Aufgabe 1 (8)

Consider an ideal gas of \( N \) particles in an isolated container with volume \( V \). A partition separates the container into two sections with volumes \( V_1 \) and \( V_2 \), respectively, such that \( V_1 + V_2 = V \). Also, there are \( N_1 \) particles in the volume \( V_1 \) and \( N_2 \) particles in the volume \( V_2 \). It is assumed that the number density is the same throughout the system

\[
\rho = \frac{N_1}{V_1} = \frac{N_2}{V_2}
\]

Figure 1: Sketch of an isolated container partitioned into two subsystems

a) Using phenomenological thermodynamics, calculate the entropy for each of the two sections.

b) The partition is now removed. Calculate the total entropy of the system in this configuration.

c) Calculate the difference between the total entropy before and after the removal of the partition. What do you conclude?

d) Calculate the difference between the total entropy before and after the removal of the partition assuming that different gases occupy initially the two sections of the container.
Aufgabe 2 (6)
The internal energy of a thermodynamic system can be expressed as a function of the extensive variables (for example, entropy, volume, number of particles, etc.):

\[ U = U(S, V, \{N_i\}, etc.). \]

Use this expression and the Legendre transformation to derive the thermodynamic potential which is expressed as a function of the following variables:

a) entropy, pressure, and number of particles;
b) temperature, volume, and number of particles;
c) temperature, pressure, and chemical potential;
d) temperature, volume, number of particles, and electric field.

Aufgabe 3 (6)
Solve the following problems involving combinatorics. Explain the derivation of your solutions.

a) In how many ways 5 persons can sit on 8 chairs?
b) How many codes for a 4-digit bike lock can be formed without using the same digit more than once?
c) In the roulette game, what is the probability that in six throws of the ball a total of 3 red and 3 black numbers are obtained (neglect the possibility of getting the “green” zero)?
d) How many 9-digit numbers contain 3 times the digit 1, 3 times the digit 2, and 3 times the digit 3?
e) In a football match Hans, Andreas, Gerd, and Stefan score a total of 8 goals. How many different distributions of the goals among them are possible?
f) From a standard Skat deck of 32 cards, 3 cards are drawn. What is the probability that among the drawn cards there is exactly one ace? What is the probability that there is at least one ace?