Final results on the strange quark mass and the Lambada parameter at $N_{\mathrm{f}} = 2$?

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talk mainly based on [arXiv:1205.5380]

Project B2

The 18[th] Meeting of SFB/TR9
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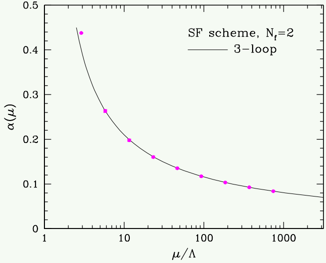
General setup

Non-perturbative Renormalization

Ingredient: Schrödinger functional as intermediate renormalization scheme

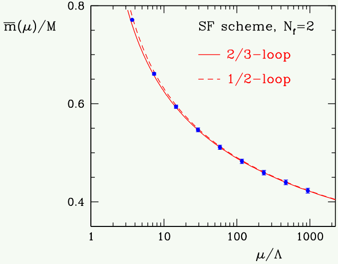
- massless, finite volume renorm. scheme in the continuum
- IR regulator on the lattice (Dirichlet b.c. in time) $\Rightarrow m = 0$ possible
- NP definition of a running coupling $\Rightarrow \bar{g}^2(\mu)$, w/ box size $L = 1/\mu$
- $N_f = 2$: QCD running coupling [ALPHA'04] and mass [ALPHA'05] known
  through finite size scaling technique

NP running coupling:



$\hookrightarrow$ Λ-parameter, low energy scale $\mu \equiv 1/L_1$

NP running mass:



$\hookrightarrow$ RGI quark masses M_i

Dynamical fermion simulations

e.g.: O7 ensemble, $64^3 \times 128$, $m_\pi \sim 270\,\mathrm{MeV}$

Lattice framework:

- plaquette gauge action
- mass-degenerate doublet of non-perturbatively improved Wilson fermions

Our criteria:

- FV effects negligible due to

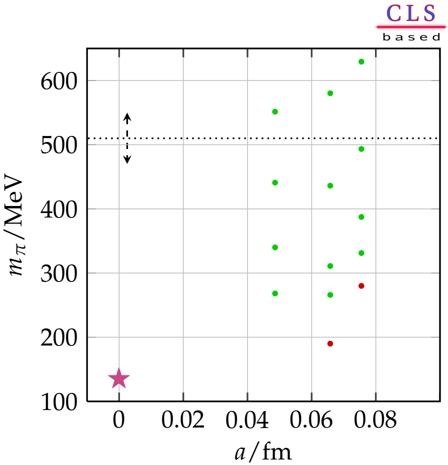
$$\boxed{Lm_\pi \geq 4.0}$$

- data for chiral extrapolation uses

$$\boxed{m_\pi \lesssim 500\,\mathrm{MeV}}$$

- three lattice spacings

$$\boxed{< 0.08\,\mathrm{fm}}$$



Goal: controlled extrapol. to physical point ★

Two strategies for chiral extrapolations

Setting the scale

Standard procedure, still room for improvements though

- calibrate lattice spacing a through dimensionful reference quantity Q:

$$a^{-1}[\mathrm{MeV}] = \frac{Q\big|_{\mathrm{exp}}[\mathrm{MeV}]}{[aQ]_{\mathrm{latt}}}\,, \quad Q \in \{f_\pi, f_\mathrm{K}, m_\mathrm{N}, m_\pi, \ldots\}$$

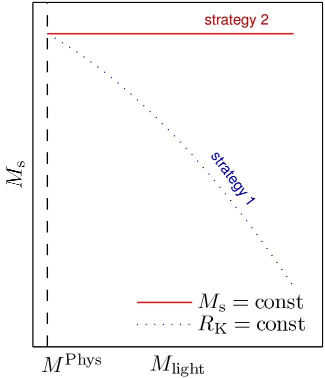
  choose well behaved quantity Q according to

  - experimentally available input
  - reasonable signal-to-noise ratio
  - well-controlled and understood chiral behaviour
  - mild cut-off effects
  - . . .

Our choice: kaon decay constant f_K

- milder chiral extrapolation compared to f_π
- strange quark is still quenched, i.e., no contribution to sea
- better control over systematic errors from chiral extrapolation (2 strategies)

Two chiral extrapolations



Strategy 1

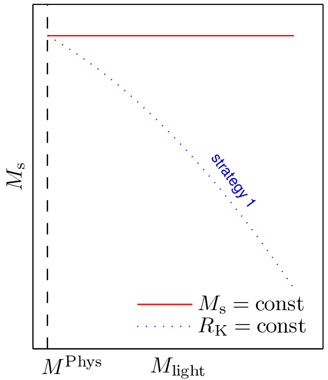
fix ratio R_{K}:

$$\left.\frac{m_{\mathrm{K}}^2(m_{\mathrm{sea}}, m_{\mathrm{s}})}{f_{\mathrm{K}}^2(m_{\mathrm{sea}}, m_{\mathrm{s}})}\right|_{\mathrm{lat}} \overset{!}{=} \frac{m_{\mathrm{K,phys}}^2}{f_{\mathrm{K,phys}}^2}$$

- trajectory with $m_{\mathrm{K}} \approx m_{\mathrm{K,phys}}$ and thus

$$\overline{m}_{\mathrm{s}} + \overline{m}_{\mathrm{light}} \approx \text{const} + \mathrm{O}(\overline{m}^2)$$

- systematic expansion in $m_\pi^2, m_{\mathrm{K}}^2 \leq m_{\mathrm{K,phys}}^2$



Strategy 1, Strategy 2

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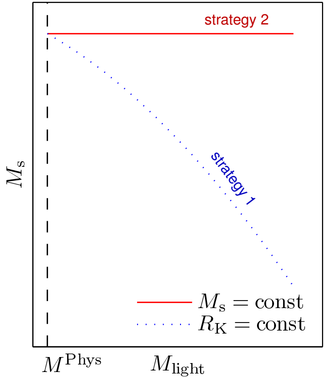
fix strange quark's PCAC mass:

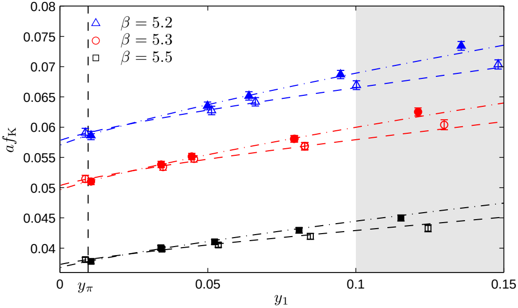
$$am_{\mathrm{s}}(m_{\mathrm{sea}}) \stackrel{!}{=} \text{const}$$



Chiral extrapolation to physical light quark mass

Strategy 1: △○□ NLO PQ-SU(3) ChPT

Strategy 2: ▲●■ NLO SU(2) ChPT



- global fit
- expansion variable:

$$y_1 \equiv \frac{m_\pi^2(m_{\rm sea})}{8\pi^2 f_K^2(m_{\rm sea}, m_s)} \leq 0.1$$

- $O(y^2)$ small

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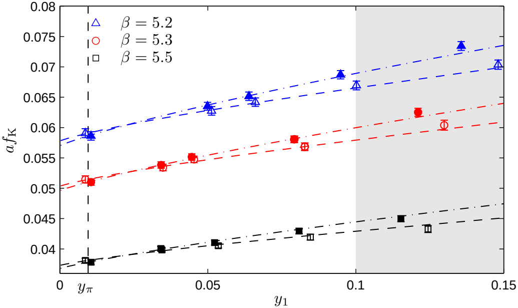

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- $\mathrm{O}(y^2)$ small

$$\Rightarrow \quad a = \left.\frac{[af_{\mathrm{K}}(y_1)]}{f_{\mathrm{K,phys}}}\right|_{y_1 = y_\pi}$$

with $f_{\mathrm{K,phys}} = 155\,\mathrm{MeV}$ [FLAG'11] $\left\{\begin{array}{l}\text{isospin symmetric limit \&}\\ \text{QED effects removed}\end{array}\right.$

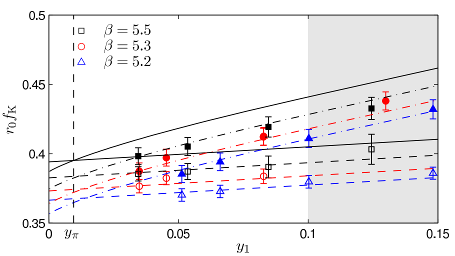
- systematic expansion includes M, $M \cdot \ln M$, a^2 [no $M \cdot a^2$, but checked]
- agreement also with simple linear extrapolation [i.e., not $M \cdot \ln M$]

The scale parameter r_0

from static potential $V(r)$ by solving

$$r^2 F(r)\Big|_{r=r_0} = 1.65\,, \qquad \text{with} \quad F(r) \equiv V'(r)\,, \qquad \Rightarrow \quad r_0/a$$

Chiral extrapolation of $r_0 f_K$:



- same expansion parameter:

$$y_1 \equiv \frac{m_\pi^2(m_{\text{sea}})}{8\pi^2 f_K^2(m_{\text{sea}}, m_s)}$$

- △○□ r_0, f_K at finite quark mass linear in y_1
- ▲●■ r_0 in χ-limit, f_K as before includes leading log

- both (independent) extrapolations in very good agreement
- smallness of $y_1 \cdot a^2$ checked

$\Rightarrow r_0 = 0.503(10)\ \text{fm}$

RESULTS
Λ, $M_{\rm s}$

Master formula: [ALPHA'05]

$$\frac{\Lambda^{(2)}_{\overline{\mathrm{MS}}}}{f_\mathrm{K}} = \frac{1}{[f_\mathrm{K} L_1]_\mathrm{cont}} \times [L_1 \Lambda^{(2)}_\mathrm{SF}]_\mathrm{cont} \times \frac{\Lambda^{(2)}_{\overline{\mathrm{MS}}}}{\Lambda^{(2)}_\mathrm{SF}}$$

Renorm. scale set through SF coupling:

- $\bar{g}^2(L_1) = 4.484, \qquad \mu_1 = 1/L_1$

Exact relation between schemes:

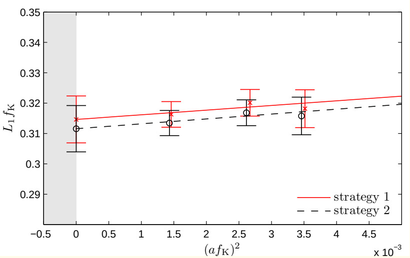
- $\Lambda^{(2)}_{\overline{\mathrm{MS}}} / \Lambda^{(2)}_\mathrm{SF} = 2.382035(3)$

Non-perturbative running coupling:

- $L_1 \Lambda^{(2)}_\mathrm{SF} = 0.264(15)$

Missing piece:

- $[f_\mathrm{K} L_1]_\mathrm{cont}$



The Λ parameter of $N_\mathrm{f} = 2$ QCD

Master formula: [ALPHA'05]

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using result from strategy 1

$$L_1 f_\mathrm{K} = 0.315(8)(2)$$

$$\Downarrow$$

$$\Lambda^{(2)}_\mathrm{SF} / f_\mathrm{K} = 0.84(6)$$

$$\Downarrow$$

$$\boxed{\Lambda^{(2)}_{\overline{\mathrm{MS}}} = 310(20)\,\mathrm{MeV}}$$

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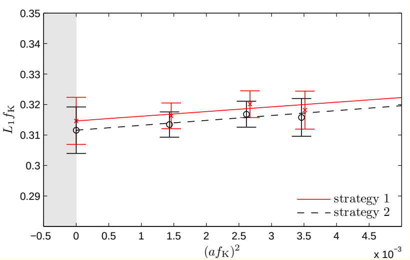

The strange quark mass in $N_\mathrm{f} = 2$ QCD

RGI strange quark mass:

$$M_\mathrm{s} = Z_\mathrm{M} m_\mathrm{s} = \frac{M}{\overline{m}(\mu_1)} \times \overline{m}_\mathrm{s}(\mu_1)$$
$$= \frac{M}{\overline{m}(\mu_1)} \times \frac{Z_\mathrm{A}}{Z_\mathrm{P}(\mu_1)} \times m_\mathrm{s}$$

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- $\bar{g}^2(L_1) = 4.484, \qquad \mu_1 = 1/L_1$

Non-perturbative running mass: [ALPHA'05]

- $M/\overline{m}(\mu_1) = 1.308(16)$

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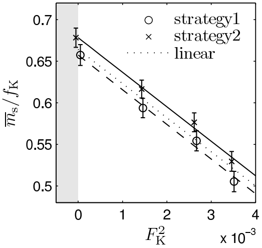


Strategy 1:

- only combination $\overline{m}_\mathrm{s} + \hat{\overline{m}}$ directly accessible
- remove average light quark mass $\hat{\overline{m}}$
  $\rightsquigarrow$ additional systematic uncertainty
- combination of LEC α_4 & α_6 from constrained global fit

Strategy 2: (*conceptually preferred*)

- no additional LEC's involved
- $\overline{m}_\mathrm{s}/f_\mathrm{K}$ = 0.678(12)(5)

$$M_\mathrm{s} = 138(3)(1)\,\mathrm{MeV}\,, \qquad \overline{m}_\mathrm{s}^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 102(3)(1)\,\mathrm{MeV}$$

Summary

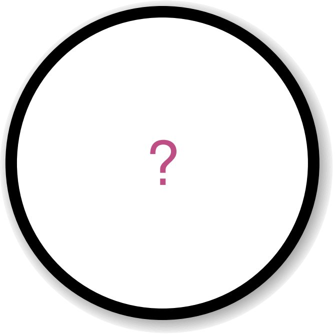
- *complete analysis of CLS based* $\mathrm{N_f} = 2$ *ensembles*;
  $a = (0.05 - 0.08)\,\mathrm{fm}$
- Conservative error estimates through autocorrelation analysis
- Scale setting with f_{K}
  - Two strategies for chiral extrapolation in agreement
  - simple linear extrapolation also agrees within errors
  - small cut-off effects

$\rightsquigarrow$ control over systematic effects in scale setting

- Main results

$$\boxed{\Lambda^{(2)}_{\overline{\mathrm{MS}}} = 310(20)\,\mathrm{MeV}\,, \qquad M_{\mathrm{s}} = 138(3)(1)\,\mathrm{MeV} \qquad \overline{m}_{\mathrm{s}}^{\overline{\mathrm{MS}}}\big(\mu = 2\,\mathrm{GeV}\big) = 102(3)(1)\,\mathrm{MeV}}$$

  - controlled reduction of statistical & systematic error achieved (since 2005)



Comparison to other collaborations

Note: two flavour results expressed in units of r_0 to make it unambigous

$r_0\Lambda^{(2)}_{\overline{\mathrm{MS}}}$	method	reference
$0.790(51)$	NP running coupling, PT matching @ $\sim 100\,\mathrm{GeV}$	[ALPHA 2012]
$0.72(5)$	ghost-gluon-v., PT matching @ $\sim 3\,\mathrm{GeV}$	[BlossierEtAl 2010]
$0.658(55)$	static potential, PT matching @ $\sim 1\,\mathrm{GeV}$	[ETMC 2011]
$0.59(2)(^{+4}_{-0})$	Adler functions, PT matching @ $\sim 1\,\mathrm{GeV}$	[JLQCD 2009]

Do some systematic errors not show up in standard criteria used?

Questions to be asked:

- impact of the matching scale ?
- different methods $\rightarrow$ different systematic errors; good estimates ?
- Quenching errors? presumably small from $N_\mathrm{f} = 2 \rightarrow 3$
- ...

THANK YOU FOR YOUR ATTENTION!

And special thanks to . . .

my colleagues @HU and @DESY-Zeuthen