Quantum Information and Quantum Computers

SS 2018 **4 . Exercise** **17.05.2018**

Task 1

As will be seen in the future, qubit rotations play an important role in quantum information. In their formal description the application of Pauli matrices is very helpful. In the context of this task the relation between Pauli matrices and rotations should be refreshed.

a) Determine the eigenvectors, the eigenvalues, and the spectral decomposition of the three Pauli matrices $\hat{\sigma}_x, \hat{\sigma}_y$, and $\hat{\sigma}_z$.

b) To which points on the Bloch sphere do the (normalized) eigenvectors of the Pauli matrices correspond?

c) Show that the so-called $\hat{\mathrm{T}}$ operator

$$\hat{\mathrm{T}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$

which we will also encounter as T or "$\pi/8$" quantum gate corresponds, except for a phase factor, to a rotation around the z axis.

d) Show that the operators $e^{-i\theta\hat{\sigma}_x/2}$, $e^{-i\theta\hat{\sigma}_y/2}$, and $e^{-i\theta\hat{\sigma}_z/2}$ correspond to rotations around the x, y, and z axis, respectively.

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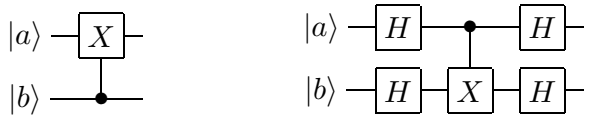
Task 2

Are the following pairs of circuits equivalent to each other?

a) Exchange of the input qubits in the controlled Z gate:

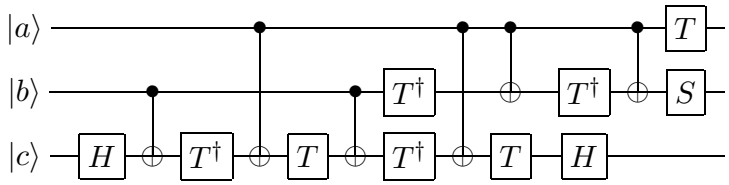
b) Exchange of the input qubits in the controlled X gate and additional Hadamard operations:



c) Measurement of the control qubit before or after the application of a controlled U gate ($\hat{\mathrm{U}}$ represents an arbitrary unitary transformation):

Task 3

The Toffoli gate had been introduced earlier as a universal and reversible *classical* gate. A universal quantum gate can on the other hand be formed with the aid of the 2-qubit gate $CNOT$ and the 1-qubit gates H (Hadamard), S (phase), and T ("$\pi/8$"). If this is true, it must be possible to emulate with the aid of these gates a Toffoli gate, since every universal quantum computer must be able to emulate a classical computer. Check whether the following circuit corresponds to a Toffoli gate:



Here, $T^\dagger$ is the adjoint transformation of T. How can it be emulated with the aid of the universal gate given above?