Quantum Information and Quantum Computers

SS 2018 **3. Exercise** **03.05.2018**

Task 1

Entangled state possess a very prominent role in quantum information. Assume that Alice is a brilliant experimentalist and thus capable of generating entangled states without serious difficulties. On the other hand, the experimental performance of Bob is quite limited and he cannot generate entangled states. When Bob visited Alice last time, both fell in love with each other. Therefore, before his departure he asked Alice to generate two entangled 2-qubit states of the type $|\,\beta_{11}\,\rangle = \frac{1}{\sqrt{2}}(|\,01\,\rangle - |\,10\,\rangle)$ and two give him as a remembrance the second qubit of each of the two 2-qubit states. In this way they would stay connected even after Bob has returned home (romance of physicists!?!). He asked Alice for two qubits (and thus to generate two 2-qubit states) in order to “make assurance double sure”. However, despite this twofold assurance the relation between Bob and Alice turned into a normal friendship shortly thereafter (as Bob had met Claire after he returned home). Since Bob is a pragmatic person (a physicist) he wonders whether he can make use of the qubits after they lost their romantic touch. After some thinking (or, what appears more likely, after asking Claire) he requests the following favour from Alice. She should perform a measurement on her qubits with respect to the Bell basis states

$$
\begin{aligned}
|\,\beta_{00}\,\rangle &= \frac{1}{\sqrt{2}}\,(\,|\,00\,\rangle + |\,11\,\rangle\,) \\
|\,\beta_{01}\,\rangle &= \frac{1}{\sqrt{2}}\,(\,|\,01\,\rangle + |\,10\,\rangle\,) \\
|\,\beta_{10}\,\rangle &= \frac{1}{\sqrt{2}}\,(\,|\,00\,\rangle - |\,11\,\rangle\,) \\
|\,\beta_{11}\,\rangle &= \frac{1}{\sqrt{2}}\,(\,|\,01\,\rangle - |\,10\,\rangle\,)
\end{aligned}
$$

and tell him the outcome of the measurement.

a) What has Bob achieved with this measurement of Alice?

b) What happens, if Alice made an error during the preparation of the two 2-qubit states and generated one singlet state (as wished), but the other qubit pair was generated in the state $|\,\psi\,\rangle = \cos(\theta)\,|\,01\,\rangle - \sin(\theta)\,|\,10\,\rangle$?

c) What happens, if Alice generated in both cases the state $|\,\psi\,\rangle = \cos(\theta)\,|\,01\,\rangle - \sin(\theta)\,|\,10\,\rangle$ (with the same θ) instead of singlet states?

Task 2

The Deutsch algorithm and its n-qubit extension, the Deutsch-Josza algorithm that is shown below for $n = 4$, are supposed to be the first quantum-computer algorithms. In the shown circuit, H is the Hadamard gate and U_f computes the classical Boolean function $f(x)$ with $f : \{0, 1\}^n \rightarrow \{0, 1\}$ for the n input qubits ($x \equiv x_1, x_2, \ldots, x_n$), i. e. a single bit result 0 or 1 is assigned to every state of the bit register (or qubit basis state) $x = x_1\, x_2\, \ldots\, x_n$. This result ($f(x)$) is coded by U_f in qubit y in the form $y \oplus f(x)$, while the remaining input qubits ($x_1, x_2, \ldots, x_n$) remain unchanged by U_f. Demonstrate that the Deutsch-Josza algorithm is capable to distinguish with a single call of U_f whether function f is *constant* (always 0 or always 1, independently of x) or *balanced* (in exactly half of the possible cases, i. e. for one half of the possible input states x, f has the value 0 and for the other half it has the value 1). What happens, if f is neither strictly constant nor strictly balanced?