Statistische Physik

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Aufgabe 1 (8)

The cycle of a highly idealized gasoline engine can be approximated by the Otto cycle (shown in Fig. 1) where $1 \to 2$ and $3 \to 4$ are adiabatic compression and expansion, respectively; while $2 \to 3$ and $4 \to 1$ are isochoric processes. Consider the working medium as an ideal gas with constant $\gamma = C_P/C_V$.

a) Compute the efficiency (*Wirkungsgrad*), i. e. the ratio $\eta = W_{\text{out}}/Q_{\text{in}}$ of the (useful) work W_{out} performed by the motor and the amount of heat Q_{in} fed into the motor (absorbed by the gas), of one cycle for $\gamma = 1.5$ and the compression ratio $V_i/V_f = 10$ between the initial and the final volumes $V_i = V_1$ and $V_f = V_2$, respectively.

b) Calculate the work done on the gas in the compression process $1 \to 2$ assuming the initial volume $V_i = 3\,\text{l}$ and $P_i = 1\,\text{atm}$.

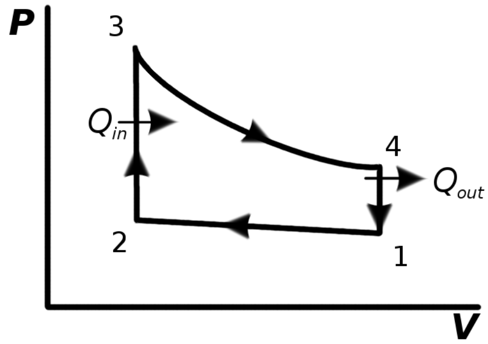

Figure 1: Otto cycle

Aufgabe 2 (6)

The heat capacity at constant volume is defined as $C_V = (\partial U/\partial T)_V$, while the heat capacity at constant pressure is $C_P = (\partial H/\partial T)_P$.

a) Show the validity of

$$C_P - C_V = T \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \quad .$$

Calculate $C_P - C_V$ for

b) the ideal gas,

c) a van der Waals gas obeying the equation of state $(P - \frac{n^2 a}{V^2})(V - nb) = nRT$ where a and b are specific constants for a given gas, $n = N/N_A$ the number of moles (equal to the number of particles N divided by the Avogadro constant N_A) and R is the universal gas constant.

Aufgabe 3 (6)

a) Starting with the total differential of the inner energy,

$$\mathrm{d}U = T\mathrm{d}S - P\mathrm{d}V + \sum_{i=1}^{I} \mu_i \, \mathrm{d}N_i \quad ,$$

and using the fact that $\mathrm{d}U$ depends solely on total differentials of *extensive* quantities, i. e. quantities whose values scale with the amount of material or size of the system, show that for the Gibbs free energy

$$G = \sum_{i=1}^{I} \mu_i \, N_i$$

where N_i is the number of particles of component i, μ_i is the chemical potential for this component, and I is the total number of components.

b) Derive

$$\sum_{i=1}^{I} N_i \, \mathrm{d}\mu_i = -S \, \mathrm{d}T + V \, \mathrm{d}P$$

using the relation for G shown in a).

c) What does the relation derived in b) imply for a two-component system at constant temperature and pressure?