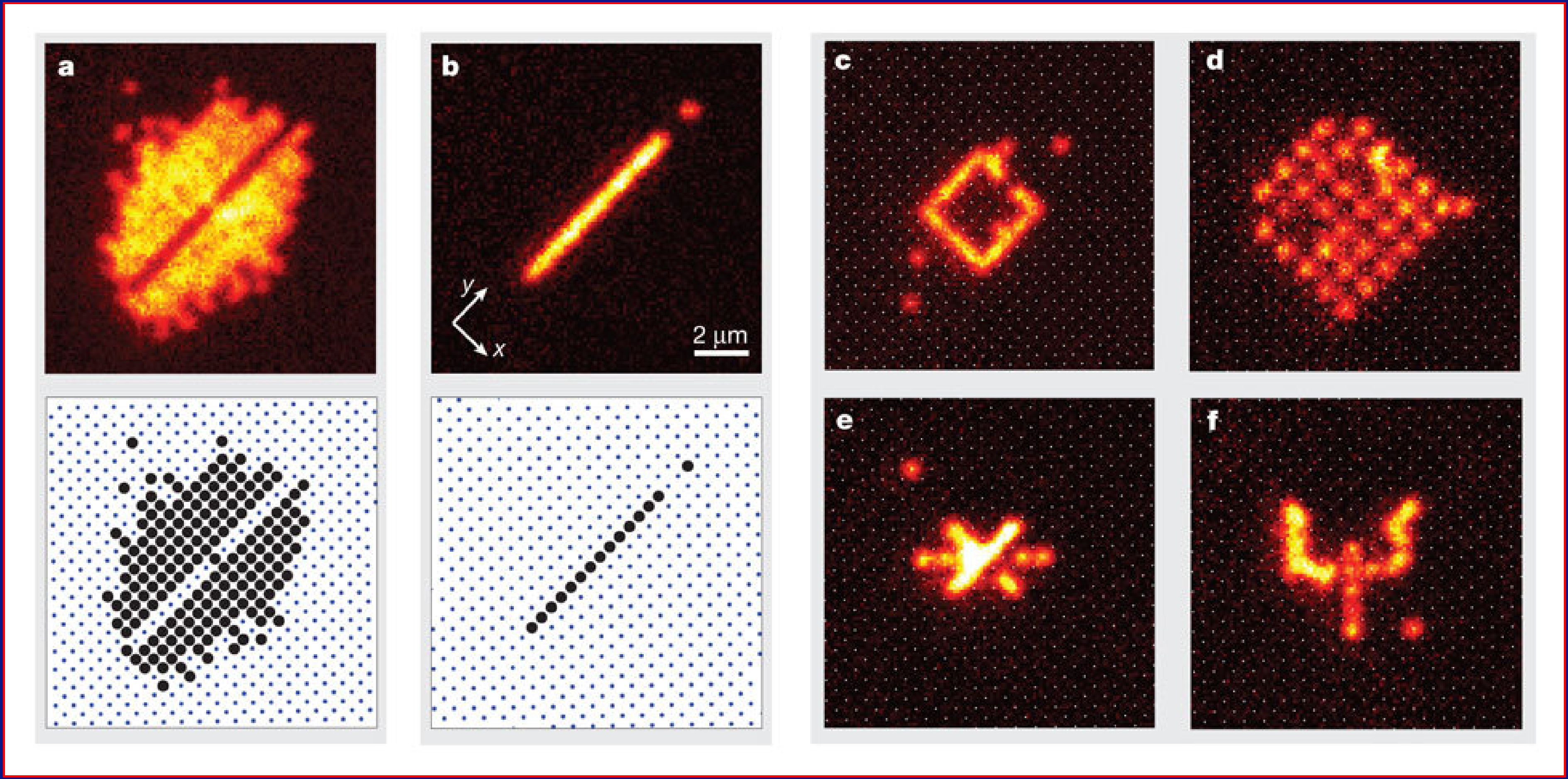
Sherson *et al.*, *Nature* **467**, 68 (2010)

Single-site addressing (I)



Weitenberg *et al.*, *Nature* **471**, 319 (2011)

Single-site addressing (II)



Weitenberg *et al.*, *Nature* **471**, 319 (2011)

Bose-Hubbard model of the OL

N-Boson Hamiltonian with additional external confinement $V_{\mathrm{conf}}(\mathbf{r})$

$$H_{\mathrm{OL}} = \sum_{n=1}^{N} \left(\frac{p_n^2}{2m} + V_{\mathrm{OL}}(\mathbf{r}_n) + V_{\mathrm{conf}}(\mathbf{r}_n) \right) + \sum_{n<m} \hat{V}_{\mathrm{int}}(\mathbf{r}_n - \mathbf{r}_m)$$

is rewritten in basis of Wannier functions $w_i(\mathbf{r})$ (superpositions of Bloch solutions localized at lattice site i) of the first Bloch band as

$$\hat{H}_{BH} = -J \sum_{<i,j>} b_i^\dagger b_j + \sum_i \epsilon_i \hat{n}_i + U \sum_i \frac{\hat{n}_i(\hat{n}_i - 1)}{2}$$

Bose-Hubbard model of the OL

N-Boson Hamiltonian with additional external confinement $V_{\rm conf}(\mathbf{r})$

$$H_{\rm OL} = \sum_{n=1}^{N} \left(\frac{p_n^2}{2m} + V_{\rm OL}(\mathbf{r}_n) + V_{\rm conf}(\mathbf{r}_n) \right) + \sum_{n<m} \hat{V}_{\rm int}(\mathbf{r}_n - \mathbf{r}_m)$$

is rewritten in basis of Wannier functions $w_i(\mathbf{r})$ (superpositions of Bloch solutions localized at lattice site i) of the first Bloch band as

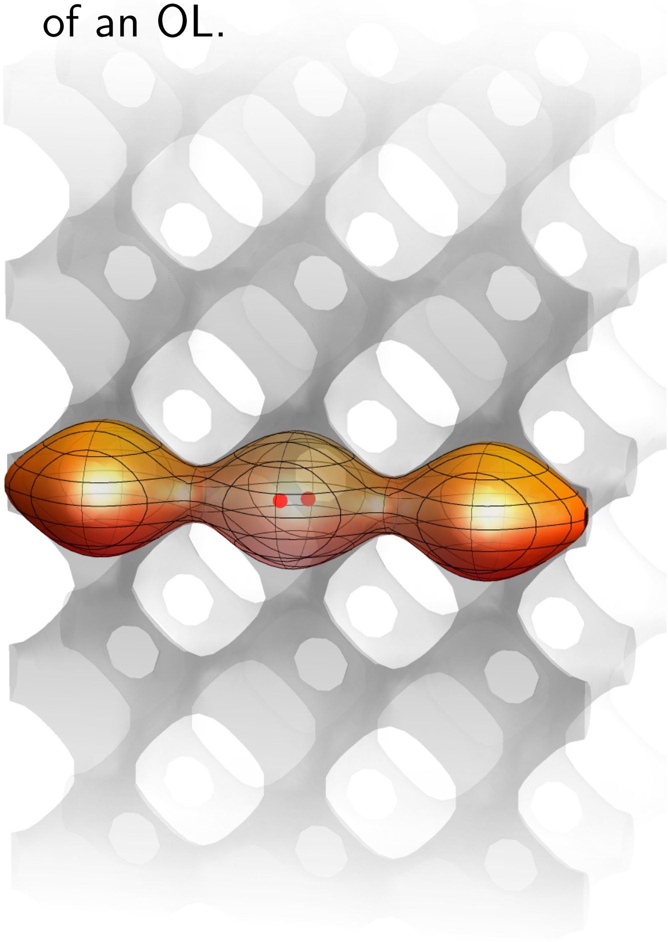
$$\hat{H}_{BH} = -J \sum_{<i,j>} b_i^\dagger b_j + \sum_i \epsilon_i \hat{n}_i + U \sum_i \frac{\hat{n}_i(\hat{n}_i - 1)}{2}$$

with $J = -\left\langle w_0 \middle| \frac{\hat{p}}{2m} + \hat{V}_{\rm OL} \middle| w_1 \right\rangle$, $\epsilon_i = \left\langle w_i \middle| \frac{\hat{p}}{2m} + \hat{V}_{\rm OL} + \hat{V}_{\rm conf} \middle| w_i \right\rangle$

and $U = \langle w_0 | \langle w_0 | \hat{V}_{\rm Int} | w_0 \rangle | w_0 \rangle$

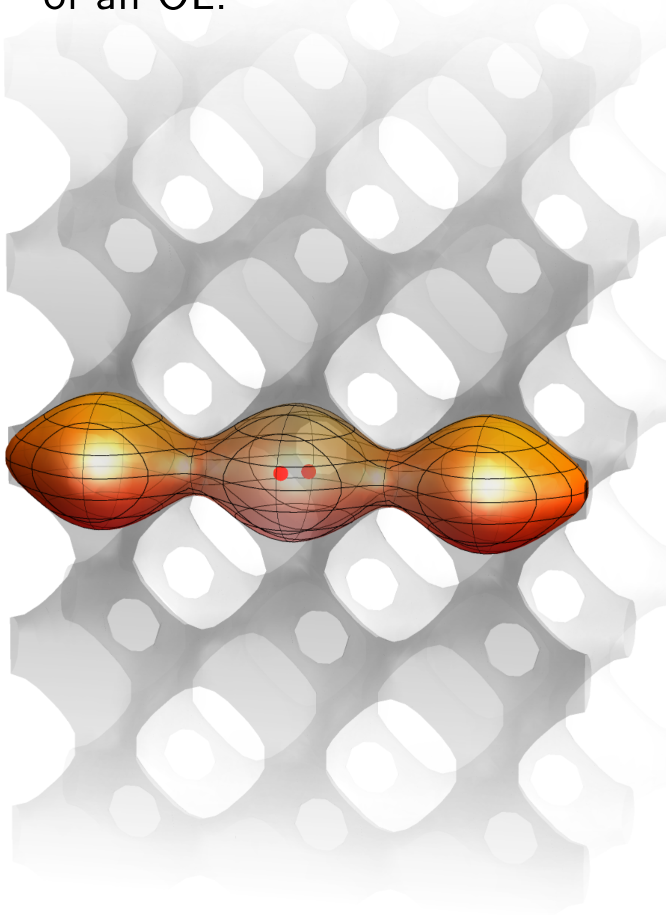
Two atoms in a triple well

We obtain **exact solutions** for two interacting atoms in 3 wells of an OL.



Two atoms in a triple well

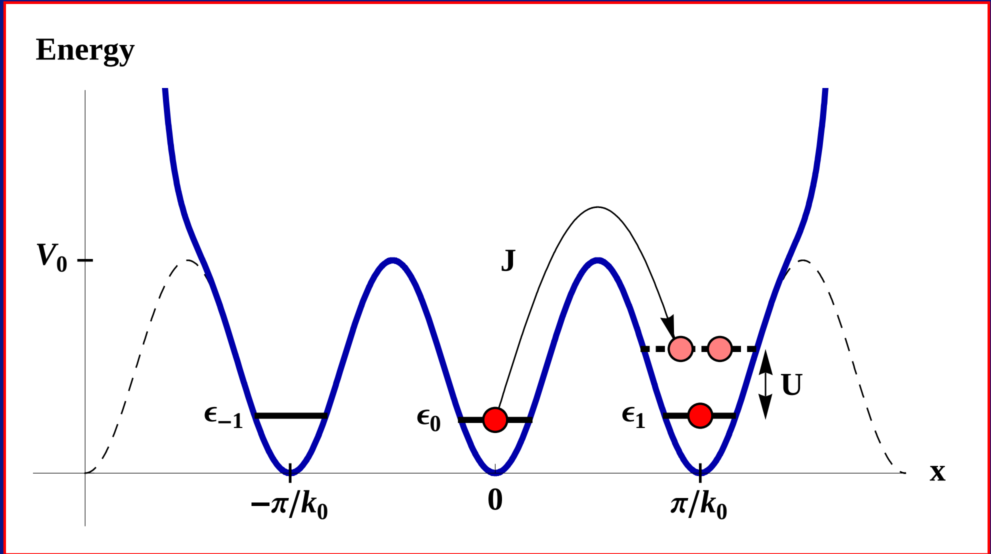
We obtain **exact solutions** for two interacting atoms in 3 wells of an OL.



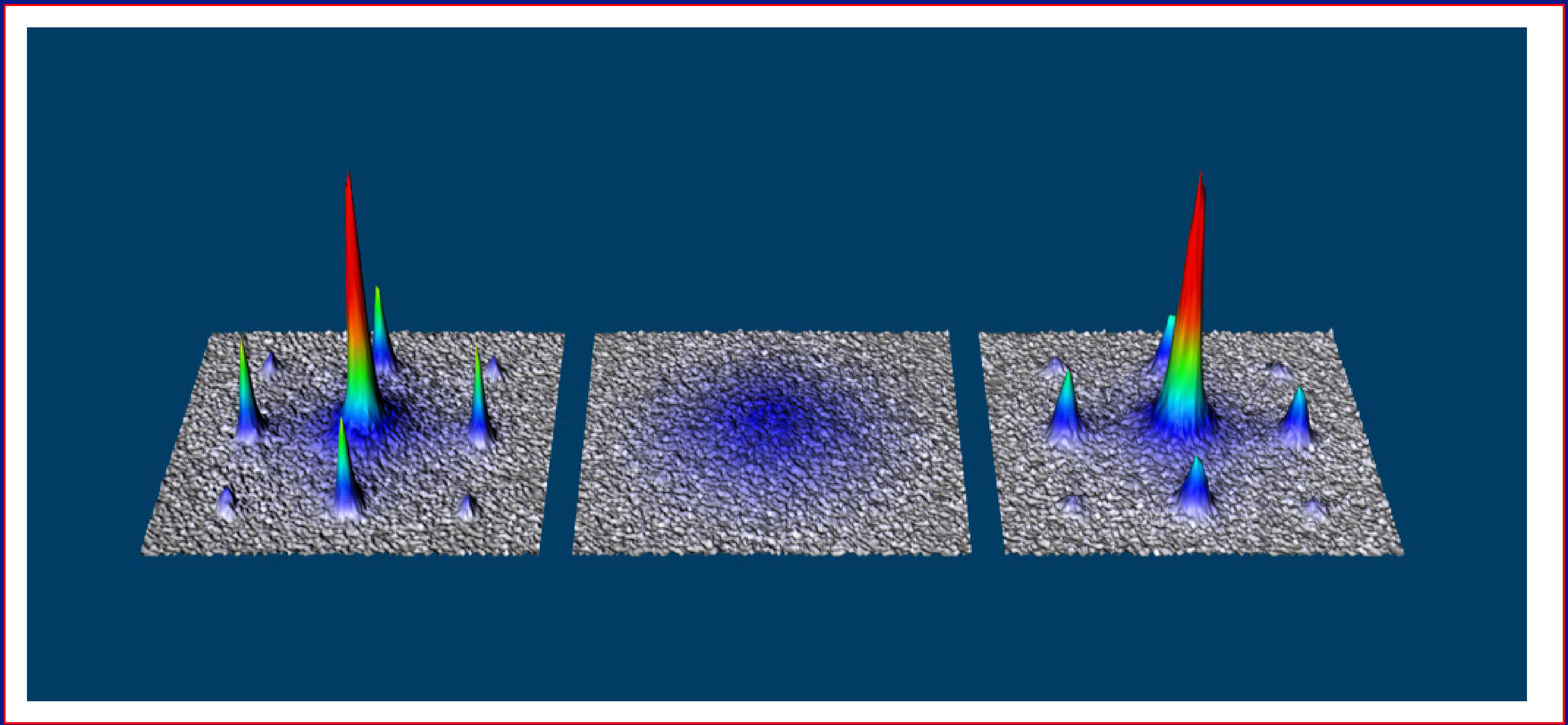
- Comparison with **BH model** with Hamiltonian

$$\hat{H}_{\rm BH} = J \sum_{<i,j>} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \epsilon_i \hat{b}_i^\dagger \hat{b}_i$$

yields **optimal BH parameters** $J^{\rm opt}, U^{\rm opt}, \epsilon_i^{\rm opt}$
and **validity range of BH model**
[P.I. Schneider *et al.*, *Phys. Rev. A* **80**, 013404 (2009)]



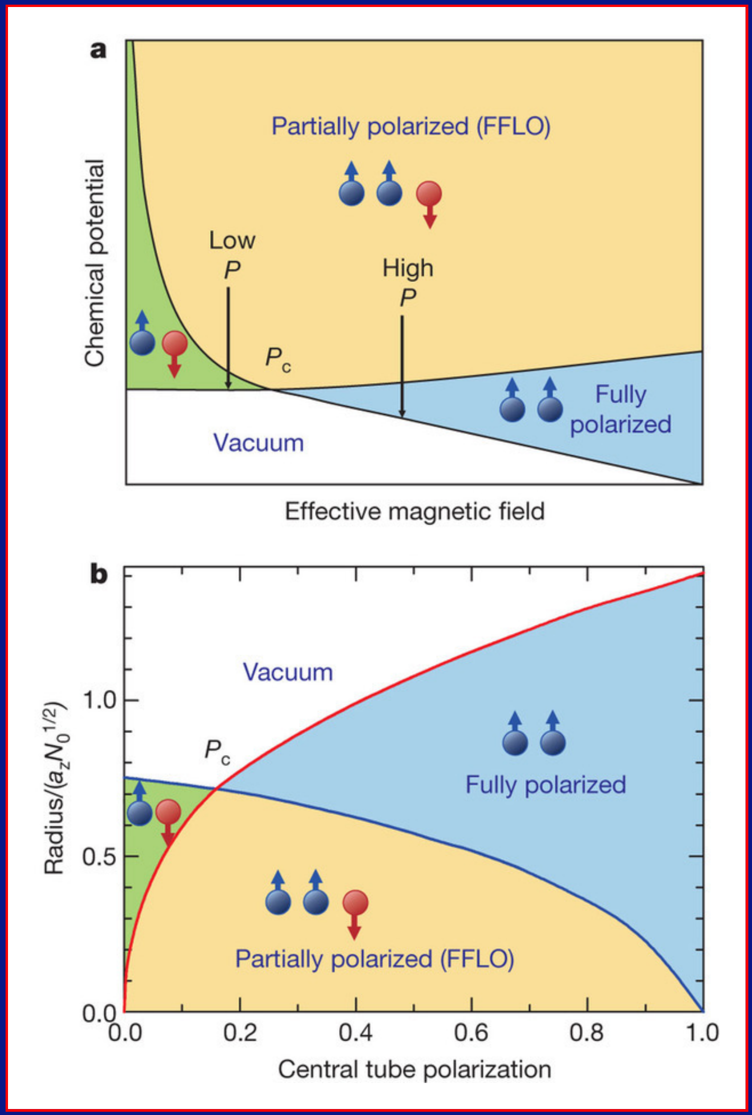
Mott to superfluid transition

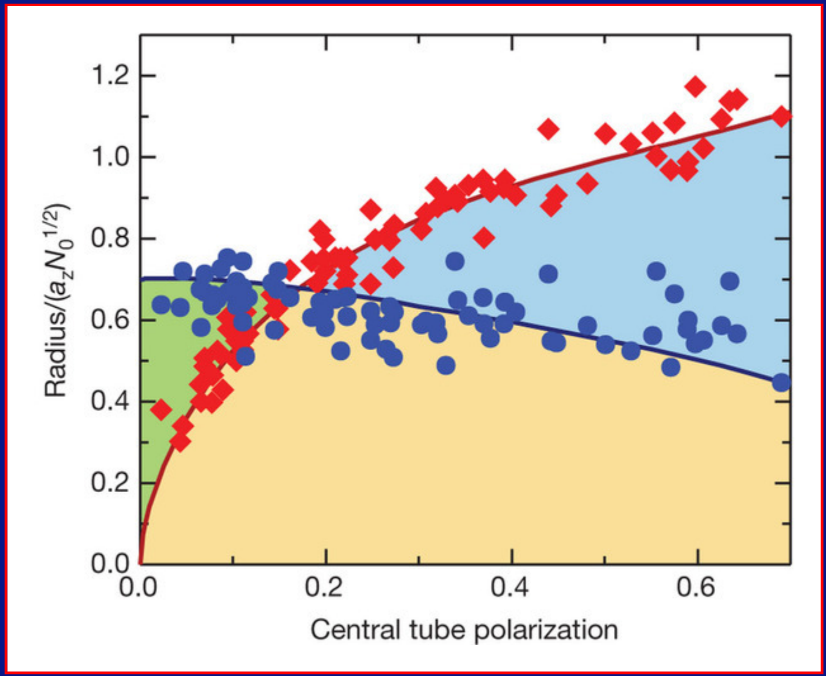


super-fluid phase (BEC) | **insulating phase (incoherent)** | **super-fluid phase (BEC)**

(s. Greiner et al., *Nature* **415** 39 (2002))

Phase diagram (spin system)





Liao *et al.*, *Nature* **467**, 567 (2010)