Triviality of ϕ_4^4 in the broken phase revisited

by Tomasz Korzec and Ulli Wolff , HU Berlin

LAT14, Columbia U. New York , June 24, 2014

Background

Euclidean action, lattice ϕ^4 theory:

$$S[\phi] = a^4 \sum_x \left\{ \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\mu_0^2}{2}\phi^2 + \frac{g_0}{4!}\phi^4 \right\}$$

Triviality:

- define from correlation (ratios):
  renormalized mass m_R ($\leftrightarrow$particles) and coupling g_R ($\leftrightarrow$scattering)
- mapping: $(a\mu_0, g_0) \to (am_R, g_R)$
- $g_R \searrow 0$ whenever $am_R \searrow 0$ = no interaction in the continuum limit
- this happens, no matter how we tune (am_0, g_0)
- proven for $D > 4$ (Aizenman, Fröhlich)
- true in Perturbation Theory and conjectured beyond PT in $D = 4$
- confirmed in simulations of varying degree of persuasiveness

Broken Phase

- naive: for $\mu_0^2 < 0$ S has minima at $\phi = \pm\overline{\phi}$, $\overline{\phi}^{\,2} = -6\mu_0^2/g_0$,
- fluctuations around $+\overline{\phi}$ (picked by small external field)
- 2-point correlation for small momenta ($p^2 \to 0$) is a way to define m_R and v_R

$$Z^{-1}\big\langle \tilde{\phi}(p)\tilde{\phi}(-p)\big\rangle = \delta^{(4)}(p)v_R^2 + \frac{1}{p^2 + m_R^2} \quad (*), \qquad\qquad v_R = Z^{-1/2}\langle\phi\rangle$$

$$g_R = \frac{3m_R^2}{v_R^2} = g_0 + \mathrm{O}(g_0^2)$$

Finite torus L^4

- discrete momenta, $\delta^{(4)}(p) \to L^4\delta_{p,0}$
- three special momenta: $p = 0$, $p_* = \frac{2\pi}{L}(1,0,0,0)$, $p_{**} = \frac{2\pi}{L}(1,1,0,0)$
- enforce $(*)$ at these three momenta, solve for $m_R, v_R, (Z)$
- take $z = m_R L$ fixed (later $z^2 = 10$) [finite size scaling], $w = v_R L$
- no thermodynamic limit, use all accessible L/a to vary a

Lattice

reparameterize the bare parameters:

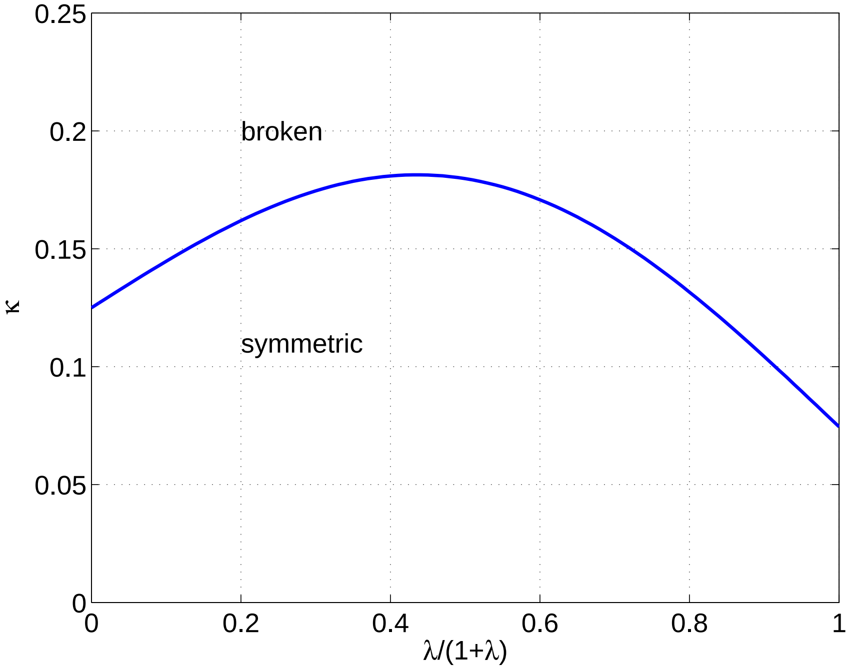
$$S = \sum_x \left[\varphi(x)^2 + \lambda(\varphi(x)^2 - 1)^2\right] - 2\kappa \sum_{x\mu} \varphi(x)\varphi(x+\hat{\mu})$$

with

$$a\phi = \sqrt{2\kappa}\,\varphi, \qquad a^2\mu_0^2 = \frac{1-2\lambda}{\kappa} - 8, \qquad g_0 = \frac{6\lambda}{\kappa^2}$$

$\lambda = \infty$: Ising limit

phase diagram $\longrightarrow$



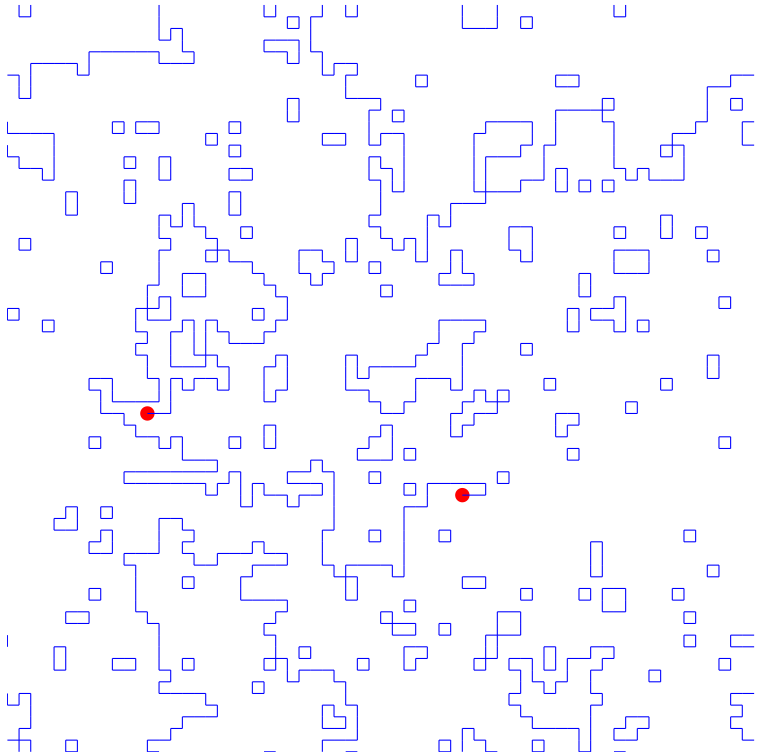
Two point function from worms

$$Z(u,v) = \sum_{\varphi(x)=\pm 1} \mathrm{e}^{2\kappa \sum_{x,\mu} \varphi(x)\varphi(x+\hat{\mu})} \, \varphi(u)\varphi(v) \qquad (\lambda = \infty)$$

Worm/defect ensemble:

$$\mathcal{Z} = \sum_{s.c.\text{graphs with 2 defects}} [\tanh(2\kappa)]^{\#\mathrm{on-links}} = c \sum_{u,v} Z(u,v)$$

typical graph (equilibrium) $\longrightarrow$



$$\langle \phi(x)\phi(0)\rangle \propto \langle\langle \delta_{x,u-v}\rangle\rangle$$

$$\tilde{G}(p) = \langle \tilde{\phi}(p)\tilde{\phi}(-p)\rangle \propto \langle\langle \mathrm{e}^{ip(u-v)}\rangle\rangle$$

$$= \langle\langle \prod_{\mu=0,\ldots,3} \cos[p_\mu (u-v)_\mu]\rangle\rangle$$

RG, triviality

- pick $\lambda = \infty$, $z^2 = 10$, some $L/a \Rightarrow$ determine: $[\kappa]$, v_R and hence $g_R(L/a)$

Callan Symanzik in perturbation theory

$$-\frac{\partial g_R}{\partial \ln (L/a)} = b_1 g_R^2 + b_2 g_R^3 + b_{3,z^2}\, g_R^4 + \mathrm{O}(g_R^5)$$

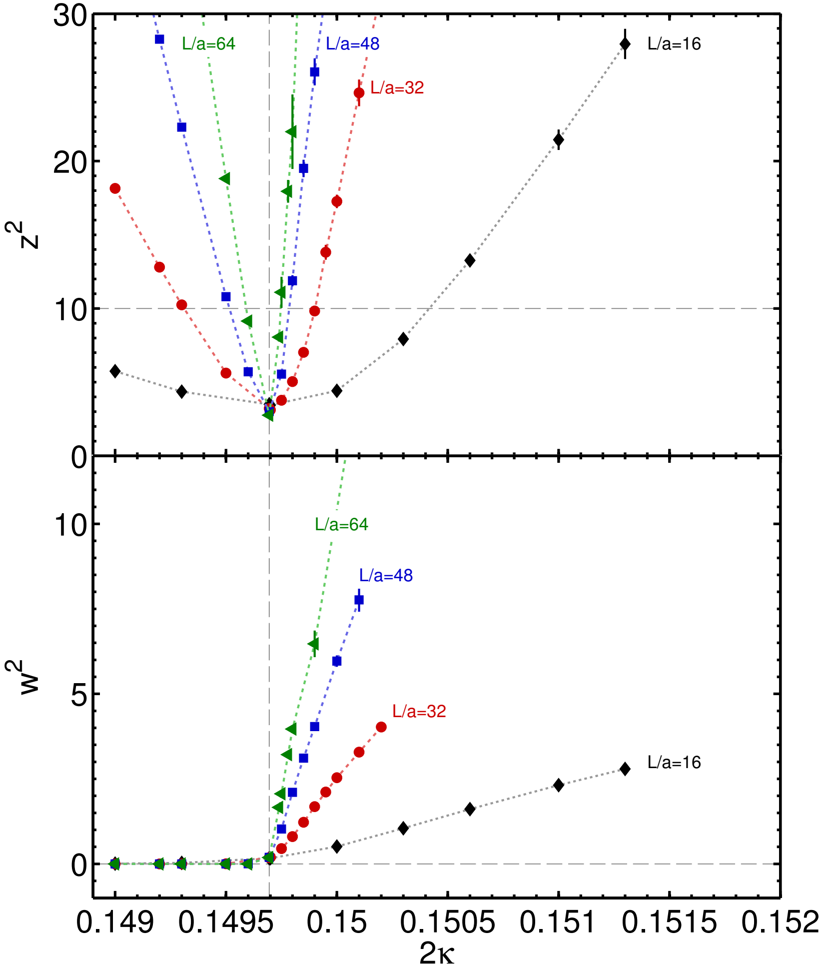
with

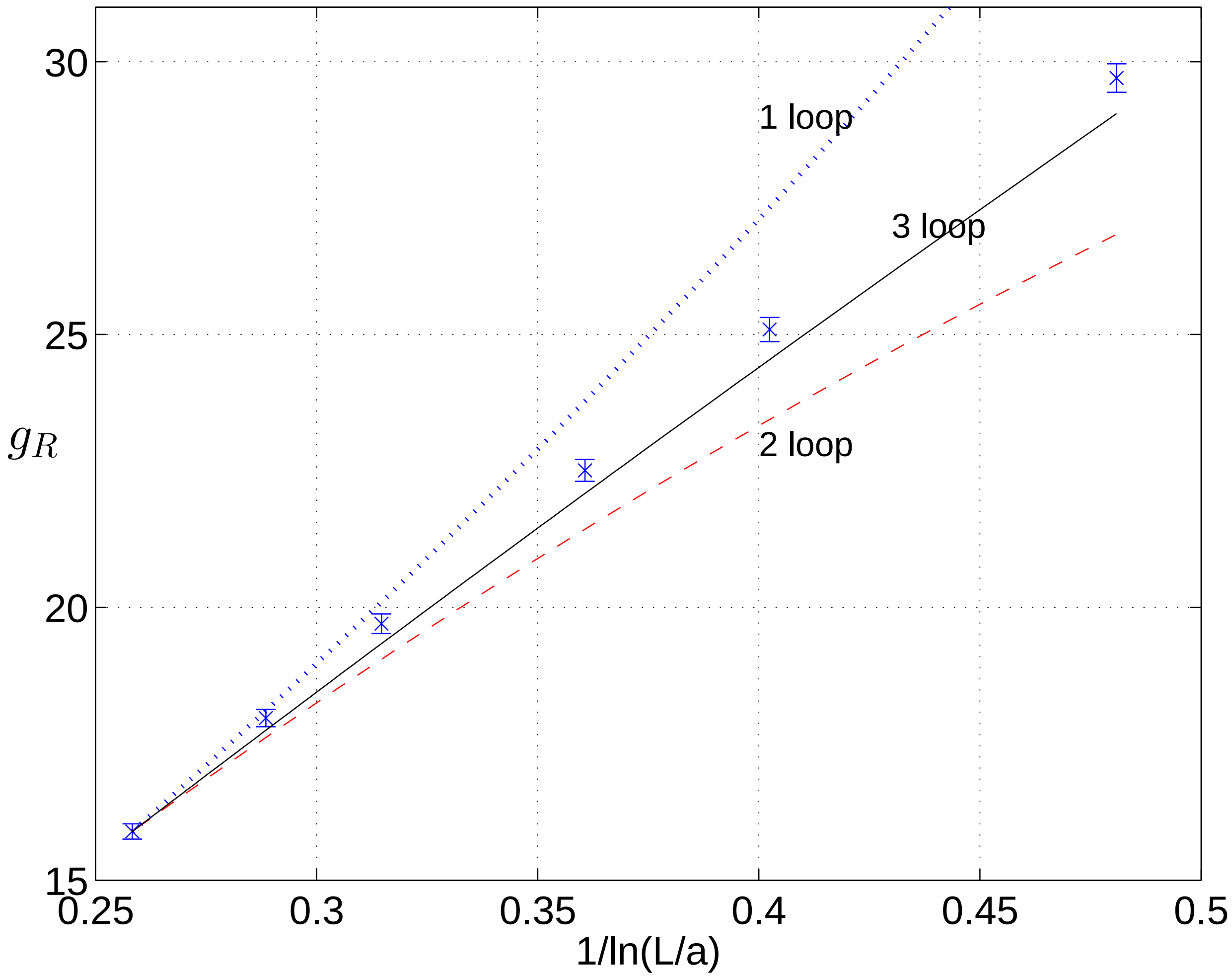
$$b_1 = \frac{3}{(4\pi)^2}, \quad b_2 = -\frac{17}{3(4\pi)^4}, \quad b_{3,z^2=\infty} = \frac{14.715616}{(4\pi)^6}, \quad [b_{3,z^2=10}\ \text{not (yet) known}]$$

ONCE PT is applicable $\Rightarrow$ triviality:

$$g_R(L/a) \simeq \frac{1}{b_1 \ln(L/a)}$$

Simulation results





- data $L/a = 64, 48, ..., 8$ at $z^2 = 10$ $g_R = \frac{3m_R^2}{v_R^2} = \frac{3z^2}{w^2}$
  (master thesis Johannes Siefert, HU 2013, additional data by Tomasz Korzec)
- under way (→proceedings): more data and $b_{3,z^2=10}$
- Conclusion: triviality supported also in broken phase FSS scheme