Quantum Information and Quantum Computer

SS 2018 **Exercise sheet 1** **19.04.2018**

Task 1

Which of the following states are possible (and sensibly defined) qubit states?

(a) $\phi = \frac{1}{\sqrt{2}} \left(|\, 0\,\rangle + |\, 1\,\rangle \right)$

(b) $\phi = \frac{\sqrt{3}}{2} \,|\, 0\,\rangle - \frac{1}{2} \,|\, 1\,\rangle$

(c) $\phi = 0{,}7 \,|\, 0\,\rangle + 0{,}3 \,|\, 1\,\rangle$

(d) $\phi = 0{,}8 \,|\, 0\,\rangle + 0{,}6 \,|\, 1\,\rangle$

(e) $\phi = \cos\theta \,|\, 0\,\rangle + i \sin\theta \,|\, 1\,\rangle$

(f) $\phi = \cos^2\theta \,|\, 0\,\rangle - \sin^2\theta \,|\, 1\,\rangle$

Considering only the valid qubit states, what is the probability that a measurement in the so-called (standard) computer basis $\{|\, 0\,\rangle, |\, 1\,\rangle\}$ yields the values $+1$ or -1, respectively?

Which probabilities are found, if the measurement is not performed in the standard computer basis but in the basis $\left\{ \frac{1}{\sqrt{2}} \left(|\, 0\,\rangle + |\, 1\,\rangle \right), \frac{1}{\sqrt{2}} \left(|\, 0\,\rangle - |\, 1\,\rangle \right) \right\}$?

Task 2

Express the operator $\hat{\mathrm{H}}$ with the matrix representation

$$\mathbf{H} \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

in terms of the standard computer basis $\{|\, 0\,\rangle\langle\, 0\,|, |\, 0\,\rangle\langle\, 1\,|, |\, 1\,\rangle\langle\, 0\,|, |\, 1\,\rangle\langle\, 1\,|\}$.

Task 3

Show that a unitary time-evolution operator conserves the norm.

Task 4

Use the Pauli matrices $\boldsymbol{\sigma}_x$ and $\boldsymbol{\sigma}_y$,

$$\boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad ,$$

to show that the tensor operation ($\otimes$ operator) is not commutative, i. e. $A \otimes B \neq B \otimes A$.

Task 5

An important theorem in the context of quantum information is the quantum *no-cloning theorem*. It states that an unknown quantum state cannot be duplicated, i. e. it is impossible to produce an exact copy of an unknown quantum state without destroying the initial state. Expressed in a more formal way it states: an unknown quantum state $|\psi\rangle$ cannot be cloned perfectly, i. e. there exists no unitary transformation $\hat{\mathrm{U}}$ that fulfils $\hat{\mathrm{U}}(|\psi\rangle \otimes |i\rangle) = |\psi\rangle \otimes |\psi\rangle$ where $|i\rangle$ denotes some (known) state.

a) Why are the restrictions *unknown* and *without destroying the initial state* so important?

b) If a unitary transformation operator $\hat{\mathrm{U}}$ corresponding to the cloning process would exist, then evidently it must fulfil the relations $\hat{\mathrm{U}}|0i\rangle = |00\rangle$ and $\hat{\mathrm{U}}|1i\rangle = |11\rangle$. Show that such an operator $\hat{\mathrm{U}}$, acting on an arbitrary state $|\psi i\rangle = |\psi\rangle \otimes |i\rangle$ with $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, cannot yield the wanted result.

c) Proof that a transformation $\hat{\mathrm{U}}$ that would allow for quantum cloning is not unitary.