Quantum Information and Quantum Computer

SS 2018 **6 . Exercise** **31.05.2018**

Task 1

Show that the phase-estimation algorithm (introduced in the lecture) applied to the *NOT* operator ($\hat{\sigma}_x$) generates the corresponding eigenvalues, if a single qubit is used in the first register (i. e. $t = 1$). Note, the phase φ is encoded as binary fraction (and not as the decimal representation of a binary number), i. e. $\varphi := 0.\varphi_1\varphi_2 \dots \varphi_t = 2^{-1}\varphi_1 + 2^{-2}\varphi_2 + \dots + 2^{-t}\varphi_t$. For the demonstration, follow these steps:

a) How can the eigenvectors of $\hat{\sigma}_x$ be prepared in the second register? (How many qubits are needed?)

b) Give the matrix representation and the corresponding circuit for a 1-qubit Fourier transform.

c) Give the complete circuit (using the universal set of gates adopted regularly in this course) for the phase estimation of the *NOT* operator.

d) Verify your circuit with the aid of its action on the different eigenstates of the *NOT* operator.

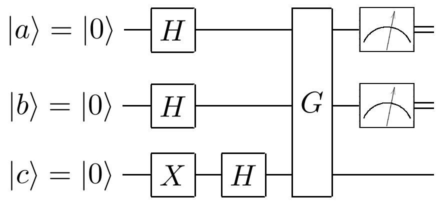
Task 2

Determine the eigenvalues of the Grover iteration operator $\hat{\mathrm{G}}$. Use your result for explaining how the phase-estimation algorithm can be used to estimate the number of hits M contained in a list of N elements. Sketch the corresponding circuit.

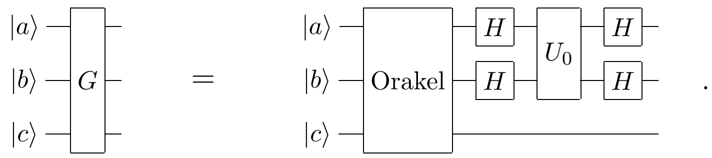
(Please, turn the page.)

Task 3

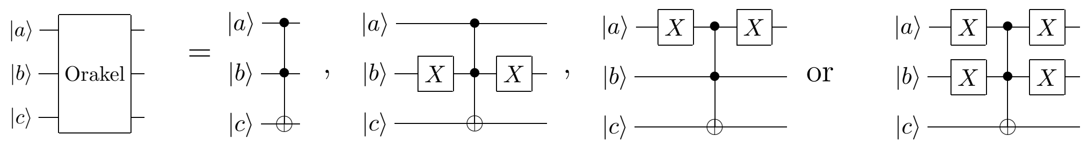
As an illustration, the Grover search algorithm for a list of $N = 4$ elements may be realised by the circuit



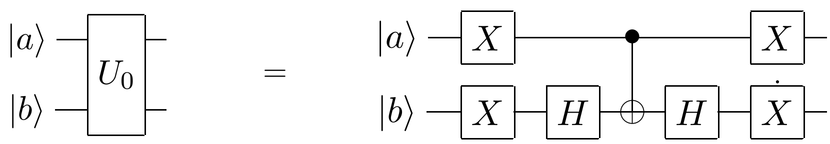
in which the two qubits $|\,a\,\rangle$ and $|\,b\,\rangle$ encode the elements of the list (coded in the four basis states $|\,0\,\rangle, |\,1\,\rangle, |\,2\,\rangle$, and $|\,3\,\rangle$). The qubit $|\,c\,\rangle$ represents the oracle qubit and $\hat{\mathrm{G}}$ is the Grover operator



In contrast to a (sensible) realistic application of the Grover algorithm for data-bank searches, in the present example the list elements are identical to their indices and the property "hit" (or "no hit") is encoded in the oracle. For the present 2-qubit example the circuits



are thus possible implementations of the oracle, if there exists exactly one hit. The phase-shift operator $\hat{\mathrm{U}}_0$ can (in this example) be realised with the circuit



a) Which basis state (list element) represents the respective hit in the four oracle circuits shown above?

b) Which measurement result is obtained, if the Grover operator is applied exactly once, as is shown in the circuit above? In order to answer this question, analyse the circuit step by step and assume that $|\,3\,\rangle$ represents the only hit.