

# Update on $V_{ub}$ from $B \rightarrow \tau \nu$

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(B2)



**ALPHA**  
Collaboration

**FlaviA**  
net



March 20<sup>th</sup>  
17<sup>th</sup> Meeting of SFB/TR9, Karlsruhe

# Motivation

Couplings of flavor-changing *weak interactions*:

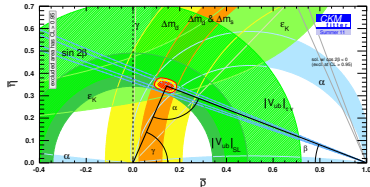
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

processes with  $b \rightarrow u$  transitions

- Inclusive  $B \rightarrow X_u \ell \nu$   
optical theorem and heavy quark expansion

Lattice input

- Exclusive  $B \rightarrow \pi \ell \nu$   
hadronic formfactor  $f_+(q^2)$
- Leptonic  $B \rightarrow \tau \nu$   
hadronic decay constant  $f_B$

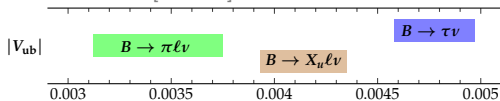


$V_{ub}$  puzzle

+

$(\mathcal{B}(B \rightarrow \tau \nu), \sin(2\beta))$  discrepancy

Summer 2011: [PDG'10]



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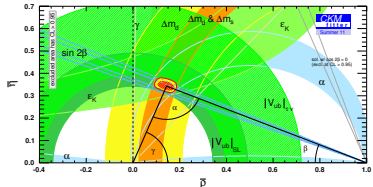
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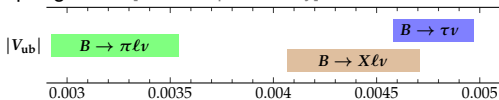
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Spring 2012: [PDG'12, preliminary]



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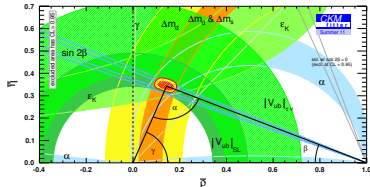
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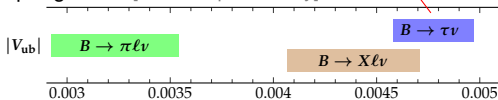
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precision??

$V_{ub}$  puzzle  
+  
( $\mathcal{B}(B \rightarrow \tau \nu), \sin(2\beta)$ ) discrepancy

Spring 2012: [PDG'12, preliminary]



# On the lattice

a variety of interfering **systematic errors** have to be controlled:

- **finite volume**  $T \times L^3$ :

exponentially small in  $m_\pi L$

- **finite lattice spacing**  $a$ :

need **various lattice spacings**, (no. of  $a$ )  $\geq 3$ , to take **CL** ( $a \rightarrow 0$ )

- **discretized action**:

**universality** allows to choose different gauge & fermion action,  
with different advantages & shortcomings

- **quark masses**:

- no. of dynamical (sea) quarks:

$$N_f = 0, 2, 2+1, 2+1+1$$

- **light quarks**:

$$m_\pi^{\text{exp}} \lesssim m_\pi \lesssim (250 - 500)\text{MeV}$$

need **chiral extrapolation**

- **heavy quarks**:

$$(am_b \sim O(1)) \not\ll 1$$

relativistic treatment of b-quark in large volume not feasible

$\hookrightarrow$  use of effective theories, NRQCD, ..., **HQET**

# HQET on the lattice

**HQET:** effective theory of QCD, expansion in powers of  $1/m_b$

HQET on the lattice:

operator mixing induces power divergences in  $a^{-1}$   
 $\Rightarrow$  subtractions need to be performed NP'ly

Our approach:

NP matching of HQET and QCD in small physical volume



- b-quark can be simulated relativistically ( $L \sim 0.4\text{fm}$ )
- running coupling & mass known NP'ly  
 $\Rightarrow$  contact to large volume physics
- power divergences subtracted NP'ly (at fixed  $a$ )  
 $\Rightarrow$  NP parameters of effective theory guarantee renormalizability
- only **one input parameter**,  $m_B^{\text{exp}}$ , to setup effective theory  
 $\Rightarrow$  to static ( $n = 0$ ) or next-to-leading ( $n = 1$ ) order

# Dynamical fermion simulations

criteria for subsequent data analysis:

- FV effects small by construction

$$Lm_\pi \geq 4.0$$

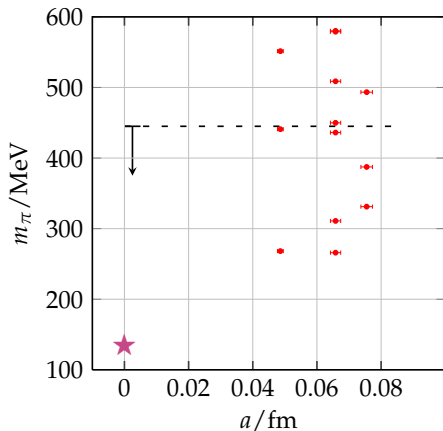
- data for chiral extrapolation uses

$$(250 \lesssim m_\pi \lesssim 400 - 450) \text{ MeV}$$

- lattice spacings

$$(0.048, 0.065, 0.075 < 0.1) \text{ fm}$$

CLS  
based



7 simulations fulfill our current criteria

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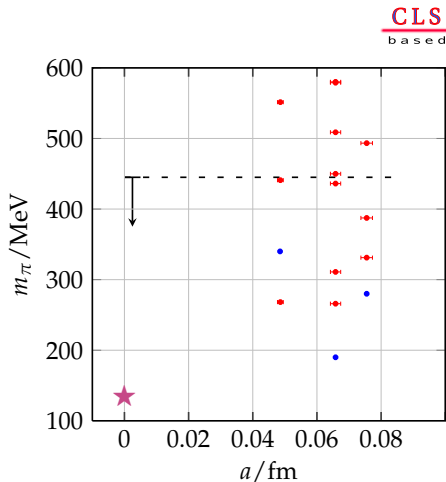
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+ 3 more by end of this year

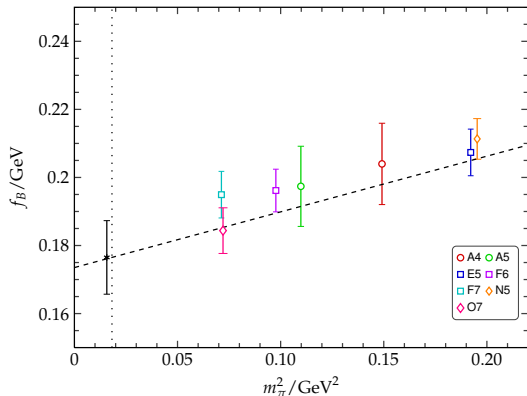




# The B-meson decay constant $f_B$

extrapolation to the physical point  $f_B \equiv \lim_{(m_\pi, a) \rightarrow (m_\pi^{\text{exp}}, 0)} f_B(m_\pi, a)$  through

$$f_B(m_\pi, a) = b + cm_\pi^2 + da^2 \quad (\text{LO})$$



■ no mixed term  $(am_\pi)^2 \lesssim 0.02$

■  $f_B = 176(11)(5)_a$

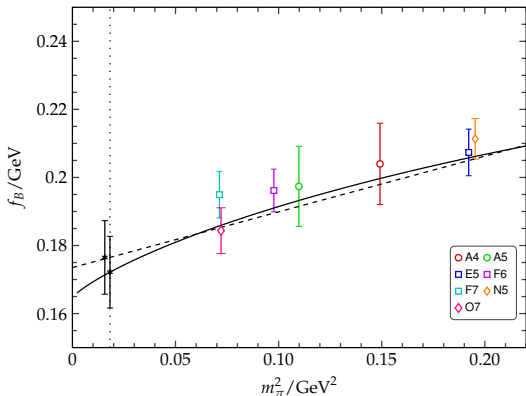
LO

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- $f_B = 176(11)(5)_a$  LO

- $f_B = 172(11)(5)_a$  HM $\chi$ PT

$$f_\pi = f_\pi^{\text{exp}}, \hat{g} = 0.51(2)$$

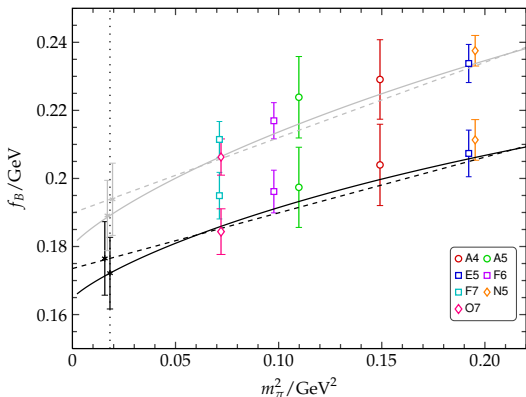
[PoS-Lat'10:BulavaETAL]

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[PoS-Lat'10:BulavaETAL]

- static theory

$$f_B = 194(11)(5)_a \quad \text{LO}$$

$$f_B = 189(11)(5)_a \quad \text{HM}\chi\text{PT}$$

# Our estimate of $V_{ub}$

$$\mathcal{B}_{\text{SM}}(B \rightarrow \tau\nu) = f_B^2 |V_{ub}|^2 \frac{G_F^2 m_B \tau_B}{8\pi} m_\tau^2 \left[ 1 - \frac{m_\tau^2}{m_B^2} \right]$$

using PDG values and

$$f_B = 174(11)(5)_a(2)_\chi \text{ MeV}, \quad \mathcal{B}_{\text{SM}}(B \rightarrow \tau\nu)_{\text{CKMfit}} = (7.57^{+0.98}_{-0.61}) \times 10^{-5}$$

$$|V_{ub}|_{\text{lept.}} = (5.57^{+0.65}_{-0.59}) \times 10^{-3}$$

our determination

$$|V_{ub}|_{\text{lept.}} = (5.00^{+0.37}_{-0.27}) \times 10^{-3}$$

$$f_B = 194(7) \text{ MeV} [\text{Lat}'11]$$

our current value slightly increases the tension

⇒ experiment and theory have to improve further

# Summary & outlook

- possible sources of tension in  $|V_{ub}|$ :  
lattice input  $\leftrightarrow$  exp. measurements  $\leftrightarrow$  new physics
- precise  $N_f = 2$  determination of  $f_B$  in the continuum limit:

$$f_B = 174(11)(5)_a(2)_\chi \text{ MeV}$$

improvement possible in near future

- HQET observables from first principles at NLO in  $1/m_b$ , renormalized NP'ly ✓
- systematic errors seem to be well controlled ✓
- more observables from NP'ly renormalized HQET:  
 $m_b(\overline{\text{MS}}) = 4.23(13)(3)_a(6)_z \text{ GeV}$ ,  $f_+(q^2)$ ,  $f_{B_s}$ , ...
- only truncation error  $O((\Lambda/m_b)^2)$  remain (but usually negligible)

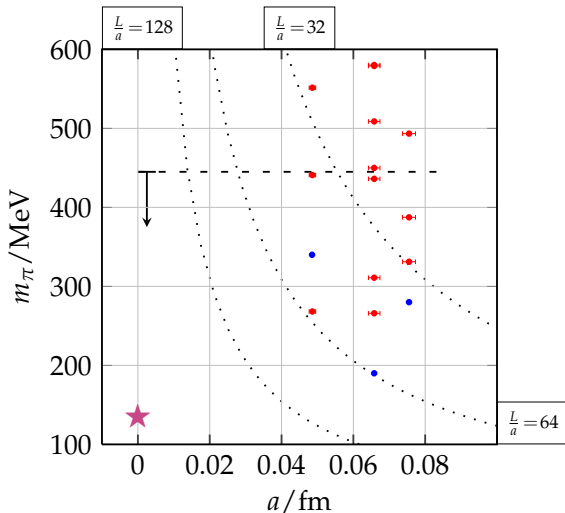
watch out for **FLAG-2** = Flavor Lattice Averaging Group (phase 2):

FLAG-2 (EU+Japan+US) will review light- and *heavy-quark* related quantities (end 2012)

# backup slides

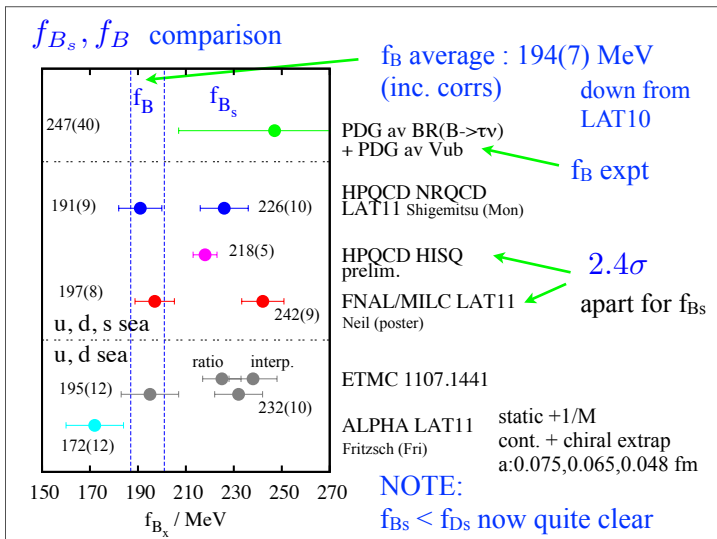
$$V_4 = L^3 \times T, T = 2L$$

(dotted curves at  $Lm_\pi \equiv 4$ )



# Results for $f_B$ @Lattice'11

Ch. Davies



Saturday, 16 July 2011