Bose-Einstein condensate of dilute atomic gases

In a perfect Bose-Einstein condensate (BEC) all ideal bosons occupy the same lowest energy state (in mean field):

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Macroscopic coherent quantum systems

with variable interparticle interactions.
Thermal cloud vs. Bose-Einstein Condensate (BEC)

[from: http://cua.mit.edu/ketterle_group/]

A. Saenz: Quantum information with ultracold quantum gases (2)

HU Berlin, 18.05.2018
Interactions in ultracold atomic gases

Interactions:

- Ultracold collisions $\rightarrow$ s-wave scattering:

\[ \sigma = 4\pi a_{l=0}^2 + O(k^2) \approx 4\pi a_{sc}^2 \]

(“hard-spheres”).

(Note: p-wave scattering for identical Fermions.)
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• Dipolar gases (Cr, diatomics, . . . ) $\rightarrow$ non-isotropic interaction ("spins").
Magnetic Feshbach resonances

Simple picture:

Only 2 channels:
- open (continuum) channel,
- closed (bound) channel.
Magnetic Feshbach resonances

Simple picture:
Only 2 channels:
- open (continuum) channel,
- closed (bound) channel.

Multichannel reality:
Example $^6\text{Li} - ^{87}\text{Rb}$: 8 coupled channels,
- very different length scales involved,
- high quality molecular potential curves required.
Optical lattices: physics on a lattice

Counterpropagating lasers: $\rightarrow$ standing light field.

**Trap potential** varies as

$$U_{\text{lat}} \sin^2(\vec{k}\vec{r})$$

with

$$k = \frac{2\pi}{\lambda}$$

$\lambda$: laser wavelength.

$$U_{\text{lat}} \propto I \alpha(\lambda)$$

with laser intensity $I$ and atomic polarizability $\alpha$.

[reproduced from I. Bloch, *Nature Physics* 1, 23 (2005)]
Applications in quantum information

Optical lattices:

- Perfect periodic structure (phonon free) with variable filling.
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+ single-site addressability:

→ Quantum computers could become possible!
1. Radio-frequency pulse (RF) changes spin state of one atom.
2. Merge the two atoms into a single well.
3. Exchange interaction induces oscillation of spin population in lower and upper vibrational state.

[Anselini et al., *Nature* **448** 452 (2007)]
Example of a two-qubit gate (SWAP) (II)

Left (Fig. a):
1. Radio-frequency pulse (RF) changes spin state of one atom.
2. Merge the two atoms into a single well.
3. Exchange interaction induces oscillation of spin population in lower and upper vibrational state.

Right (Fig. c):
Measured spin populations in upper vibrational state as a function of interaction time.

[Anderlini et al., Nature 448 452 (2007)]
Single-site resolution (I)

Sherson et al., Nature 467, 68 (2010)
Single-site resolution (II)


A. Saenz: Quantum information with ultracold quantum gases (10)
Single-site addressing (I)

Weitenberg et al., Nature 471, 319 (2011)
Single-site addressing (II)

Bose-Hubbard model of the OL

\(N\)-Boson Hamiltonian with additional external confinement \(V_{\text{conf}}(\mathbf{r})\)

\[ H_{\text{OL}} = \sum_{n=1}^{N} \left( \frac{p_n^2}{2m} + V_{\text{OL}}(\mathbf{r}_n) + V_{\text{conf}}(\mathbf{r}_n) \right) + \sum_{n<m} \hat{V}_{\text{int}}(\mathbf{r}_n - \mathbf{r}_m) \]

is rewritten in basis of Wannier functions \(w_i(\mathbf{r})\) (superpositions of Bloch solutions localized at lattice site \(i\)) of the first Bloch band as

\[ \hat{H}_{BH} = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \sum_i \epsilon_i \hat{n}_i + U \sum_i \frac{\hat{n}_i (\hat{n}_i - 1)}{2} \]
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with

\[
J = -\left\langle w_0 \left| \frac{\vec{p}}{2m} + \hat{V}_{\text{OL}} \right| w_1 \right\rangle, \quad \epsilon_i = \left\langle w_i \left| \frac{\vec{p}}{2m} + \hat{V}_{\text{OL}} + \hat{V}_{\text{conf}} \right| w_i \right\rangle
\]

and

\[
U = \left\langle w_0 \left| \left\langle w_0 \right| \hat{V}_{\text{Int}} \right| w_0 \right\rangle \right| w_0 \rangle
\]
Two atoms in a triple well

We obtain exact solutions for two interacting atoms in 3 wells of an OL.
We obtain **exact solutions** for two interacting atoms in 3 wells of an OL.

- Comparison with **BH model** with Hamiltonian

\[ \hat{H}_{\text{BH}} = J \sum_{<i,j>} \hat{b}_i^{\dagger} \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i \epsilon_i \hat{b}_i^{\dagger} \hat{b}_i \]

yields **optimal BH parameters** \( J^{\text{opt}}, U^{\text{opt}}, \epsilon_i^{\text{opt}} \)

and **validity range of BH model**

Mott to superfluid transition

super-fluid phase (BEC)

insulating phase (incoherent)

super-fluid phase (BEC)

(s. Greiner et al., Nature 415 39 (2002))
Phase diagram (spin system)