Heavy quarks, the lattice and me

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- Motivation & Introduction
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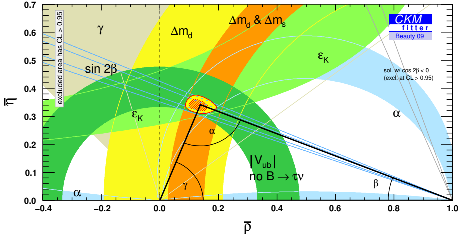
- RBC/UKQCD's heavy quark project (overview)

Content

- Why?
- I am in Berlin, the lattice capital of the world! [DeGrand]

Constraining the CKM unitarity triangle ⇝ hints for 'new physics'

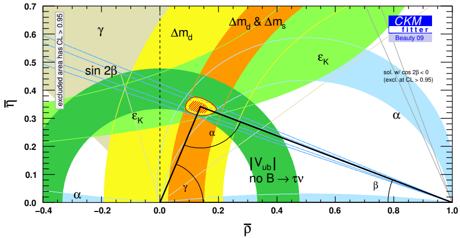
- to test standard model (QCD)



Constraining the CKM unitarity triangle $\rightsquigarrow$ hints for 'new physics'

- to test standard model (QCD)
- Δm_q's well known by EXP $\lesssim 1\%$

  [BABAR,Belle,CDF]



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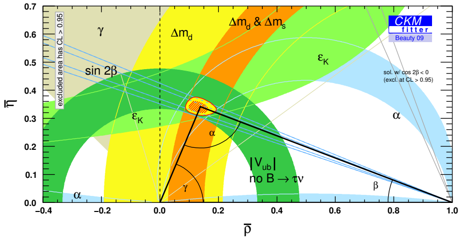

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- to test standard model (QCD)
- Δm_q's well known by EXP $\lesssim 1\%$
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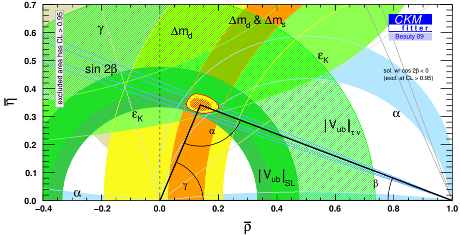
- apex of UT constrained by ratios like $\Delta m_s / \Delta m_d$
- dominant error in ξ $\sim 3\%$ (LQCD)



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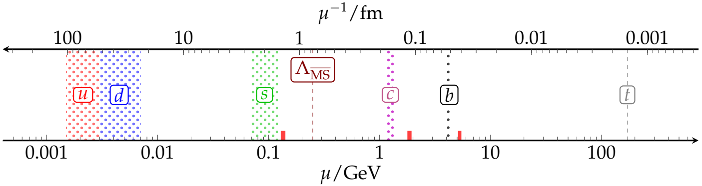
QCD, a multiple scale problem

spectrum, quark masses, bound states



measurable quantities:

$$m_\pi, \ldots, m_D, m_B$$

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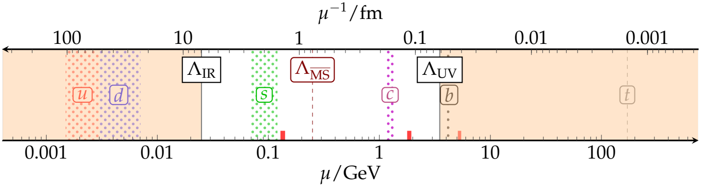

computer + LatticeQCD:

cutoff $L^{-1} \equiv \Lambda_{\text{IR}} \leq \mu \leq \Lambda_{\text{UV}} \equiv a^{-1}$

$$L^{-1} \ll m_\pi, \ldots, m_{\text{D}}, m_{\text{B}} \ll a^{-1}$$

$$O(\mathrm{e}^{-Lm_\pi}) \qquad am_{\text{D}} \lesssim 0.5$$

$$L \gtrsim 4/m_\pi \sim 6\text{fm} \qquad a \approx 0.05\text{fm}$$

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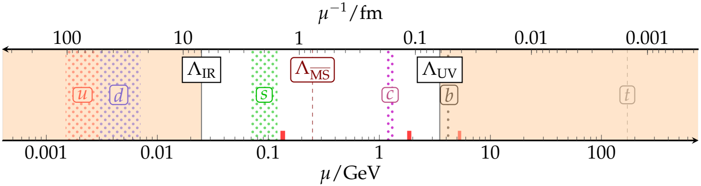

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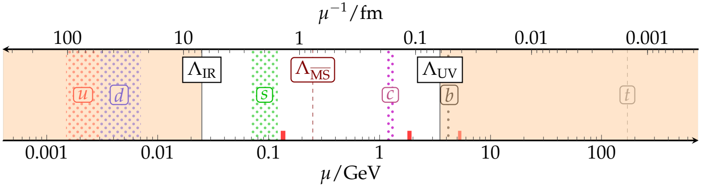

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Issue: $m_{\mathrm{b}}/m_{\mathrm{c}} \sim 4 - 5$ $\Rightarrow a \approx 0.01\mathrm{fm}$

Solution: use of effective theories; here :

Heavy Quark Effective Theory (HQET)

HQET

Heavy Quark Effective Theory

$$\mathcal{L}_{\rm HQET} = \overline{\psi}_{\rm h}\Big[\underbrace{D_0 + \delta m}_{\text{static limit (LO)}} - \omega_{\rm kin}\,\mathbf{D}^2 - \omega_{\rm spin}\,\boldsymbol{\sigma}\mathbf{B}\Big]\psi_{\rm h} + \ldots\,, \qquad \left.\begin{matrix}\omega_{\rm kin}\\ \omega_{\rm spin}\end{matrix}\right\} \sim \frac{1}{2m}$$

- systematic $1/m$-expansion of QCD (valid for $m \gg \Lambda_{\rm QCD}$), renormalizable at fixed order $1/m^n$ (also in static limit?)
- HQET is an effective theory of QCD **after** Matching of effective parameters $\{m, \omega_{\rm spin}, \ldots\} \Leftrightarrow \{$QCD Parameter$\}$
  quark mass dependence of QCD in B-meson region $\Rightarrow$ m_B
- **here:** consider HQ-expansion of QCD in $1/z \equiv 1/(LM) \ll 1$; compute (*non-perturbatively*) z-dependence of QCD observables and compare it to the behaviour expected in HQET
  - ? are systematic sources of errors under control
  - ? are interpolations to the b-quark scale using charm physics and the static limit reasonable
    $\rightsquigarrow$ important for current large volume simulations: e.g. calculation of $F_{\rm B}$
  - ? dynamical effects of internal quark loops ($N_{\rm f}=0 \to N_{\rm f}=2$ simulations)

[Eichten;Isgur,Wise;Georgi]

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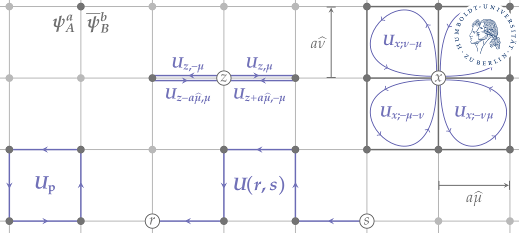
Lattice QCD

The lattice as a regulator

Minkowski continuum QFT

$\downarrow$

discretised, Euclidean QFT



Quarks: fermions $\psi(x), \overline{\psi}(x)$ defined on lattice sites $x_\mu = an_\mu$

Gluons: gauge links $U_\mu(x) \sim e^{aA_\mu(x)} \in SU(3)$ between x and $x + a\hat{\mu}$

Field strength: plaquette var. $U_{\mu\nu}(x) \equiv U_\mu(x) U_\nu(x + a\hat{\mu}) U^\dagger_\mu(x + a\hat{\nu}) U^\dagger_\nu(x)$

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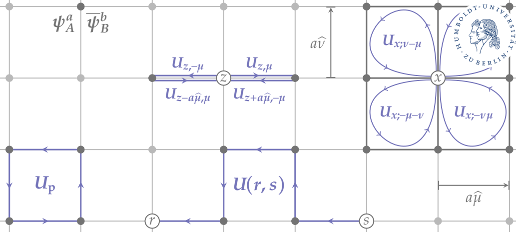

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no unique discretisation

Wilson action:

$$\begin{aligned}
\mathcal{S}_\mathrm{G}[A] &\longleftarrow & \mathcal{S}_\mathrm{p}[U] &\equiv \beta \textstyle\sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{3} \mathrm{Re\,Tr} U_{\mu\nu}(x)\right) \\
\mathcal{S}_\mathrm{F}[\psi, \overline{\psi}] &\longleftarrow & \mathcal{S}_\mathrm{W}[\psi, \overline{\psi}] &\equiv \textstyle\sum_x \overline{\psi}_x \left\{ \frac{1}{2} \left(\gamma_\mu (\nabla^*_\mu + \nabla_\mu) - a \nabla^*_\mu \nabla_\mu \right) + m_0 \right\} \psi_x \\
\{g_0^2, m_{0,f}\} &\longleftarrow & \{\beta, \kappa_f\} &\equiv \{6/g_0^2, (2am_{0,f} + 8)^{-1}\}
\end{aligned}$$

avoid 'doubling problem' with explicit χSB term for $a \neq 0$!

Lattice regularisation . . .

Consequences of explicit χSB on the lattice (Wilson action):

$\Rightarrow$ additive mass renormalization: $$m_{\mathrm{R}} = Z_{\mathrm{m}} m_{\mathrm{q}} = Z_{\mathrm{m}}(m_0 - m_{\mathrm{c}})$$

$\rightsquigarrow$ *critical mass* $m_{\mathrm{c}}(g_0)$

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$\Rightarrow$ modified bare coupling and mass for massless RS:

$$\widetilde{g}_0^2 \equiv g_0^2\left[1 + b_{\mathrm{g}}(g_0)am_{\mathrm{q}}\right]\,, \quad \widetilde{m}_{\mathrm{q}} \equiv m_{\mathrm{q}}\left[1 + b_{\mathrm{m}}(g_0)am_{\mathrm{q}}\right]$$

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⇒ more complex renormalization pattern of parameters & multiplicatively renormalizable observables ϕ:

$$g_{\rm R}^2 = \tilde{g}_0^2\, Z_{\rm g}(\tilde{g}_0^2, a\mu)\ , \quad m_{\rm R} = \tilde{m}_{\rm q}\, Z_{\rm m}(\tilde{g}_0^2, a\mu)$$

$$\phi_{\rm R} = Z_\phi(\tilde{g}_0^2, a\mu)\Big[1 + b_\phi(g_0) a m_{\rm q}\Big]\phi_{\rm I}\ , \quad \phi_{\rm I} = \phi + c_\phi(g_0) a \mathcal{O}_\phi^{{\rm D}(\phi)+1}$$

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$\rightsquigarrow$ *critical mass* $m_\mathrm{c}(g_0)$ **fine-tuning**

$\Rightarrow$ reduced convergence properties: $S_\mathrm{W} \to S_\mathrm{QCD} + \mathrm{O}(a^2)$

$\rightsquigarrow$ SW term for *on-shell* $\mathrm{O}(a)$ *improvement* with additional parameter $c_\mathrm{sw}(g_0)$ **NP known**

$\Rightarrow$ modified bare coupling and mass for massless RS:

important

$$\tilde{g}_0^2 \equiv g_0^2\left[1 + \cancel{b_\mathrm{g}(g_0) a} m_\mathrm{q}\right] , \quad \tilde{m}_\mathrm{q} \equiv m_\mathrm{q}\left[1 + b_\mathrm{m}(g_0) a m_\mathrm{q}\right]$$

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ideally: non-perturbative estimation

Consider: perturbative mass renormalization in lattice HQET

$$m_{\mathrm{b,R}} = Z(m_{\mathrm{b}}^{\mathrm{bare}} + \delta m)\,, \quad \delta m = \frac{1}{a}(f_1 g_0^2 + f_2 g_0^4 + \ldots)$$
$$\sim \Lambda_{\mathrm{QCD}} \mathrm{e}^{1/(2b_0 g_0^2)}(f_1 g_0^2 + \ldots) \quad \text{for } g_0 \to 0$$

$\Rightarrow$ HQET perturbatively not renormalizable on the lattice!

NP renormalization procedure necessary!

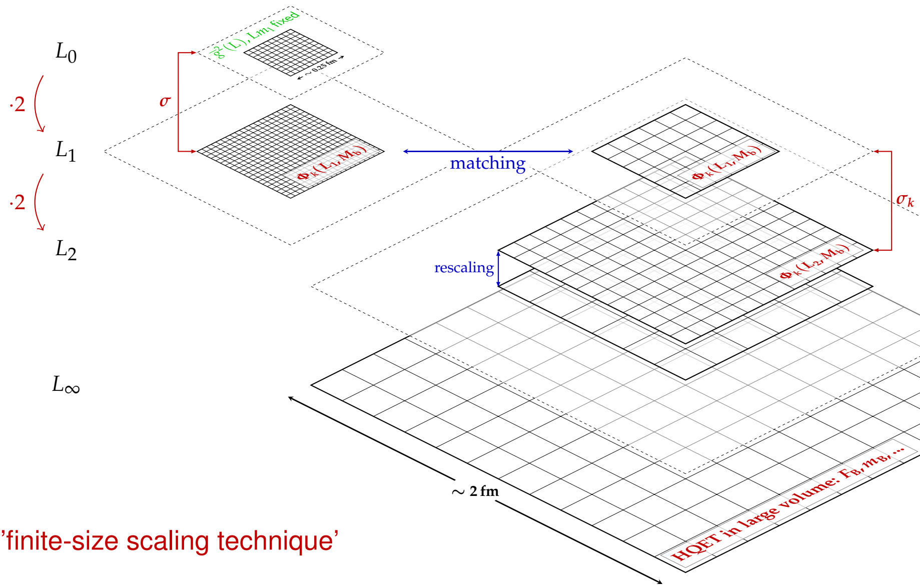
Matching of HQET parameters:

either: match to physical measurements directly; $\rightsquigarrow$ loss of predictability

or: match to QCD in finite (small) physical volume

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



'finite-size scaling technique'

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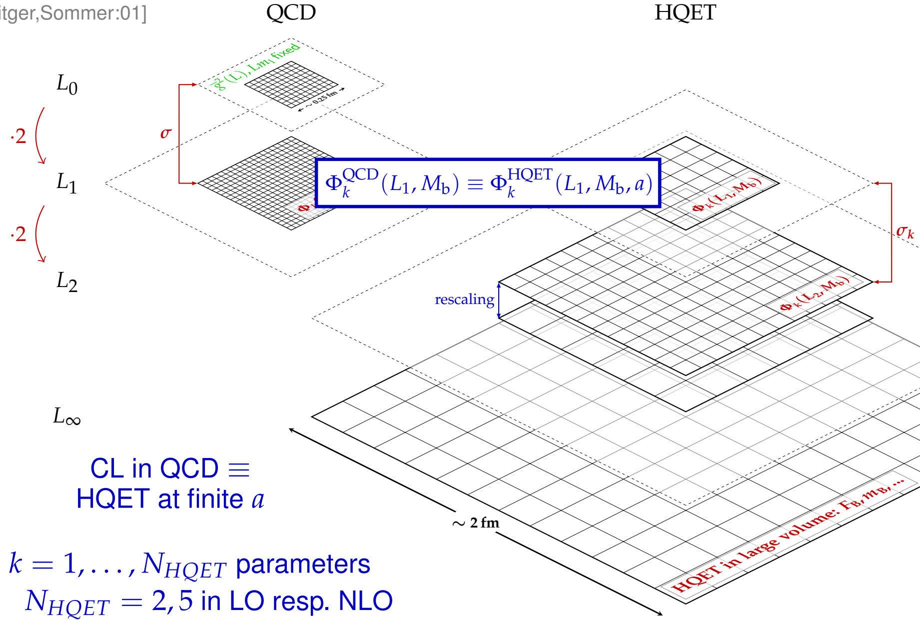


CL in QCD $\equiv$
HQET at finite a

$\Rightarrow k = 1, \ldots, N_{HQET}$ parameters

$N_{HQET} = 2, 5$ in LO resp. NLO

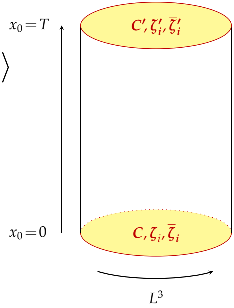
The Schrödinger functional (SF)

as (intermediate) finite volume renormalization scheme

- Euclidean partition function

$$\mathcal{Z} \equiv \int_{T\times L^3} \mathcal{D}[U,\overline{\psi},\psi]\, \mathrm{e}^{-S[U,\overline{\psi},\psi]} = \langle 0|\, \mathrm{e}^{-TH} P|0\rangle$$

with *periodic BC in L^3* and *Dirichlet BC in T*



- fermion fields periodic in L^3 up to a phase θ:

$$\psi(x+\widehat{k}L) = \mathrm{e}^{i\theta}\psi(x)$$
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$\Rightarrow$ SF parameters: $\{L, T/L, \theta, N_{\mathrm{f}} = 2\}$

- 'infrared save' $\rightsquigarrow$ simulate massless sea-quark doubletts ($m_{\mathrm{l}} \equiv 0$) with a variant of the HMC algorithm

  $\Rightarrow$ mass-independent renormalization scheme
- renormalization scale μ identified with box length L: $\boxed{\mu = 1/L}$
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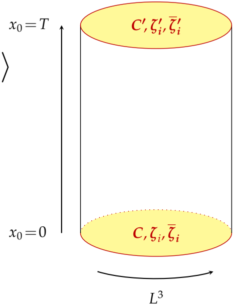

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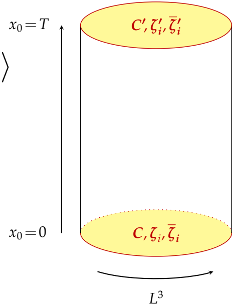

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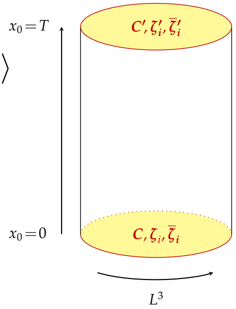

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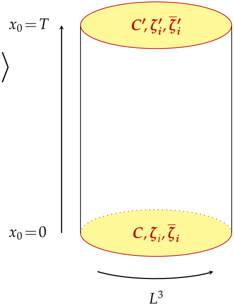


  $\Rightarrow$ SF parameters: $\{L, T/L, \theta, N_\mathrm{f} = 2\}$

- 'infrared save' $\rightsquigarrow$ simulate massless sea-quark doubletts ($m_\mathrm{l} \equiv 0$) with a variant of the HMC algorithm

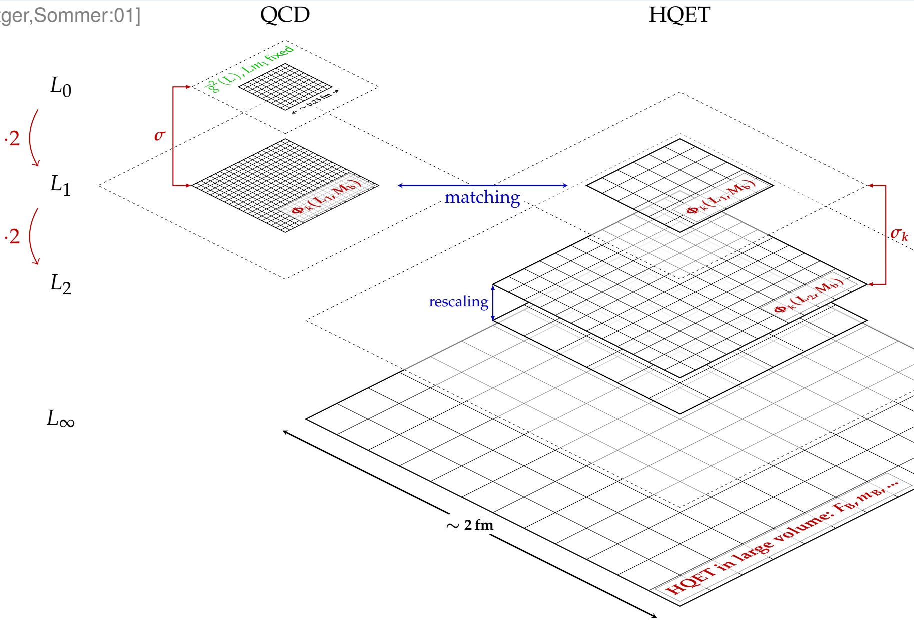
  $\Rightarrow$ mass-independent renormalization scheme

- renormalization scale μ identified with box length L: $\boxed{\mu = 1/L}$

- heavy-light meson correlation functions with a heavy (quenched) valence quark ($m_\mathrm{h} \neq 0$)

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



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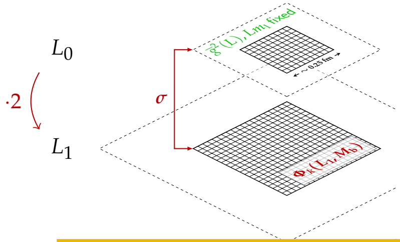
[Heitger,Sommer:01]

QCD



- $L_0/a \in \{10, 12, 16, 20\}$, $L_0 \approx 0.25\mathrm{fm}$
- $L_1/a \in \{20, 24, 32, 40\}$, $L_1 \approx 0.5\mathrm{fm}$

$\rightsquigarrow a \approx (0.025 - 0.012)\mathrm{fm}$

Definition: line of constant physics ('light' sector)

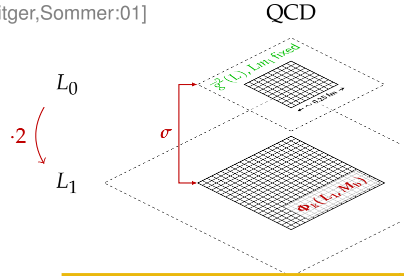
$$\boxed{\bar{g}^2(L_0) \equiv 2.989\,, \quad L_0 m_l \equiv 0} \qquad \Rightarrow (\beta, \kappa_l, L/a), \text{ fine-tuning}$$

running coupl. $\bar{g}^2(\mu)$ & mass $\overline{m}(\mu)$ NP known in the SF ($\mu = 1/L$)! [DellaMorte etal:'05]

$$\boxed{\bar{g}^2(L_1) \equiv \sigma(2.989) = 4.484(48)\,, \quad L_1 m_l \equiv 0}$$

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



- $L_0/a \in \{10, 12, 16, 20\}$, $L_0 \approx 0.25\mathrm{fm}$
- compute $b_\mathrm{m}, b_\mathrm{A} - b_\mathrm{P}$ and Z

- $L_1/a \in \{20, 24, 32, 40\}$, $L_1 \approx 0.5\mathrm{fm}$
- heavy quark mass dependence of QCD observ.

$\rightsquigarrow a \approx (0.025 - 0.012)\mathrm{fm}$

Definition: line of constant physics ('heavy' sector)

fix dimensionless heavy RGI quark mass *in the continuum*:

$$z \equiv LM = [L/a] h_{L_0} Z_\mathrm{m} (1 + b_\mathrm{m} a m_\mathrm{q}) a m_\mathrm{q} \,, \qquad Z_\mathrm{m} \equiv \frac{Z(g_0) Z_\mathrm{A}(g_0)}{Z_\mathrm{P}(g_0, \mu_0)}$$

$$a m_\mathrm{q} \equiv \tfrac{1}{2}\left[\kappa_\mathrm{h}^{-1} - \kappa_\mathrm{c}^{-1}(g_0)\right] \,, \text{ but we need } \kappa_\mathrm{h}(z, L/a)$$

- $L/a, h_{L_0}, Z_\mathrm{A}, Z_\mathrm{P}$ NP known
- estimate b_m, Z non-perturbatively

Parameters for an on-shell $\mathrm{O}(a)$ improved theory

Estimation of $b_\mathrm{m}, b_\mathrm{A} - b_\mathrm{P}$ and Z in L_0 with $T = 3L_0/2$

current quark mass from PCAC relation: $$m_{ij} = \frac{\langle (\tilde{\partial}_0 A_0^{ij}) P^{ji} \rangle}{2\langle P^{ij} P^{ji} \rangle}$$

with $A_0^{ij} = \overline{\psi}_i \gamma_0 \gamma_5 \psi_j$ and $P^{ij} = \overline{\psi}_i \gamma_5 \psi_j$

subtracted quark mass: $$am_{\mathrm{q},i} = am_{0,i} - am_\mathrm{c}(g_0)$$

$$\frac{2(2m_{12} - m_{11} - m_{22})}{(m_{11} - m_{22})(am_{\mathrm{q},1} - am_{\mathrm{q},2})} = b_\mathrm{A} - b_\mathrm{P} + O(am_{\mathrm{q},1} + am_{\mathrm{q},2})$$

$$\frac{4(m_{12} - m_{33})}{(m_{11} - m_{22})(am_{\mathrm{q},1} - am_{\mathrm{q},2})} = b_\mathrm{m} \qquad + O(am_{\mathrm{q},1} + am_{\mathrm{q},2})$$

$$\frac{m_{11} - m_{22}}{m_{\mathrm{q},1} - m_{\mathrm{q},2}} + [(b_\mathrm{A} - b_\mathrm{P}) - b_\mathrm{m}](am_{11} + am_{22}) = Z \qquad + O(a^2)$$

by construction $am_{\mathrm{q},3} \equiv \dfrac{am_{\mathrm{q},1} + am_{\mathrm{q},2}}{2}$, but free to choose $am_{\mathrm{q},1} \neq am_{\mathrm{q},2}$

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$\boxed{am_{\mathrm{q},1} \equiv 0 \text{ for dynamical quarks}}$

remaining $\mathrm{O}(a)$ ambiguity

for testing purpose:

set1: $L_0 m_{11} \approx 0\,, \quad L_0 m_{22} \approx 0.5 \quad (\star)$

set2: $L_0 m_{11} \approx 0\,, \quad L_0 m_{22} \approx 2.5$

$\Rightarrow$ 'improvement conditions' are fixed!

$\Updownarrow$

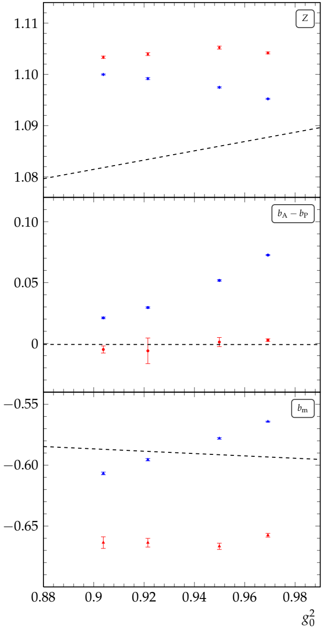
well-defined parametrisation in g_0^2

use improved lattice derivatives

$$\tilde{\partial}_0 \to \tilde{\partial}_0 \left(1 - \tfrac{1}{6} a^2 \partial_0^* \partial_0\right)$$

to compute $m_{ij} \rightsquigarrow O(g_0^2 a^2, a^4)$

- smooth dependence on g_0^2
- deviations from 1-loop PT
- quantitatively: mass-dep. cutoff-effects larger for set2



Fixing the heavy quark mass $z = L_1 M$

$L_1/a \in \{20, 24, 32, 40\}, \quad L_1 \approx 0.5\text{fm}$

$$z = L_1 M = L_1 Z_{\rm M}(1 + b_{\rm m}\, am_{\rm q,h})am_{\rm q,h}\,, \qquad Z_{\rm M} = h(L_0)\frac{Z Z_{\rm A}}{Z_{\rm P}(L_0)}$$

with $b_{\rm m}$ and Z obtained from set1

$$\kappa_{\rm h}(z, L_1) = \left[\frac{1}{\kappa_{\rm c}} - \frac{1}{b_{\rm m}}\left(1 - \sqrt{1 + z\cdot\frac{4b_{\rm m}}{[L_1/a]Z_{\rm M}}}\right)\right]^{-1}$$

choose $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $z_{\rm c} \approx 4.13$, $z_{\rm b} \approx 17.5$

universal coefficient $h(L_0) \equiv \dfrac{M}{\overline{m}(\mu_0)} = 1.521(14)$ (running of the mass)

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choose $z \in \{4, 6, 7, 9$

universal coefficient $h(L$

	z	20	24	32	40
κ_s	0	0.1360536	0.1359104	0.1355210	0.1351922
κ_1	4	0.1327278	0.1332121	0.1335643	0.1336510
κ_2	6	0.1309498	0.1317899	0.1325495	0.1328583
κ_3	7	0.1300226	0.1310561	0.1320315	0.1324556
κ_4	9	0.1280709	0.1295337	0.1309715	0.1316366
κ_5	11	0.1259456	0.1279214	0.1298749	0.1307974
κ_6	13	0.1235550	0.1261898	0.1287348	0.1299352
κ_7	15	*0.1206872*	0.1242898	0.1275422	0.1290468
κ_8	18	—	*0.1208919*	0.1256259	0.1276559
κ_9	21	—	*0.1151926*	0.1234913	0.1261774

QCD observables in small volume

...and their asymptotics for large heavy quark masses $z = LM$

leading asymptotics

effective meson masses: $\begin{cases} \Gamma_{\mathrm{PS}}(L,z) = -\widetilde{\partial}_0 \ln\left[\langle\Omega|A_0|B\rangle\right] \\ \Gamma_{\mathrm{V}}(L,z) = -\widetilde{\partial}_0 \ln\left[\langle\Omega|V_k|B^*\rangle\right] \end{cases}$

HQET-QCD conversion functions

$$m_B^{\mathrm{av}} \leftarrow L\Gamma_{\mathrm{av}} \equiv \frac{1}{4}[L\Gamma_{\mathrm{PS}} + 3L\Gamma_{\mathrm{V}}] \xrightarrow{z\to\infty} C_{\mathrm{mass}}(z) \cdot z \cdot \left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

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HQET-QCD conversion functions $C_X(z)$

Definition (example: heavy light axial current $A_\mu(x) = \overline{\psi}_\mathrm{h}(x)\gamma_\mu\gamma_5\psi_\mathrm{l}(x)$)

corresponding matrix element: $\Phi(m_\mathrm{B}) = \langle \beta, b | A_\mu(x) | \alpha \rangle$

$m_\mathrm{l} = 0$, only one large scale: m_B $\rightsquigarrow$ m_B-dependence of Φ?

RGE in a massless scheme: $\mu \frac{\partial \overline{g}}{\partial \mu} = \beta(\overline{g}), \quad \frac{\mu}{\overline{m}} \frac{\partial \overline{m}}{\partial \mu} = \tau(\overline{g})$

scale is fixed: $\mu = m_\star \equiv \overline{m}(m_\star), \quad g_\star \equiv \overline{g}(m_\star)$

$\Rightarrow$ mass dependence given by RGE of Φ:

$$\frac{m_\star}{\Phi} \frac{\partial \Phi}{\partial m_\star} \equiv \gamma^\Phi_\mathrm{match}(g_\star) \quad \overset{m_\star \to \infty}{\sim} \quad -g_\star^2 \cdot \gamma_0 + \mathrm{O}(g_\star^4)$$

factorization in effective theory:

$$\gamma^\Phi_\mathrm{match}(g_\star) = \gamma_\mathrm{match}(g_\star) + \mathrm{O}(\Lambda/m_\star)$$

is scheme dependent $\to$ use RGI's: Λ, M

HQET-QCD conversion functions $C_X(z)$

mass dependence in QCD

$$\Lambda = m_\star \exp\left\{-\int^{g_\star} \frac{\mathrm{d}g}{\beta(g)}\right\}, \qquad M = m_\star \exp\left\{-\int^{g_\star} \mathrm{d}g \frac{\tau(g)}{\beta(g)}\right\},$$

thus

$$\frac{M}{\Phi}\frac{\partial \Phi}{\partial M} \equiv \gamma_{\mathrm{PS}}^{(M)}(M/\Lambda) + \mathrm{O}(\Lambda/M)$$

$$\gamma_{\mathrm{PS}}^{(M)}(M/\Lambda) \equiv \frac{\gamma_{\mathrm{match}}(M/\Lambda)}{1-\tau(M/\Lambda)}$$

gives

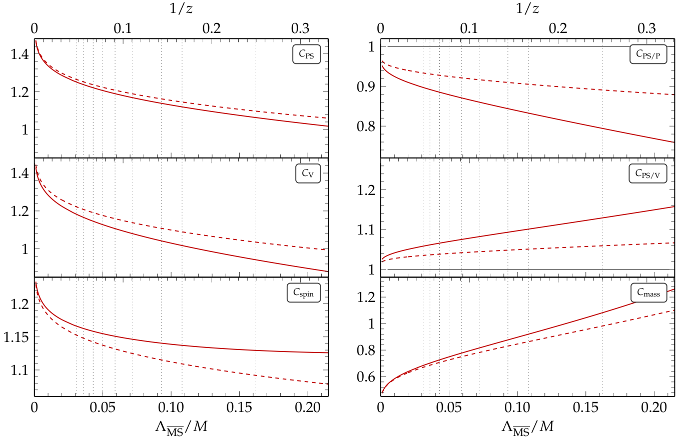
$$\Phi(M,\Lambda) = C_{\mathrm{PS}}\left(\frac{M}{\Lambda}\right)\Phi_{\mathrm{RGI}} + \mathrm{O}\left(\frac{\Lambda}{M}\right), \quad C_{\mathrm{PS}} = \exp\left\{\int^{g_\star(M/\Lambda)} \mathrm{d}g \frac{\gamma_{\mathrm{PS}}^{(M)}(g)}{\beta(g)}\right\}$$

matrix element Φ_{RGI} unambiguous, computable in effective theory, mass independent

HQET-QCD conversion functions $C_X(z)$

$z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$

these PT conv. functions only appear in some of our testobs.

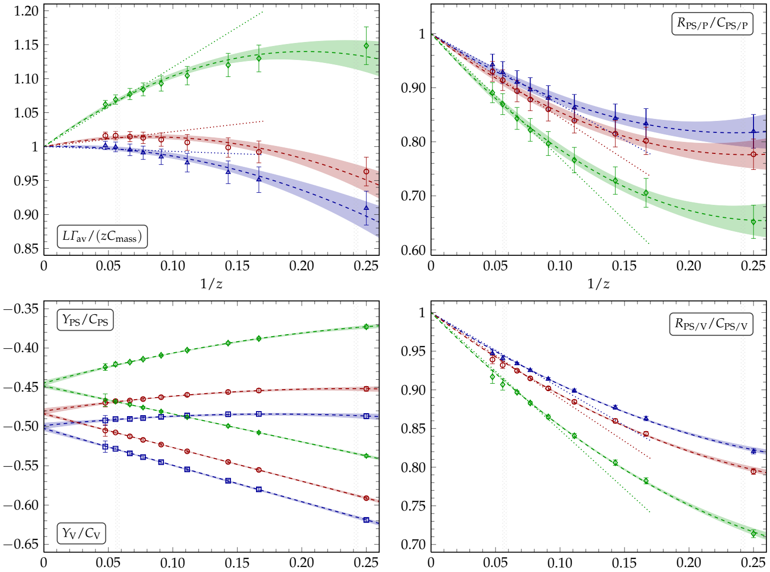


usual problems: how to estimate systematic error

PT expansion of coupling reliable enough

Results, part 2

mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $\theta \in \{0, 0.5, 1\}$

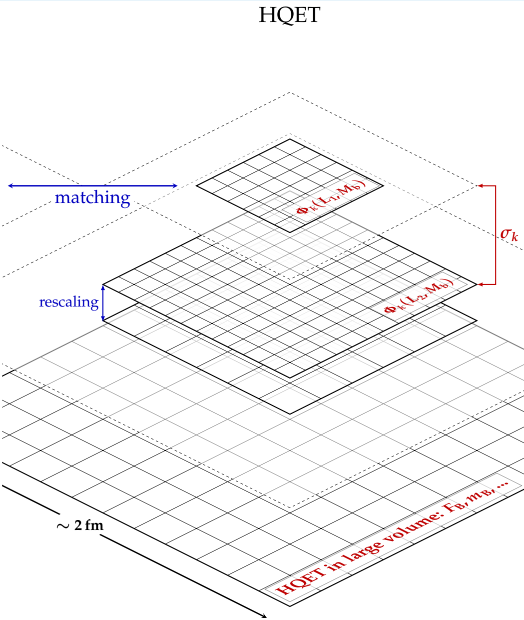


HQET part of our simulations

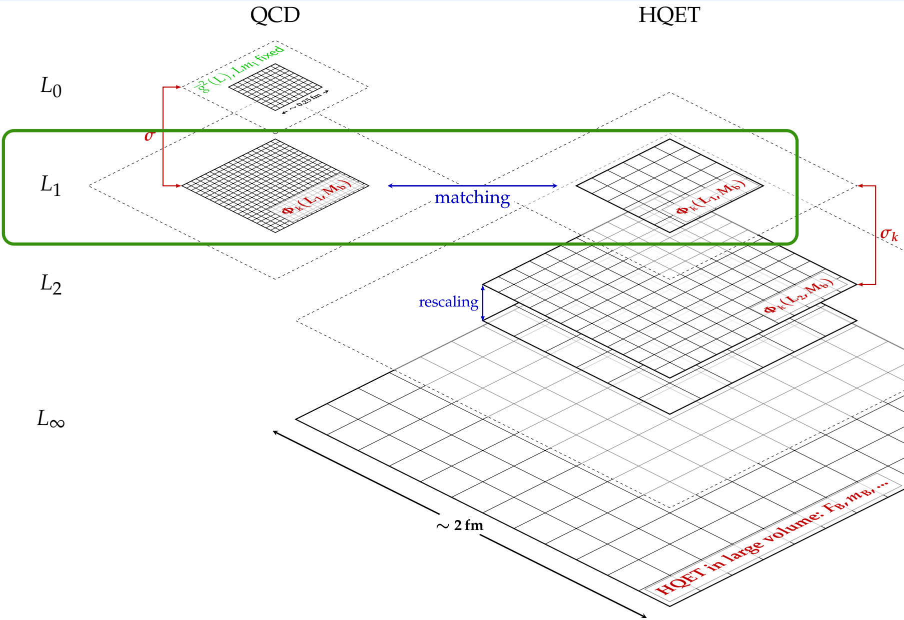
- $L_1 \approx 0.5\text{fm}$
  $L_1/a \in \{6, 8, 10, 12, 16\}$
  with $T = L$ and $T = L/2$
  production & measurements done
  target: 8000 configs each

- $L_2 \approx 1\text{fm}$
  $L_2/a \in \{12, 16, 20, 24, 32\}$
  with $T = L$ and $T = L/2$
  in progress

- $L_\infty \approx 2\text{fm}$
  CLS configurations
  (Coordinate Lattice Simulation)
  issue: dynamical fermion updating of topological sectors
  [Schäfer:PoS-Lattice'09]



HQET part of our simulations



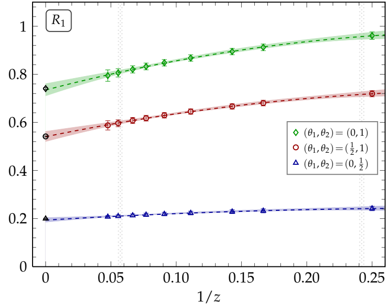
Results without any perturbative uncertainty

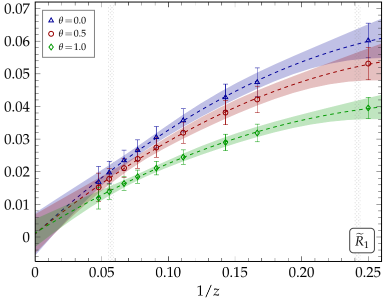
mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$

in QCD:

$$R_1 = \frac{1}{4} \ln \left(\frac{f_1(\theta_1) k_1(\theta_1)}{f_1(\theta_2) k_1(\theta_2)} \right) , \qquad \widetilde{R_1} = \frac{3}{4} \ln \left(\frac{f_1(\theta)}{k_1(\theta)} \right)$$

their HQET expansion contains no conversion functions at LO





free quadratic fits in $1/z$ (static limit at $1/z = 0$)

computations in HQET & QCD absolutely independent and purely NP!

Summary I

ALPHA Collaboration B-physics project

- setup of $O(a)$ improved lattice theory in small volume ✓
- strong evidence that lattice HQET is renormalizable and works (that's non-trivial) ✓
- confidence in existence of HQET CL in static approximation ✓
- full $N_{\rm f} = 2$ matching calculations in progress
- physical applications are waiting

Summary I

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high precision lattice HQET possible after years of development

- lattice HQET (LO&NLO) [Eichten&Hill:88-90]
- fully NP HQET-QCD matching procedure [Heitger,Sommer:04]
- suitable choice of HQET action [DellaMorte,Shindler:05]
- 'all-to-all' propagators [Foley,Juge,O'Cais,Peardon,Ryan,Skullerud:05]
- GEVP method [Blossier,DellaMorte,vonHippel,Mendes,Sommer:08]

RBC/UKQCD's heavy quark project

RBC/UKQCD heavy quark project

Actions:

- gauge field configurations with 2+1 light dynamical DWF on $L/a \in \{24, 32\}$ lattices, $T/a = 64$, $a^{-1} \in \{2.28, 1.73\}\mathrm{GeV}$
- heavy valence quarks with Relativistic Heavy Quark (RHQ) action

Projects:

- non-perturbative tuning of RHQ parameters (am_0, c_P, ζ) (for charm and bottom quark)
- B-Meson decay constant & $B_0 - \overline{B}_0$-mixing (allows to determine CKM matrix elements)
- determining the $D^\star D\pi$ and $B^\star B\pi$ effective couplings (appear in $\mathrm{HM}\chi\mathcal{L}$ and constrain chiral extrapolations)
- neutral B-Meson mixing & decay constant with static quarks

  [Witzel:Latt09,Aoki:Latt10]

Tools:

- USQCD software suites CPS and Chroma as well as UKhadron

RBC/UKQCD heavy quark project

field configurations with Iwasaki gauge action and 2+1 light dynamical DWF

- Domain Wall Fermion (DWF) action [Kaplan'92,Shamir'93]
  $(L_s/a = 16, aM_5 = 1.8, T/a = 64)$
  - 5dim formulation with approximate chiral symmetry
    $\rightsquigarrow$ simplified renormalization pattern
  - RI-MOM [MartinelliEtAl'95] and related renormalization schemes used
- Iwasaki gauge action [Iwasaki'83]
  - further reduces residual quark mass (and thus χSB) if combined with dynamical DWFs [AokiEtAl'04]
- some parameters and phys. scales:

L/a	am_l	m_π/MeV	am_s	L/fm	a^{-1}/GeV	a/fm
24	$0.005, 0.01, 0.02$	$331, 419, 558$	0.04	2.75	1.732(29)	~ 0.11
32	$0.004, 0.006, 0.008$	$307, 366, 418$	0.03	2.72	2.284(25)	~ 0.08

RBC/UKQCD heavy quark project

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almost equal

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common range

ok

RBC/UKQCD heavy quark project

heavy valence quarks with Relativistic Heavy Quark (RHQ) action

we use a variant of the Fermilab action [El-Khadra,Kronfeld,Mackenzie], the ...

RHQ action

$$S = \sum_{n,n'} \overline{\psi}_n \, \mathcal{K}_{n,n'} \, \psi_{n'} + \mathrm{O}\left[(a\Lambda)^2\right] \,,$$

$$\mathcal{K} = m_0 + \gamma_0 D_0 - \tfrac{a}{2} D_0^2 + \zeta \left[\boldsymbol{\gamma} \mathbf{D} - \tfrac{a}{2} \mathbf{D}^2 \right] + a c_P \tfrac{\mathrm{i}}{4} \sigma_{\mu\nu} F_{\mu\nu}$$

claim: accurate to all orders in $(am_{\mathrm{h}})^n$ [Christ,Li,Lin]

note: 'in contradiction' to previous approach, where $am_{\mathrm{h}} \ll 1$ was necessary

- explicitly breaks 4d hyper-cubic symmetry into 3+1 (alike HQET)
- 3 parameters: $\{am_0, \zeta(g_0^2, am_0), c_P(g_0^2, am_0)\}$
- non-perturbative parameter tuning required

- at fixed a, match theory in SU(2) χ-limit to physical spectrum (PDG), either by matching
  1. heavy-light system or
  2. heavy-heavy system

  i.e. $(B_s, B_s^*, \ldots)$ or $(\eta_b, \Upsilon, \ldots)$ to cover b-quark sector

- one needs 3 constraints at least (per heavy quark flavour)

- 2 heavy meson masses + relativistic dispersion relation $(E^2 = m^2 + \mathbf{p}^2)$
  $\Rightarrow$ ideally two real input parameters only
  - pseudo-scalar (m_P) + vector meson mass (m_V) or
  - avg. mass (m_{av}) + spin-splitting (Δm), with

$$m_{\mathrm{av}} = \frac{1}{4}(m_P + 3m_V), \quad \Delta m = m_V - m_P$$

(as motivated by HQET)

the procedure

1. start with sensible (physics motivated) guess for (am_0, ζ, c_P)
2. choose a reasonable step size for each direction in parameter space $(\Delta[am_0], \Delta[\zeta], \Delta[c_P])$
3. compute observables for all seven sets of parameters $(am_0, \zeta, c_P) \pm (0, \Delta[am_0], \Delta[\zeta], \Delta[c_P])$ and several momenta
4. perform χ^2-minimisation using lattice and continuum data
   $\Rightarrow$ resulting (am_0, ζ, c_P) depend on light sea quark mass am_l
   [if parameters outside region go to 1]
5. repeat this for several am_l
6. perform chiral extrapolation of (am_0, ζ, c_P)

done on 24^3 for charm [Li,Lin:Latt'07] and bottom [Li:Latt'08] sector

Ongoing discussion:

- one subtle point ignored in previous parameter estimation on 24^3: different systematic errors in describing hl/hh systems
  - $O\left[(a\Lambda)^2\right]$ in heavy-light systems
  - $O\left[(\alpha_s\, am_h)^2\right]$ in heavy-heavy systems
- charm sector: matching used hl + hh data
- bottom sector: here hh data only enters through disp. relation

$$E_{hh}^2(\mathbf{p}) = m_{hh}^2 + k_{hh} \cdot \mathbf{p}^2 \,, \qquad hh \in \{\eta_b, \Upsilon\}$$

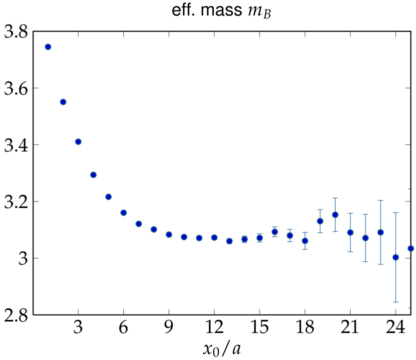
- thumb rule: better precision from $m_{\rm av}$, Δm

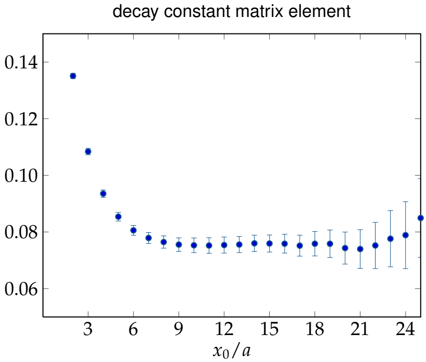
impact of different combinations is currently under investigation, studying stability, correlations, accuracy, ...

...to do high precision heavy-light physics.

of course we have to apply further techniques as well

- smearing propagator source and/or sink
  Gaussian smearing at src with 'radius' $\rho \in \{0.0, \quad , \quad \}$





Speaking of . . .

. . . high precision B-physics

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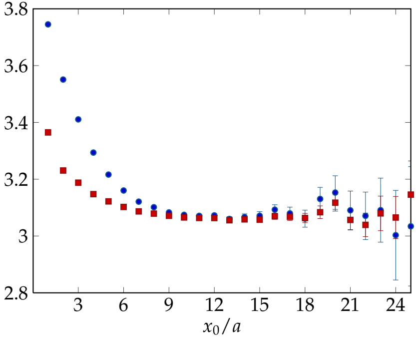
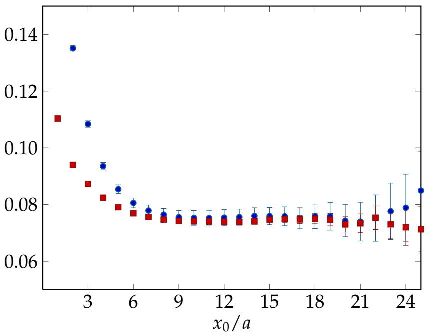
eff. mass m_B



decay constant matrix element



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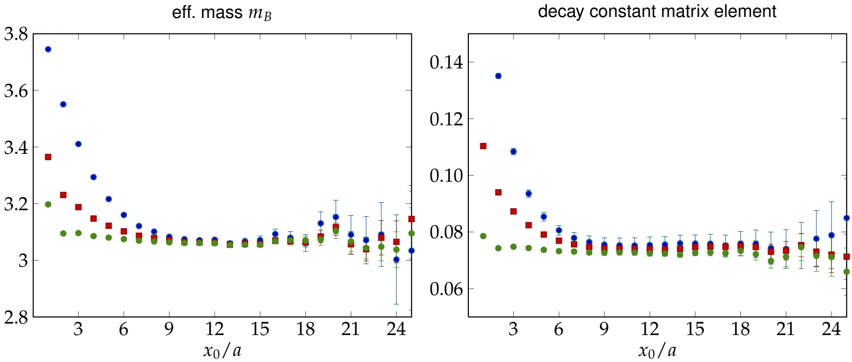

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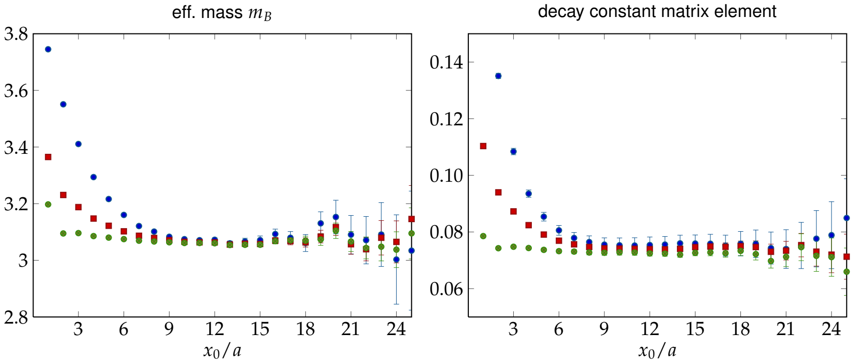

- stochastic sources, . . .
- any gain through 3d HYP-smearing ?

Conclusions II

RBC/UKQCD's heavy quark project . . .

- . . . is on a good way
- we spend more time to assure reliability of tuning method
  [light quark props stored on disk; producing RHQ props is cheap]
- so far the spectrum is reproduced quite well
- predictions can be made; using parameters from hs matching on 24^3:
  am_B in agreement with PDG value, stat. error of $O(0.1\%)$ [Witzel:Latt'10]
- we try to pin down possible systematic errors
- a lot of things still have to be studied . . .
  ($O(a)$ improvement of axial current)
- we are certain to reach the goal of doing high precision B-physics with relativistic heavy quarks

The End

thanks go to

- all collaborators I was able to work with during the past years
- I do not mentioned anybody explicitly here, because I am sure I would have forgotten somebody.
- the audience, you, for paying attention (at least for some minutes of my talk)

I am looking forward to the work that is going to come.