Precision B-physics from non-perturbative matching of Heavy Quark Effective Theory and QCD

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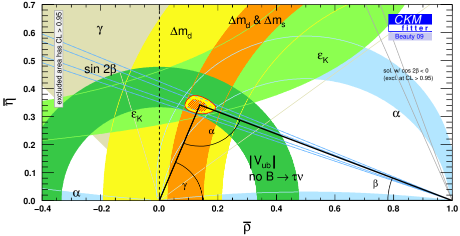
Constraining the CKM unitarity triangle ⇝ hints for 'new physics'

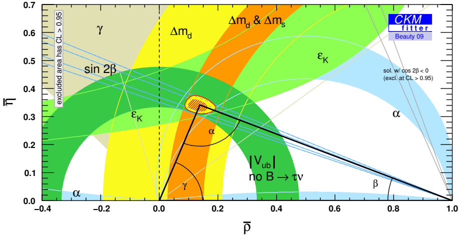
- to test standard model (QCD)



Constraining the CKM unitarity triangle $\leftrightsquigarrow$ hints for 'new physics'

- to test standard model (QCD)
- Δm_q's well known by EXP $\lesssim 1\%$

  [Cleo,BABAR,Belle,CDF,...,LHCb,...,SuperB*]



$$\Delta m_q = \frac{G_F^2 M_W^2}{6\pi^2} \eta_B S_0\left[\frac{m_t}{M_W}\right] \cdot m_{B_q} f_{B_q}^2 \widehat{B}_{B_q} |V_{tq} V_{tb}^*|^2$$

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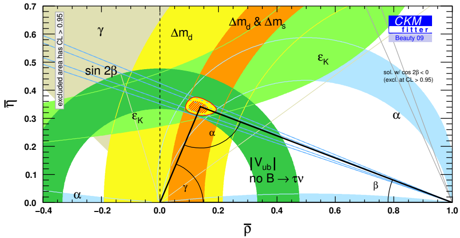
- apex of UT constrained by ratios like $\Delta m_s / \Delta m_d$



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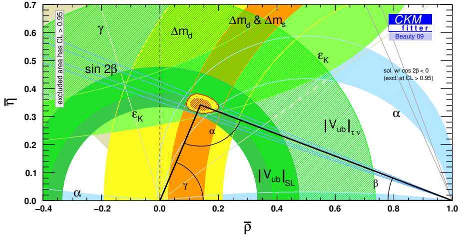
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \cdot \xi^2 \cdot \frac{|V_{ts}|^2}{|V_{td}|^2}$$

Motivation

Why B-physics?

Constraining the CKM unitarity triangle $\rightsquigarrow$ hints for 'new physics'

- to test standard model (QCD)
- Δm_q's well known by EXP $\lesssim 1\%$
  [Cleo,BABAR,Belle,CDF,. . . ,LHCb,. . . ,SuperB*]
- apex of UT constrained by ratios like $\Delta m_s / \Delta m_d$
- dominant error in $\xi \ \sim 3\%$ (LQCD)

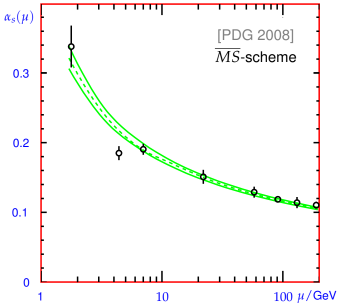


$$\Delta m_q = \frac{G_F^2 M_W^2}{6\pi^2} \eta_B S_0\left[\frac{m_t}{M_W}\right] \cdot m_{B_q} f_{B_q}^2 \widehat{B}_{B_q} |V_{tq} V_{tb}^*|^2$$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \cdot \xi^2 \cdot \frac{|V_{ts}|^2}{|V_{td}|^2} \qquad\qquad \boxed{\frac{f_{B_s}^2}{f_{B_d}^2} \frac{\widehat{B}_{B_s}}{\widehat{B}_{B_d}}}$$

$$\mathcal{L}_{\mathrm{QCD}}[g_0, m_f] = -\frac{1}{2g_0^2}\mathrm{Tr}\left\{F_{\mu\nu}F_{\mu\nu}\right\} + \sum_{f=u,d,s,\ldots} \overline{\psi}_f\left[\gamma_\mu(\partial_\mu + g_0 A_\mu) + m_{0,f}\right]\psi_f$$

- $N_{\mathrm{f}} + 1$ free (bare) parameters need renormalization to define theory
  $\Rightarrow$ scale-dependent effective coupling $\alpha_s(\mu)$ and masses $m_f(\mu)$
  due to virtual quarks & gluons ('anti-screening')



Quantum Chromodynamics (QCD)

Phenomenology

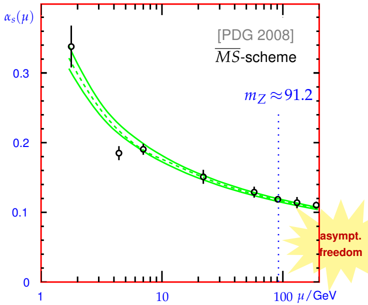
$$\mathcal{L}_{\rm QCD}[g_0,m_f] = -\frac{1}{2g_0^2}\mathrm{Tr}\,\{F_{\mu\nu}F_{\mu\nu}\} + \sum_{f=u,d,s,\ldots} \overline{\psi}_f\left[\gamma_\mu(\partial_\mu + g_0 A_\mu) + m_{0,f}\right]\psi_f$$

- $N_{\rm f}+1$ free (bare) parameters need renormalization to define theory $\Rightarrow$ scale-dependent effective coupling $\alpha_s(\mu)$ and masses $m_f(\mu)$ due to virtual quarks & gluons ('anti-screening')
- High-energy regime ($E \simeq 100\mathrm{GeV}$), QCD asymptotic free
  - weakly coupled quarks und gluons
  - $\alpha_s(m_Z) = 0.1176(20)$ means perturbation theory is applicable:

    $$\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} = 1 + \frac{1}{\pi}\alpha_s(q) + \ldots$$

  - $\alpha_s(q) = \frac{c}{\ln(q/\Lambda)} + \ldots$

    Λ: fundamental, non-perturbative scale (dimensional transmutation)



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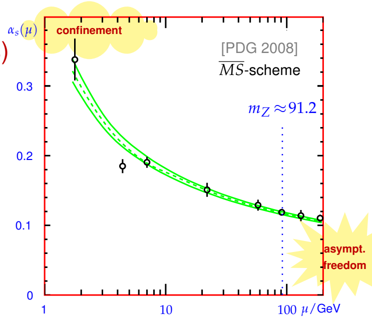
- $N_{\mathrm{f}}+1$ free (bare) parameters need renormalization to define theory $\Rightarrow$ scale-dependent effective coupling $\alpha_s(\mu)$ and masses $m_f(\mu)$ due to virtual quarks & gluons ('anti-screening')
- High-energy regime ($E \simeq 100\mathrm{GeV}$), QCD asymptotic free
- Low-energy regime ($E \simeq 250\mathrm{MeV}$), QCD strongly coupled confinement (hypothesis)
  - bound states of quarks & gluons:

    hadrons $\begin{cases} \pi, K, D, B, \ldots \\ \rho, K^*, D^*, B^*, \ldots \end{cases}$; $p, n, \ldots$
  - α_s large, perturbation theory fails
  - approach beyond PT needed to compute $m_\pi, m_{\mathrm{p}}, \ldots, F_\pi, F_{\mathrm{D}}, \ldots$



Lattice QCD

first principle calculations [Wilson 1974]

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Formulation of QCD on a space-time lattice with Euclidean metric

$$\underbrace{\begin{bmatrix} F_\pi \\ m_\pi \\ m_{\mathrm{K}} \\ m_D \\ m_B \end{bmatrix}}_{\text{experiment}} \quad \overset{\mathcal{L}_{\mathrm{QCD}}[g_0, m_f]}{\longleftrightarrow} \quad \underbrace{\begin{bmatrix} \Lambda_{\mathrm{QCD}} \\ M_u, M_d \\ M_s \\ M_c \\ M_b \end{bmatrix}}_{\text{QCD parameters (RGI)}} + \underbrace{\begin{bmatrix} F_{\mathrm{D}} \\ F_{\mathrm{B}} \\ B_{\mathrm{K}}, B_{\mathrm{B}} \\ \xi \\ \ldots \end{bmatrix}}_{\text{predictions}}$$

- preserving *gauge invariance, locality, unitarity*
- applicable at all scales
- obstacles: continuum limit, renormalization, stat. and syst. errors

$\Rightarrow$ tests of QCD; complement exp./predict exp'ly inaccessible quantitities

Lattice QCD

first principle calculations [Wilson 1974]

- minimal length scale $a \sim \Lambda_{\mathrm{UV}}^{-1}$ represents momentum cutoff
- finite volume $L^3 \times L$
- Lattice action $S[U,\overline{\psi},\psi] = S_G[U] + S_F[U,\overline{\psi},\psi]$ with

$$S_G = \frac{1}{g_0^2}\sum_p \mathrm{Tr}\{1 - U(p)\}\ , \qquad S_F = a^4 \sum_x \overline{\psi}(x) D[U] \psi(x)$$

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- Functional integral representation of expectation values:

$$\mathcal{Z} = \int \mathcal{D}[U]\mathcal{D}[\overline{\psi},\psi]\mathrm{e}^{-S[U,\overline{\psi},\psi]} = \int \mathcal{D}[U] \prod_{\mathrm{f}} \det(\not{D} + m_{\mathrm{f}}) \mathrm{e}^{-S_G[U]}$$

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \prod_{x,\mu} \mathrm{d}U_\mu(x)\ O \prod_{\mathrm{f}} \det(\not{D} + m_{\mathrm{f}}) \mathrm{e}^{-S_G[U]} \quad \text{: thermal avg.}$$

$$= \frac{1}{N}\sum_{n=1}^{N} O_n \pm \Delta O \qquad \text{: through numerical simulations}$$

Stochastic evaluation with (hybrid) *Monte Carlo methods (HMC)*

$\rightsquigarrow N_{\mathrm{f}} = 2, +1, +1$ dynamical fermion simulation ($\det \equiv 1$: quenched)

Lattice QCD

there is no unique lattice discretisation!

usually Wilson gauge action or variants thereof (Iwasaki gauge action) +

Fermionic discretisations: (increasing costs from top to bottom)

- Staggered (Kogut-Susskind) fermions [Kogut+Susskind 1975]
- Wilson fermions [Wilson 1974/75]
- Overlap/Domain Wall fermions [Kaplan '92, Furman+Shamir '96, Neuberger '98]
- 'Perfect'/Fixed point actions [Hasenfratz+Niedermaier '93/'98]

3+4 = Ginsparg-Wilson fermions [Ginsparg+Wilson 1982, Lüscher 1998]

Cost figure for $N_{\mathrm{f}} = 2$ QCD with $O(a)$ improved Wilson quarks:
# of operations [in TFlops $\times$ year] for an ensemble of 100 gauge fields

$$\sim 0.05 \left[\frac{20\mathrm{MeV}}{m}\right]^{1} \left[\frac{L}{3\mathrm{fm}}\right]^{5} \left[\frac{0.1\mathrm{fm}}{a}\right]^{6}$$

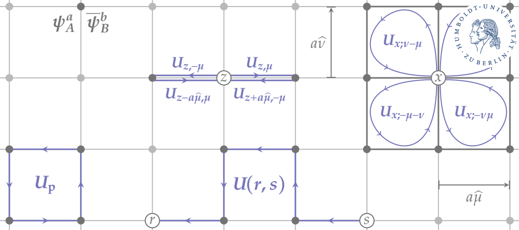
[Giusti, Lattice'06]

every one is appealing in its own, and each formulation has another weakness

Lattice QCD

The lattice as a regulator

Minkowski continuum QFT
$\downarrow$
discretised, Euclidean QFT



Quarks: fermions $\psi(x), \overline{\psi}(x)$ defined on lattice sites $x_\mu = an_\mu$

Gluons: gauge links $U_\mu(x) \sim e^{aA_\mu(x)} \in \mathrm{SU}(3)$ between x and $x + a\hat{\mu}$

Field strength: plaquette var. $U_{\mu\nu}(x) \equiv U_\mu(x)U_\nu(x+a\hat{\mu})U^\dagger_\mu(x+a\hat{\nu})U^\dagger_\nu(x)$

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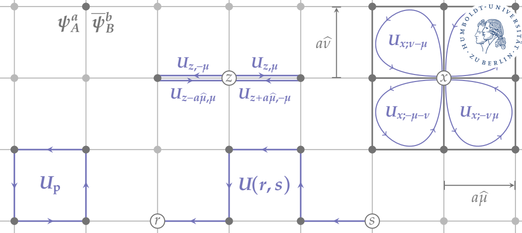

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Wilson action:

$$
\begin{aligned}
\mathcal{S}_\mathrm{G}[A] \quad &\longleftarrow \quad \mathcal{S}_\mathrm{p}[U] \quad \equiv \beta \textstyle\sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{3} \mathrm{Re\,Tr} U_{\mu\nu}(x)\right) \\
\mathcal{S}_\mathrm{F}[\psi, \overline{\psi}] \quad &\longleftarrow \quad \mathcal{S}_\mathrm{W}[\psi, \overline{\psi}] \quad \equiv \textstyle\sum_x \overline{\psi}_x \left\{ \frac{1}{2} \left(\gamma_\mu (\nabla^*_\mu + \nabla_\mu) - a \nabla^*_\mu \nabla_\mu \right) + m_0 \right\} \psi_x \\
\{g_0, m_{0,f}\} \quad &\longleftarrow \quad \{\beta, \kappa_f\} \quad \equiv \{6/g_0^2, (2am_{0,f} + 8)^{-1}\}
\end{aligned}
$$

avoid 'doubling problem' (unphysical spectrum) with explicit χSB term for $a \neq 0$!

∇_μ's: covariant (finite difference) lattice derivatives

parameters in lattice calculations: β, κ_f

Wilson action as lattice regularisation . . .

. . . and consequences of explicit χSB:

$\Rightarrow$ additive mass renormalization: $\quad m_{\mathrm{R}} = Z_{\mathrm{m}} m_{\mathrm{q}} = Z_{\mathrm{m}}(m_0 - m_{\mathrm{c}})$

$\rightsquigarrow$ *critical mass* $m_{\mathrm{c}}(g_0)$

Wilson action as lattice regularisation …

… and consequences of explicit χSB:

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  $\rightsquigarrow$ *critical mass* $m_{\mathrm{c}}(g_0)$
- $\Rightarrow$ reduced convergence properties: $S_{\mathrm{W}} \to S_{\mathrm{QCD}} + \mathrm{O}(a) + \mathrm{O}(a^2)$

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$\Rightarrow$ modified bare coupling and mass for massless RS:

$$\widetilde{g}_0^2 \equiv g_0^2 \Big[1 + b_{\mathrm{g}}(g_0) a m_{\mathrm{q}}\Big] \,, \quad \widetilde{m}_{\mathrm{q}} \equiv m_{\mathrm{q}} \Big[1 + b_{\mathrm{m}}(g_0) a m_{\mathrm{q}}\Big]$$

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- $\Rightarrow$ more complex renormalization pattern of parameters & multiplicatively renormalizable observables ϕ:

$$g_{\mathrm{R}}^2 = \tilde{g}_0^2\, Z_{\mathrm{g}}(\tilde{g}_0^2, a\mu), \quad m_{\mathrm{R}} = \tilde{m}_{\mathrm{q}}\, Z_{\mathrm{m}}(\tilde{g}_0^2, a\mu)$$

$$\phi_{\mathrm{R}} = Z_{\phi}(\tilde{g}_0^2, a\mu)\left[1 + b_{\phi}(g_0) a m_{\mathrm{q}}\right]\phi_{\mathrm{I}}, \quad \phi_{\mathrm{I}} = \phi + c_{\phi}(g_0) a \mathcal{O}_{\phi}^{\mathrm{D}(\phi)+1}$$

Wilson action as lattice regularisation …

…and consequences of explicit χSB:

⇒ additive mass renormalization: $m_\mathrm{R} = Z_\mathrm{m} m_\mathrm{q} = Z_\mathrm{m}(m_0 - m_\mathrm{c})$
⇝ *critical mass* $m_\mathrm{c}(g_0)$ **tuning**

⇒ reduced convergence properties: $S_\mathrm{W} \to S_\mathrm{QCD} \quad + \mathrm{O}(a^2)$
⇝ clover term for *on-shell* $\mathrm{O}(a)$ *improvement* with additional parameter $c_\mathrm{sw}(g_0)$ **NP known**

⇒ modified bare coupling and mass for massless RS:

important!

$$\tilde{g}_0^2 \equiv g_0^2 \left[1 + b_\mathrm{g} \cancel{(g_0)} a m_\mathrm{q}\right] , \quad \tilde{m}_\mathrm{q} \equiv m_\mathrm{q} \left[1 + b_\mathrm{m}(g_0) a m_\mathrm{q}\right]$$

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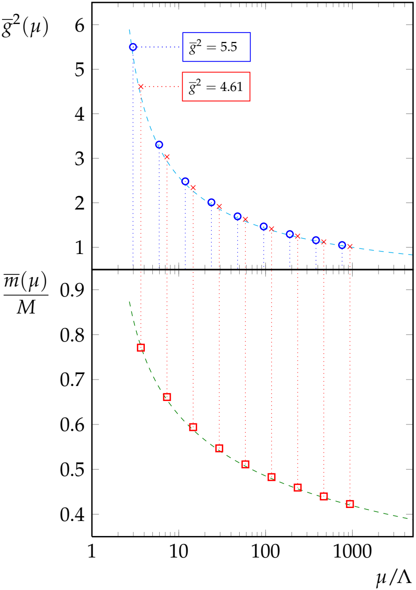
$$\phi_\mathrm{R} = \underline{Z_\phi(\tilde{g}_0^2, a\mu)} \left[1 + \underline{b_\phi(g_0)} a m_\mathrm{q}\right] \phi_\mathrm{I} \, , \quad \phi_\mathrm{I} = \phi + c_\phi(g_0) a \mathcal{O}_\phi^{D(\phi)+1}$$

ideally: non-perturbative estimation

Scale dependence of QCD parameters

Running coupling and mass,



Renormalization group (RG) equations

1. coupling

$$\mu \frac{\partial \bar{g}}{\partial \mu} = \beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3(b_0 + b_1\bar{g}^2 + \dots)$$

2. mass

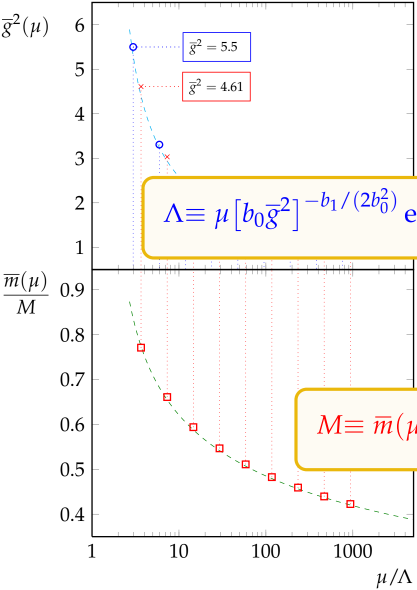
$$\frac{\mu}{\bar{m}} \frac{\partial \bar{m}}{\partial \mu} = \tau(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^2(d_0 + d_1\bar{g}^2 + \dots)$$

in a massless scheme, b_0, b_1, d_0 universal

Solution leads to *exact* equations in mass-independent scheme

Scale dependence of QCD parameters

Running coupling and mass, Renormalization Group Invariants (RGI)



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$$\boxed{\Lambda \equiv \mu\left[b_0\overline{g}^2\right]^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\overline{g}^2)} \exp\left\{-\int_0^{\overline{g}} \mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}}$$

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$$\boxed{M \equiv \overline{m}(\mu)\left[2b_0\overline{g}^2\right]^{-d_0/(2b_0)} \exp\left\{-\int_0^{\overline{g}} \mathrm{d}g\left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}}$$

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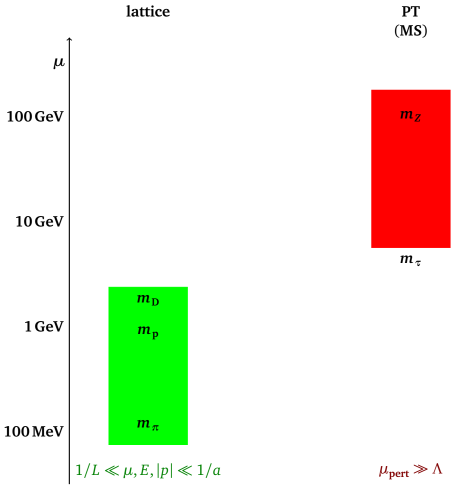
Generic strategy

…to connect low- & high-energy regime NP'ly

one more important ingredient:

How to connect hadronic observables from low-energies to the widely used $\overline{MS}$-scheme?



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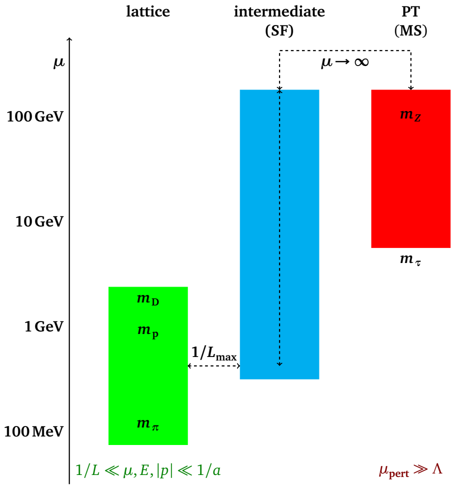
intermediate renorm. scheme

$\equiv$

Schrödinger functional

(finite volume, continuum scheme)



- RG scale evolution solved NP'ly
- thus continuum limit needs to be well controled (small cutoff effects)
- low-energy scale fixed by imposing $\bar{g}^2(L_{max}) \equiv u_{max}$

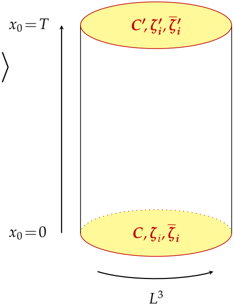
The Schrödinger functional (SF)

as (intermediate) finite volume renormalization scheme

- Euclidean partition function

$$\mathcal{Z} \equiv \int_{T\times L^3} \mathcal{D}[U,\overline{\psi},\psi]\, \mathrm{e}^{-S[U,\overline{\psi},\psi]} = \langle 0|\,\mathrm{e}^{-TH}P|0\rangle$$

with *periodic BC in L^3* and *Dirichlet BC in T*



- fermion fields periodic in L^3 up to a phase θ:

$$\psi(x+\hat{k}L) = \mathrm{e}^{i\theta}\psi(x)$$

$$\overline{\psi}(x+\hat{k}L) = \mathrm{e}^{-i\theta}\overline{\psi}(x)$$

$\Rightarrow$ SF parameters: $\{L, T/L, \theta, N_\mathrm{f}\}$

- 'IR save' $\rightsquigarrow$ simulate massless sea-quark doubletts ($m_\mathrm{l} \equiv 0$) with a variant of the HMC algorithm

$\Rightarrow$ mass-independent renormalization scheme

- renormalization scale μ identified with box length L: $\boxed{\mu = 1/L}$
- heavy-light meson correlation functions with a heavy (quenched) valence quark ($m_\mathrm{h} \neq 0$)

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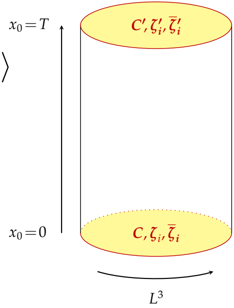

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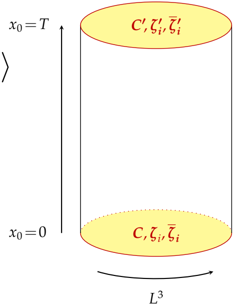

- 'IR save' $\rightsquigarrow$ simulate massless sea-quark doubletts ($m_1 \equiv 0$) with a variant of the HMC algorithm

  $\Rightarrow$ mass-independent renormalization scheme

- renormalization scale μ identified with box length L: $\boxed{\mu = 1/L}$
- heavy-light meson correlation functions with a heavy (quenched) valence quark ($m_\mathrm{h} \neq 0$)

The Schrödinger functional (SF)

as (intermediate) finite volume renormalization scheme

- Euclidean partition function

$$\mathcal{Z} \equiv \int_{T\times L^3} \mathcal{D}[U,\overline{\psi},\psi]\, \mathrm{e}^{-S[U,\overline{\psi},\psi]} = \langle 0|\,\mathrm{e}^{-TH}P|0\rangle$$

  with *periodic BC in L^3* and *Dirichlet BC in T*

- fermion fields periodic in L^3 up to a phase θ:

$$\psi(x+\widehat{k}L) = \mathrm{e}^{i\theta}\psi(x)$$
$$\overline{\psi}(x+\widehat{k}L) = \mathrm{e}^{-i\theta}\overline{\psi}(x)$$

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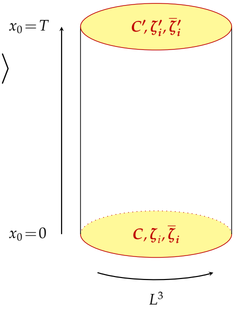

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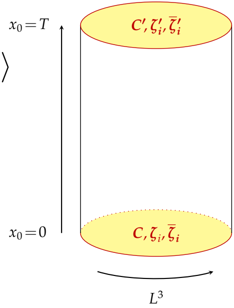

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PCAC relation as a mass definition:

$$\partial_\nu A_\nu^{su} = (m_u + m_s) P^{su}$$

$$[\overline{m}_u + \overline{m}_s] = \frac{Z_A}{Z_P} \underbrace{\frac{\langle 0|\partial_\nu A_\nu^{su}|K^-(\mathbf{p}=0)\rangle}{\langle 0|P^{su}|K^-(\mathbf{p}=0)\rangle}}_{m_u+m_s\text{: bare}}$$

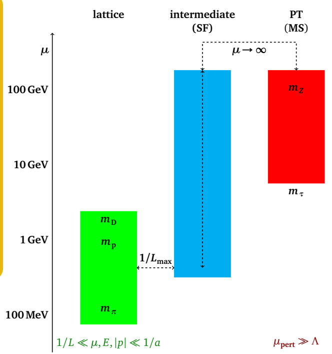
In lattice regularized theory, $g_0 \leftrightarrow a$:

$$Z_A(g_0)\,, Z_P(g_0,\mu) \quad \rightsquigarrow \quad [\overline{m}_u + \overline{m}_s](\mu)$$

scale & scheme dependence from Z_P, poorly convergent in PT

$\rightsquigarrow$ NP determination

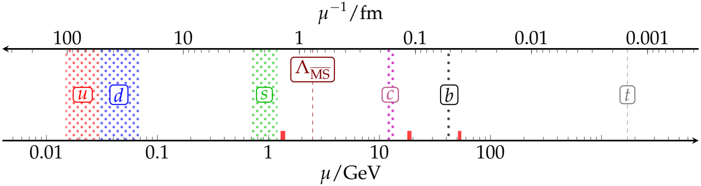
[ALPHA: 1999 ($N_f = 0$) & 2005 ($N_f = 2$)]



$$\underbrace{\underbrace{\frac{Z_A(g_0)}{Z_P(g_0,\mu_{had})}}_{\text{lattice}} \underbrace{\frac{\overline{m}(\mu_{pert})}{\overline{m}(\mu_{had})}}_{\text{NP \& CL}} \underbrace{\frac{M}{\overline{m}(\mu_{pert})}}_{\text{PT}}}_{M_i = Z_M(g_0)\, m_i(g_0)}$$

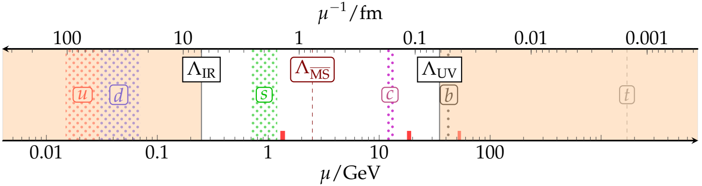
QCD, a multiple scale problem

spectrum, quark masses, bound states



measurable quantities:

$$m_\pi, \ldots, m_\text{D}, m_\text{B}$$



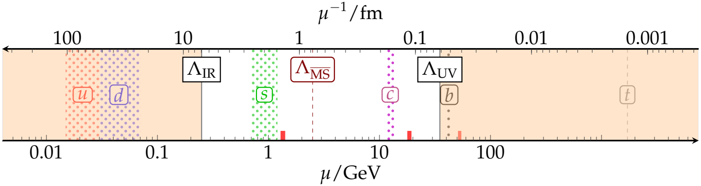
computer + LatticeQCD:

cutoff $L^{-1} \equiv \Lambda_{\mathrm{IR}} \leq \mu \leq \Lambda_{\mathrm{UV}} \equiv a^{-1}$

$$L^{-1} \ll m_\pi, \ldots, m_{\mathrm{D}}, m_{\mathrm{B}} \ll a^{-1}$$

$$O(\mathrm{e}^{-Lm_\pi}) \qquad am_{\mathrm{D}} \lesssim 0.5$$

$$L \gtrsim 4/m_\pi \sim 6\mathrm{fm} \qquad a \approx 0.05\mathrm{fm}$$



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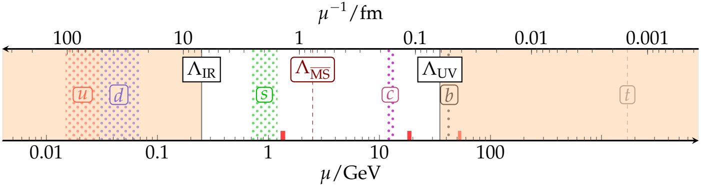
$$L \gtrsim 4/m_\pi \sim 6\mathrm{fm} \Rightarrow \boxed{L/a \sim 96-120} \Leftarrow a \approx 0.05\mathrm{fm}$$

charm just doable, but lattice artifacts may be substancial

QCD, a multiple scale problem

spectrum, quark masses, bound states



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charm just doable, but lattice artifacts may be substancial

Issue: $m_{\mathrm{b}} / m_{\mathrm{c}} \sim 4 - 5$ $\Rightarrow a \approx 0.01\mathrm{fm}$

Solution: separate b-quark scale in a theoretically sound way

use of effective theories: **Heavy Quark Effective Theory (HQET)**

Heavy Quark Effective Theory

Expansion in inverse heavy quark mass $1/m$ [Eichten; Isgur+Wise; Georgi]

$$\mathcal{L}_{\mathrm{HQET}} = \overline{\psi}_{\mathrm{h}} \Big[\underbrace{D_0 + \delta m}_{\text{static limit (LO)}} \underbrace{- \omega_{\mathrm{kin}}\, \mathbf{D}^2 - \omega_{\mathrm{spin}}\, \boldsymbol{\sigma}\mathbf{B}}_{\text{NLO, O(1/m)}} \Big] \psi_{\mathrm{h}} + \ldots\,, \qquad \left.\begin{matrix} \omega_{\mathrm{kin}} \\ \omega_{\mathrm{spin}} \end{matrix}\right\} \sim \frac{1}{2m}$$

operator $\mathcal{O}_{\mathrm{kin}} \equiv -\overline{\psi}_{\mathrm{h}} \mathbf{D}^2 \psi_{\mathrm{h}}$ — kinetic energy from residual motion of heavy quark

operator $\mathcal{O}_{\mathrm{spin}} \equiv -\overline{\psi}_{\mathrm{h}} \boldsymbol{\sigma}\mathbf{B} \psi_{\mathrm{h}}$ — chromomagnetic interaction with gluon field

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With $\mathcal{L}_{\mathrm{HQET}} = \mathcal{L}_{\mathrm{stat}} + \sum_{n\geq 1}\mathcal{L}^{(n)}$, expand integrand in functional integral repres.

$$\langle\mathcal{O}\rangle = \mathcal{Z}^{-1}\int\mathcal{D}[\phi]\,\mathcal{O}[\phi]\,\mathrm{e}^{-\mathcal{S}_{\mathrm{rel}}-\mathcal{S}_{\mathrm{HQET}}}\,, \qquad \mathcal{Z} = \int\mathcal{D}[\phi]\,\mathrm{e}^{-\mathcal{S}_{\mathrm{rel}}-\mathcal{S}_{\mathrm{HQET}}}\,,$$

as a power series in $1/m$:

$$\mathrm{e}^{-\mathcal{S}_{\mathrm{HQET}}} = \exp\left\{-a^4\sum_x\mathcal{L}_{\mathrm{stat}}(x)\right\}\times$$
$$\left\{1-a^4\sum_x\mathcal{L}^{(1)}(x) + \frac{1}{2}\left[a^4\sum_x\mathcal{L}^{(1)}(x)\right]^2 - a^4\sum_x\mathcal{L}^{(2)}(x) + \ldots\right\}$$

Heavy Quark Effective Theory

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$$\mathcal{L}_{\mathrm{HQET}} = \overline{\psi}_{\mathrm{h}} \Big[\underbrace{D_0 + \delta m}_{\text{static limit (LO)}} \underbrace{-\omega_{\mathrm{kin}}\,\mathbf{D}^2 - \omega_{\mathrm{spin}}\,\sigma\mathbf{B}}_{\text{NLO, O(1/m)}} \Big] \psi_{\mathrm{h}} + \ldots\,, \qquad \left.\begin{matrix}\omega_{\mathrm{kin}} \\ \omega_{\mathrm{spin}}\end{matrix}\right\} \sim \frac{1}{2m}$$

This definition of HQET implies:

- $1/m$-terms appear as insertions of local operators only
  $\Rightarrow$ power counting: Renormalizability to each order in $1/m$
  $\Leftrightarrow$ $\exists$ continuum limit & universality (in contrast to NRQCD)
  (remark: **not** rigorously proven for *static theory to all orders in* g)
- Effective theory = (continuum) asymptotic expansion of QCD in $1/m$
- interaction with light d.o.f's still non-perturbatively (in contrast to χPT)

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{stat}}}\, \mathcal{O} \Big\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots \Big\}$$

- originally formulated by [Eichten+Hill '88-'90]: $D_0 + \delta m \to \nabla_0 + \delta m$
- again different discretisations: APE-,HYP-smeared actions

  mainly to cure bad $\frac{\text{signal}}{\text{noise}} \propto \exp[-E_{\rm stat} x_0] \sim \exp[-(c g_0^2/a) x_0]$

Explicitly: EV in HQET to subleading order

$$\begin{aligned}\langle \mathcal{O} \rangle &= \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin}\; a^4 \sum_x \langle \mathcal{O}\mathcal{O}_{\rm kin}(x) \rangle_{\rm stat} + \omega_{\rm spin}\; a^4 \sum_x \langle \mathcal{O}\mathcal{O}_{\rm spin}(x) \rangle_{\rm stat} \\ &\equiv \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin}\end{aligned}$$

with

$$\langle O \rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\text{fields}} O\; \exp\left\{ -a^4 \sum\nolimits_x \left[\mathcal{L}_{\rm light}(x) + \mathcal{L}_{\rm h}^{\rm stat}(x) \right] \right\}$$

Heavy Quark Effective Theory on the lattice

The Problem: power divergences [Maiani,Martinelli,Sachrajda '92]

mixing of operators of different dim. in $\mathcal{L}_{\mathrm{HQET}}$ induces power divergences

- **Example:** Mass renormalization pattern at static order of HQET

  mixing of $\overline{\psi}_{\mathrm{h}} D_0 \psi_{\mathrm{h}}$ and $\overline{\psi}_{\mathrm{h}} \psi_{\mathrm{h}}$ $\rightsquigarrow$ linear divergence: $\delta m \propto a^{-1}$

$$\overline{m}_{\mathrm{b}}^{\overline{MS}} = Z_{\mathrm{pole}}^{\overline{MS}} \cdot m_{\mathrm{pole}} \,, \qquad m_{\mathrm{pole}} = m_{\mathrm{b}} - E_{\mathrm{stat}} - \delta m$$

$$\delta m = \frac{c(g_0)}{a} \sim \mathrm{e}^{+1/(2b_0 g_0^2)} \left\{ c_1 g_0^2 + c_1 g_0^2 + \ldots + O(g^{2n}) \right\}$$

- in PT: uncertainty = truncation error $\sim \mathrm{e}^{+1/(2b_0 g_0^2)} \cdot c_{n+1} \cdot g_0^{2n+2} \xrightarrow{g_0 \to 0} \infty$

  $\Rightarrow$ Non-perturbative $c(g_0)$ needed

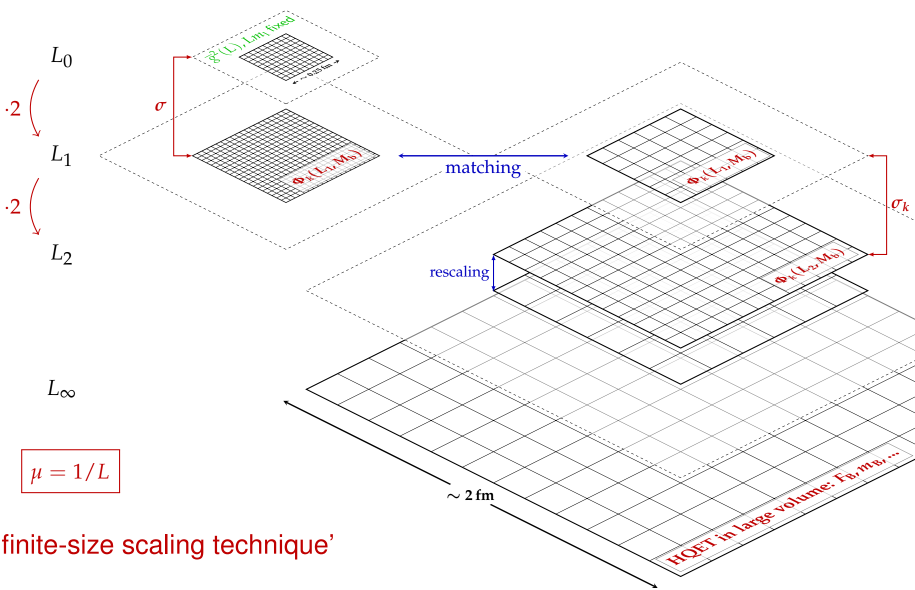
  $\Rightarrow$ NP renormalization of HQET (resp. matching to QCD) required for continuum limit to exist

- power-law divergences even worse at higher orders in $1/m$:

  LO$\rightarrow$NLO: $a^{-1} \rightarrow a^{-2}$ in coeff.s of $\omega_{\mathrm{kin}} \mathcal{O}_{\mathrm{kin}}, \omega_{\mathrm{spin}} \mathcal{O}_{\mathrm{spin}}$ in $\mathcal{L}^{(1)}$ of $\mathcal{L}_{\mathrm{HQET}}$

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]

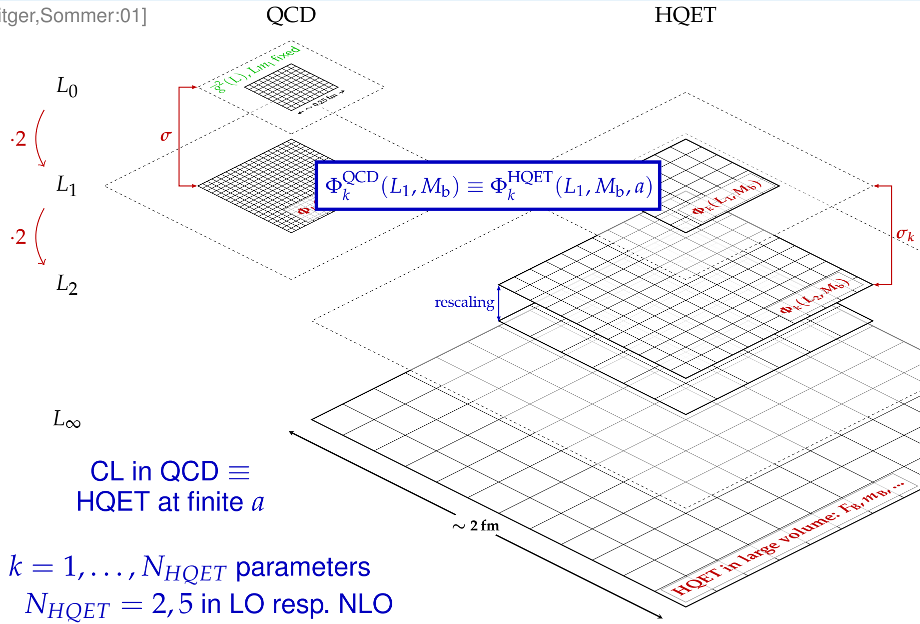


$\boxed{\mu = 1/L}$

'finite-size scaling technique'

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



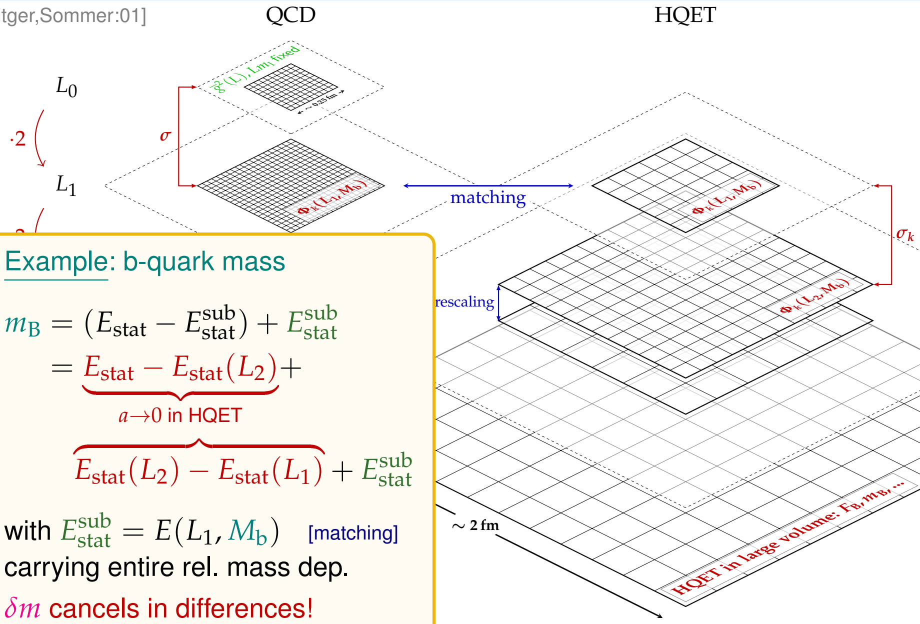
CL in QCD $\equiv$
HQET at finite a

$\Rightarrow k = 1, \ldots, N_{HQET}$ parameters

$N_{HQET} = 2, 5$ in LO resp. NLO

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



Example: b-quark mass

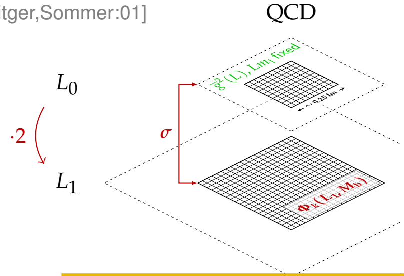
$$m_B = (E_{stat} - E_{stat}^{sub}) + E_{stat}^{sub}$$

$$= \underbrace{E_{stat} - E_{stat}(L_2)}_{a \to 0 \text{ in HQET}} + \overbrace{E_{stat}(L_2) - E_{stat}(L_1)} + E_{stat}^{sub}$$

with $E_{stat}^{sub} = E(L_1, M_b)$ [matching]
carrying entire rel. mass dep.

δm cancels in differences!

[Heitger,Sommer:01]



- $L_0/a \in \{10, 12, 16, 20\}$, $L_0 \approx 0.25\text{fm}$
- $L_1/a \in \{20, 24, 32, 40\}$, $L_1 \approx 0.5\text{fm}$

$\rightsquigarrow a \approx (0.025 - 0.012)\text{fm}$

Definition: line of constant physics ('light' sector)

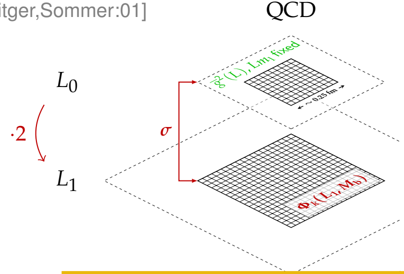
$$\boxed{\bar{g}^2(L_0) \equiv 2.989\,, \quad L_0 m_1 \equiv 0} \quad \Rightarrow (\beta, \kappa_1, L/a), \text{ fine-tuning}$$

running coupl. $\bar{g}^2(\mu)$ & mass $\bar{m}(\mu)$ NP known in the SF ($\mu = 1/L$)! [DellaMorte etal:'05]

$$\boxed{\bar{g}^2(L_1) \equiv \sigma(2.989) = 4.484(48)\,, \quad L_1 m_1 \equiv 0}$$

NP matching of HQET and QCD in a finite volume

[Heitger,Sommer:01]



- $L_0/a \in \{10, 12, 16, 20\}$, $L_0 \approx 0.25\text{fm}$
- compute $b_{\mathrm{m}}, b_{\mathrm{A}} - b_{\mathrm{P}}$ and Z

- $L_1/a \in \{20, 24, 32, 40\}$, $L_1 \approx 0.5\text{fm}$
- heavy quark mass dependence of QCD observ.

$\rightsquigarrow a \approx (0.025 - 0.012)\text{fm}$

Definition: line of constant physics ('heavy' sector)

fix dimensionless heavy RGI quark mass *in the continuum*:

$$z \equiv LM = [L/a] h_{L_0} Z_{\mathrm{m}} (1 + b_{\mathrm{m}} a m_{\mathrm{q}}) a m_{\mathrm{q}} \,, \qquad Z_{\mathrm{m}} \equiv \frac{Z(g_0) Z_{\mathrm{A}}(g_0)}{Z_{\mathrm{P}}(g_0, \mu_0)}$$

$$a m_{\mathrm{q}} \equiv \tfrac{1}{2}\left[\kappa_{\mathrm{h}}^{-1} - \kappa_{\mathrm{c}}^{-1}(g_0)\right] \,,$$ but we need $\kappa_{\mathrm{h}}(z, L/a)$

- $L/a, h_{L_0}, Z_{\mathrm{A}}, Z_{\mathrm{P}}$ NP known
- estimate b_{m}, Z non-perturbatively

[P.F.,Heitger,Tantalo'10]

Fixing the heavy quark mass $z = L_1 M$

$L_1/a \in \{20, 24, 32, 40\}, \quad L_1 \approx 0.5\mathrm{fm}$

$$z = L_1 M = L_1 Z_{\mathrm{M}}(1 + b_{\mathrm{m}}\, am_{\mathrm{q,h}})am_{\mathrm{q,h}}\,, \qquad Z_{\mathrm{M}} = h(L_0)\frac{Z Z_{\mathrm{A}}}{Z_{\mathrm{P}}(L_0)}$$

with b_{m} and Z invert for

$$\kappa_{\mathrm{h}}(z, L_1) = \left[\frac{1}{\kappa_{\mathrm{c}}} - \frac{1}{b_{\mathrm{m}}}\left(1 - \sqrt{1 + z \cdot \frac{4b_{\mathrm{m}}}{[L_1/a]Z_{\mathrm{M}}}}\right)\right]^{-1}$$

choose $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $\quad z_{\mathrm{c}} \approx 4.13,\ z_{\mathrm{b}} \approx 17.5$

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choose $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $\quad z_\mathrm{c} \approx 4.13$, $z_\mathrm{b} \approx 17.5$

	z	20	24	32	40
κ_s	0	0.1360536	0.1359104	0.1355210	0.1351922
κ_1	4	0.1327278	0.1332121	0.1335643	0.1336510
κ_2	6	0.1309498	0.1317899	0.1325495	0.1328583
κ_3	7	0.1300226	0.1310561	0.1320315	0.1324556
κ_4	9	0.1280709	0.1295337	0.1309715	0.1316366
κ_5	11	0.1259456	0.1279214	0.1298749	0.1307974
κ_6	13	0.1235550	0.1261898	0.1287348	0.1299352
κ_7	15	*0.1206872*	0.1242898	0.1275422	0.1290468
κ_8	18	—	*0.1208919*	0.1256259	0.1276559

Stop & relax for a moment

take a breath

Strategy explained, lattice setup fixed !

Give me some more minutes to define observables and to present results!

QCD observables in small volume

. . . and their asymptotics for large heavy quark masses $z = LM$

leading asymptotics

effective meson masses: $\begin{cases} \Gamma_{\mathrm{PS}}(L,z) = -\widetilde{\partial}_0 \ln\left[\langle\Omega|A_0|B\rangle\right] \\ \Gamma_{\mathrm{V}}(L,z) = -\widetilde{\partial}_0 \ln\left[\langle\Omega|V_k|B^*\rangle\right] \end{cases}$

HQET-QCD conversion functions

$$m_B^{\mathrm{av}} \leftarrow L\Gamma_{\mathrm{av}} \equiv \tfrac{1}{4}[L\Gamma_{\mathrm{PS}} + 3L\Gamma_{\mathrm{V}}] \xrightarrow{z\to\infty} C_{\mathrm{mass}}(z)\cdot z\cdot\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$f_B \xleftarrow{L\to\infty} Y_{\mathrm{PS}} \equiv \frac{\langle\Omega|A_0|B\rangle}{\||\Omega\rangle\|\cdot\||B\rangle\|} \xrightarrow{z\to\infty} C_{\mathrm{PS}}(z)\Phi_{\mathrm{RGI}}(L)\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$f_{B^*} \xleftarrow{L\to\infty} Y_{\mathrm{V}} \equiv \frac{\langle\Omega|V_k|B^*\rangle}{\||\Omega\rangle\|\cdot\||B^*\rangle\|} \xrightarrow{z\to\infty} C_{\mathrm{V}}(z)\Phi_{\mathrm{RGI}}(L)\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$R_{\mathrm{PS/P}} \equiv \frac{\langle\Omega|A_0|B\rangle}{\langle\Omega|P|B\rangle} \xrightarrow{z\to\infty} C_{\mathrm{PS/P}}(z)\cdot 1\cdot\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$R_{\mathrm{PS/V}} \equiv \frac{\langle\Omega|A_0|B\rangle}{\langle\Omega|V_k|B^*\rangle} \xrightarrow{z\to\infty} C_{\mathrm{PS/V}}(z)\cdot 1\cdot\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$\Delta m_B \xleftarrow{L\to\infty} R_{\mathrm{spin}} \equiv \tfrac{1}{4}\ln\frac{\langle B|B\rangle}{\langle B^*|B^*\rangle} \xrightarrow{z\to\infty} C_{\mathrm{spin}}(z)\frac{\Phi_{\mathrm{RGI}}^{\mathrm{spin}}(L)}{z}\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

QCD observables in small volume

...and their asymptotics for large heavy quark masses $z = LM$

leading asymptotics

effective meson masses: $\begin{cases} \Gamma_{\mathrm{PS}}(L,z) = -\tilde{\partial}_0 \ln\left[\langle\Omega|A_0|B\rangle\right] \\ \Gamma_{\mathrm{V}}(L,z) = -\tilde{\partial}_0 \ln\left[\langle\Omega|V_k|B^*\rangle\right] \end{cases}$

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$$f_B \quad \xleftarrow{L\to\infty} \quad Y_{\mathrm{PS}} \equiv \frac{\langle\Omega|A_0|B\rangle}{\||\Omega\rangle\|\cdot\||B\rangle\|} \xrightarrow{z\to\infty} C_{\mathrm{PS}}(z)\Phi_{\mathrm{RGI}}(L)\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$f_{B^*} \quad \xleftarrow{L\to\infty} \quad Y_{\mathrm{V}} \equiv \frac{\langle\Omega|V_k|B^*\rangle}{\||\Omega\rangle\|\cdot\||B^*\rangle\|} \xrightarrow{z\to\infty} C_{\mathrm{V}}(z)\Phi_{\mathrm{RGI}}(L)\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

$$R_{\mathrm{PS/P}} \equiv \frac{\langle\Omega|A_0|B\rangle}{\langle\Omega|P|B\rangle} \xrightarrow{z\to\infty} C_{\mathrm{PS/P}}(z)\cdot 1\cdot\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

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$$\Delta m_B \quad \xleftarrow{L\to\infty} \quad R_{\mathrm{spin}} \equiv \frac{1}{4}\ln\frac{\langle B|B\rangle}{\langle B^*|B^*\rangle} \xrightarrow{z\to\infty} C_{\mathrm{spin}}(z)\frac{\Phi_{\mathrm{RGI}}^{\mathrm{spin}}(L)}{z}\left[1+\mathrm{O}\left(\tfrac{1}{z}\right)\right]$$

QCD observables in small volume

...and their asymptotics for large heavy quark masses $z = LM$

leading asymptotics

effective meson masses: $\begin{cases} \Gamma_{\mathrm{PS}}(L,z) = -\widetilde{\partial}_0 \ln\left[\langle\Omega|A_0|B\rangle\right] \\ \Gamma_{\mathrm{V}}(L,z) = -\widetilde{\partial}_0 \ln\left[\langle\Omega|V_k|B^*\rangle\right] \end{cases}$

HQET-QCD conversion functions

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$m_B^{\rm av}$	$\leftarrow$	$L\Gamma_{\rm av} \equiv \frac{1}{4}[L\Gamma_{\rm PS} + 3L\Gamma_{\rm V}]$	$\xrightarrow{z\to\infty}$	$C_{\rm mass}(z)\cdot z\cdot\left[1+{\rm O}\left(\frac{1}{z}\right)\right]$
f_B	$\xleftarrow{L\to\infty}$	$Y_{\rm PS} \equiv \frac{\langle\Omega\|A_0\|B\rangle}{\\|\|\Omega\rangle\\|\cdot\\|\|B\rangle\\|}$	$\xrightarrow{z\to\infty}$	$C_{\rm PS}(z)\Phi_{\rm RGI}(L)\left[1+{\rm O}\left(\frac{1}{z}\right)\right]$
f_{B^*}	$\xleftarrow{L\to\infty}$	$Y_{\rm V} \equiv \frac{\langle\Omega\|V_k\|B^*\rangle}{\\|\|\Omega\rangle\\|\cdot\\|\|B^*\rangle\\|}$	$\xrightarrow{z\to\infty}$	$C_{\rm V}(z)\Phi_{\rm RGI}(L)\left[1+{\rm O}\left(\frac{1}{z}\right)\right]$
		$R_{\rm PS/P} \equiv \frac{\langle\Omega\|A_0\|B\rangle}{\langle\Omega\|P\|B\rangle}$	$\xrightarrow{z\to\infty}$	$C_{\rm PS/P}(z)\cdot 1\cdot\left[1+{\rm O}\left(\frac{1}{z}\right)\right]$
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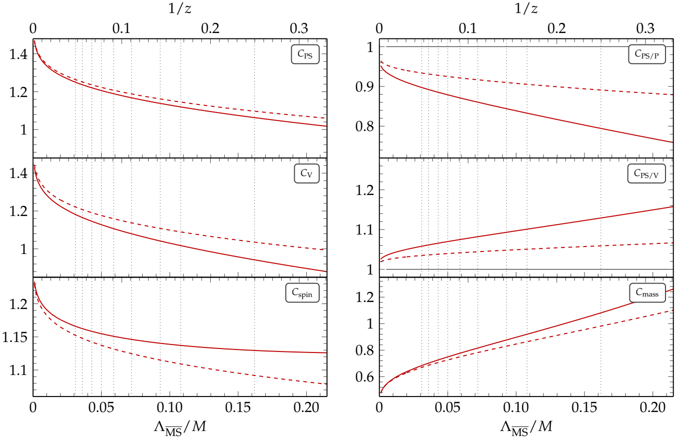
Δm_B	$\xleftarrow{L\to\infty}$	$R_{\rm spin} \equiv \frac{1}{4}\ln\frac{\langle B\|B\rangle}{\langle B^*\|B^*\rangle}$	$\xrightarrow{z\to\infty}$	$C_{\rm spin}(z)\frac{\Phi_{\rm RGI}^{\rm spin}(L)}{z}\left[1+{\rm O}\left(\frac{1}{z}\right)\right]$

$z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$

these PT conv. functions only appear in some of our testobs.



$\gamma_0^{PS} = -1/(4\pi^2)$: [Shifman&Voloshin'87], [Politzer&Wise'88]

γ_1^{PS} : [Ji&Musolf'91], [Broadhurst&Grozin'91], [Gimenez'92]

γ_2^{PS} : [Chetyrkin&Grozin'03]

...

$\gamma_0^{spin} = -2/(4\pi)^2$: [Eichten&Hill'90], [Falk,Grinstein&Luke'88]

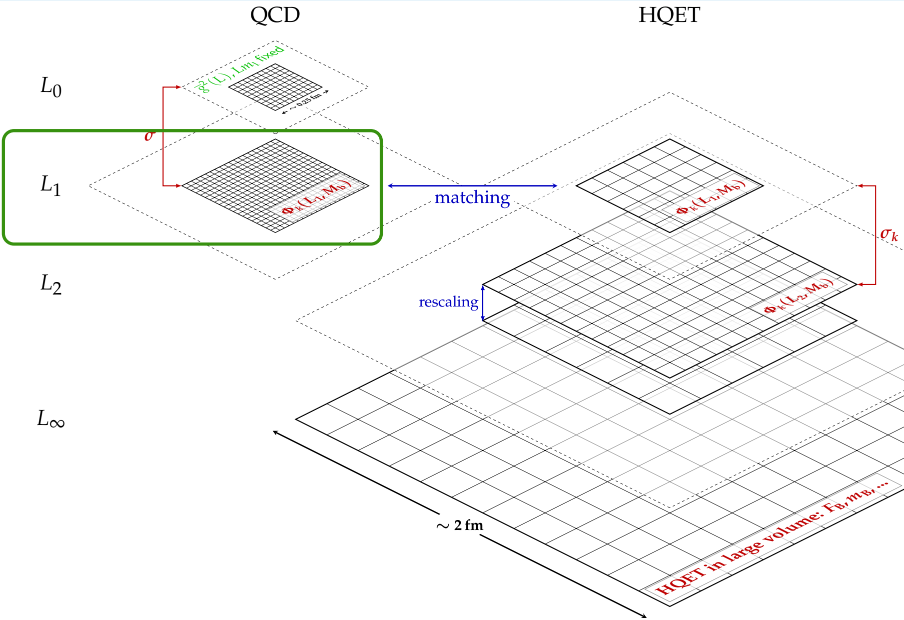
γ_1^{spin} : [Czarnecki&Grozin'97], [Amoros,Beneke&Neubert'97]

γ_2^{spin} : [GrozinEtAl'08]

usual problems: how to estimate systematic error

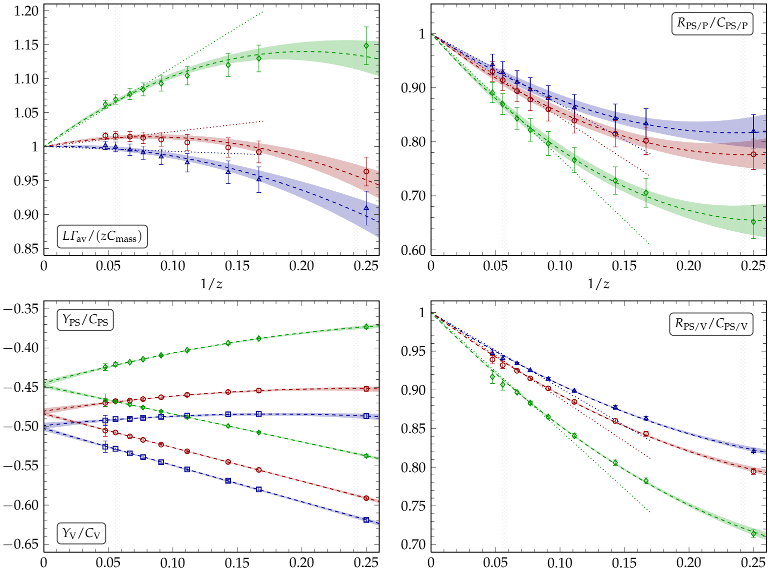
PT expansion of coupling reliable enough

Continuum QCD results in small volume



QCD-Results converted to HQET

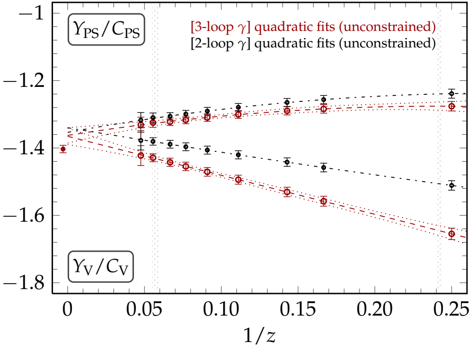
mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $\theta \in \{0, 0.5, 1\}$



QCD-Results converted to HQET, decay constant

mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$, $\theta \in \{0.5\}$

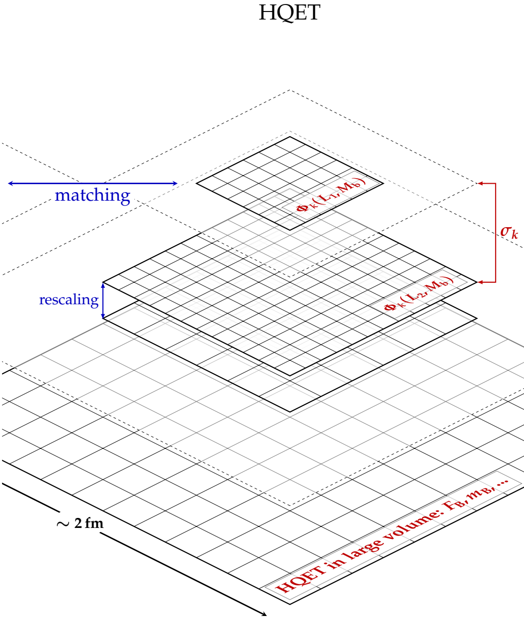
- impact of conversion function C_{PS} with 2- or 3-loop anomalous dimension



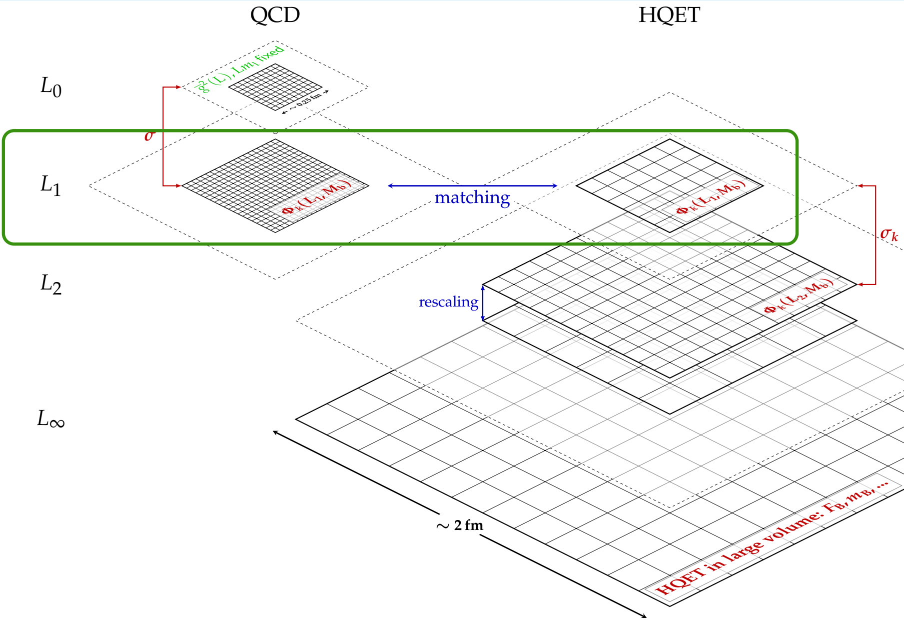
- barely agrees with our result at static order in HQET [DellaMorte,P.F.,Heitger'05]
- Missmatch a result of perturbative C_{PS}? 4-loops, ...?
- NP matching removes this perturbative uncertainty!

HQET part of our simulations

- $L_1 \approx 0.5\text{fm}$
  $L_1/a \in \{6, 8, 10, 12, 16\}$
  with $T = L$ and $T = L/2$
  production & measurements done
  target: 8000 configs each
- $L_2 \approx 1\text{fm}$
  $L_2/a \in \{12, 16, 20, 24, 32\}$
  with $T = L$ and $T = L/2$
  in progress
- $L_\infty \approx 2\text{fm}$
  CLS
  based configurations
  Coordinated Lattice Simulations
  Consortium of different European lattice groups
  Goal: reach large volumes & small a



Comparing HQET and QCD in the matching volume



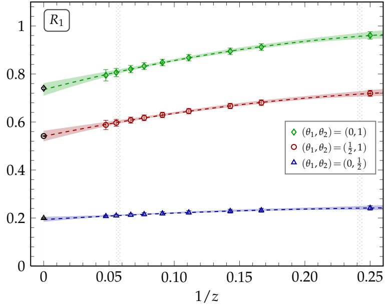
Results without any perturbative uncertainty

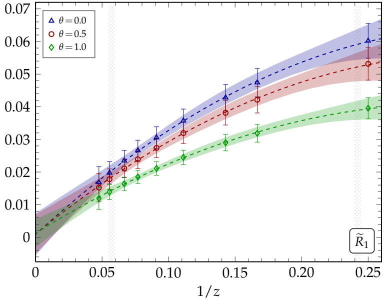
mass-dependence in the continuum, $z \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$

in QCD: $f_1 = \mathcal{Z}^{-1}\langle B(L)|B(L)\rangle$ $k_1 = \mathcal{Z}^{-1}\langle B^*(L)|B^*(L)\rangle$

$$R_1 = \frac{1}{4}\ln\left(\frac{f_1(\theta_1)k_1^3(\theta_1)}{f_1(\theta_2)k_1^3(\theta_2)}\right), \qquad \widetilde{R_1} = \frac{3}{4}\ln\left(\frac{f_1(\theta)}{k_1(\theta)}\right) \sim \omega_{\rm spin}$$

their HQET expansion contains no conversion functions at LO



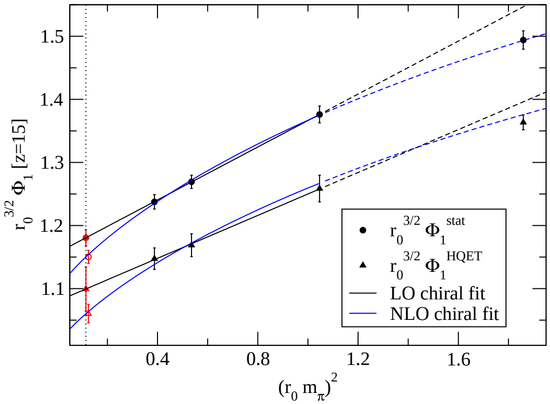


free quadratic fits in $1/z$ (static limit at $1/z = 0$)

computations in HQET & QCD absolutely independent and purely NP!

First results from large volume calculations

B meson decay constant from $N_\mathrm{f} = 2$ HQET



preliminary:

$$f_\mathrm{B}\big|_{N_\mathrm{f}=2}^{\mathrm{HQET}} = 178(16)_{\mathrm{stat}}(?)_a\mathrm{MeV}$$

- lin. / lin.+leading-log(HMχPT) fit (3 data points used)
- only one lattice spacing yet ($a = 0.07\mathrm{fm}$)
  $\Rightarrow$ include more in the future and do continuum limit

Summary

ALPHA Collaboration B-physics project

- setup of $O(a)$ improved lattice theory in small volume $\checkmark$
- strong evidence that lattice HQET is renormalizable and works (that's non-trivial) $\checkmark$
- confidence in existence of HQET CL in static approximation $\checkmark$
- full $N_\mathrm{f} = 2$ matching calculations in progress
- physical applications are waiting

Summary

ALPHA Collaboration B-physics project

- setup of $O(a)$ improved lattice theory in small volume ✓
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- confidence in existence of HQET CL in static approximation ✓
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- physical applications are waiting

high precision lattice HQET possible after years of development

- lattice HQET (LO&NLO) [Eichten&Hill:88-90]
- fully NP HQET-QCD matching procedure [Heitger,Sommer:04]
- suitable choice of HQET action [DellaMorte,Shindler:05]
- 'all-to-all' propagators [Foley,Juge,O'Cais,Peardon,Ryan,Skullerud:05]
- GEVP method [Blossier,DellaMorte,vonHippel,Mendes,Sommer:08]

Talk in particular based on:

ALPHA Collaboration

Thank You!

- Non-perturbative heavy quark effective theory
  J.Heitger and R.Sommer, JHEP 0402 (2004) 022
- Effective heavy-light meson energies in small-volume quenched QCD
  J.Heitger and J.Wennekers, JHEP 0402 (2004) 064
- Heavy Quark Effective Theory computation of the mass of the bottom quark
  M.Della Morte, N.Garron, M.Papinutto and R.Sommer, JHEP 0701 (2007) 007
- On the generalized eigenvalue method for energies and matrix elements in lattice field theory
  B.Blossier, M.Della Morte, G.von Hippel, T.Mendes and R.Sommer, JHEP 0904 (2009) 094
- Spectroscopy & Decay Constants from Non-pert. HQET at O(1/m),
  B.Blossier, M.Della Morte, N.Garron, G.von Hippel, T.Mendes, H.Simma and R.Sommer, PoS LAT2009 (2009) 106
- HQET at O(1/m): I. Non-perturbative parameters in the quenched approximation
  B.Blossier, M.Della Morte, N.Garron and R.Sommer, JHEP 06 (2010) 002 [hep-lat]
- Towards a non-perturbative matching of HQET and QCD with dynamical light quarks
  M.Della Morte, P.F., J.Heitger, H.Meyer, H.Simma and R.Sommer, PoS LAT2007 (2007) 246
- Non-perturbative quark mass dependence in the heavy-light sector of two-flavour QCD
  M.Della Morte, P.F., J.Heitger and R.Sommer, PoS LATTICE2008 (2008) 226
- Non-perturbative improvement of quark mass renormalization in two-flavour lattice QCD
  P.F., J.Heitger and N.Tantalo, JHEP 08 (2010) 074
- Non-perturbative test of HQET in small volume two-flavour QCD
  P.F., N.Garron and J.Heitger, in preparation

and work in progress within CLS: B.Blossier, G.De Divitiis, M.Della Morte, M.Donnellan, P.F., N.Garron, J.Heitger, G.von Hippel, T.Mendes, R.Petronzio, H.Simma, R.Sommer, N.Tantalo, . . .

backup slides

Results for Z

for testing purpose:

set1: $L_0 m_{11} \approx 0\,, \quad L_0 m_{22} \approx 0.5 \quad (\star)$

set2: $L_0 m_{11} \approx 0\,, \quad L_0 m_{22} \approx 2.5$

$\Rightarrow$ 'improvement conditions' are fixed!

$\Updownarrow$

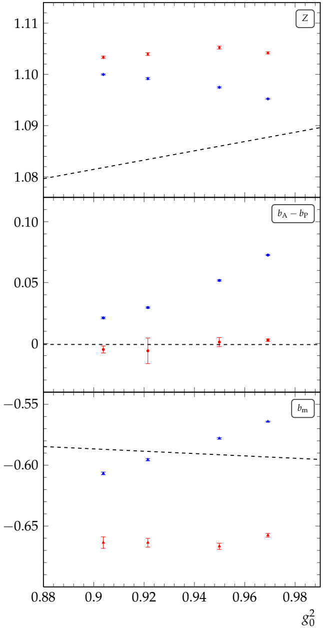
well-defined parametrisation in g_0^2

use improved lattice derivatives

$$\widetilde{\partial}_0 \rightarrow \widetilde{\partial}_0 \left(1 - \tfrac{1}{6} a^2 \partial_0^* \partial_0\right)$$

to compute $m_{ij} \rightsquigarrow O(g_0^2 a^2, a^4)$

- smooth dependence on g_0^2
- deviations from 1-loop PT
- quantitatively: mass-dep. cutoff-effects larger for set2



HQET-QCD conversion functions $C_X(z)$

Definition (example: heavy light axial current $A_\mu(x) = \overline{\psi}_{\rm h}(x)\gamma_\mu\gamma_5\psi_{\rm l}(x)$)

corresponding matrix element: $\Phi(m_{\rm B}) = \langle \beta, b | A_\mu(x) | \alpha \rangle$

$m_{\rm l} = 0$, only one large scale: $m_{\rm B}$ $\rightsquigarrow$ $m_{\rm B}$-dependence of Φ?

RGE in a massless scheme: $\mu \frac{\partial \overline{g}}{\partial \mu} = \beta(\overline{g}), \quad \frac{\mu}{\overline{m}} \frac{\partial \overline{m}}{\partial \mu} = \tau(\overline{g})$

scale is fixed: $\mu = m_\star \equiv \overline{m}(m_\star), \quad g_\star \equiv \overline{g}(m_\star)$

$\Rightarrow$ mass dependence given by RGE of Φ:

$$\frac{m_\star}{\Phi} \frac{\partial \Phi}{\partial m_\star} \equiv \gamma^{\Phi}_{\rm match}(g_\star) \quad \overset{m_\star \to \infty}{\sim} \quad -g_\star^2 \cdot \gamma_0 + {\rm O}(g_\star^4)$$

factorization in effective theory:

$$\gamma^{\Phi}_{\rm match}(g_\star) = \gamma_{\rm match}(g_\star) + {\rm O}(\Lambda/m_\star)$$

is scheme dependent $\to$ use RGI's: Λ, M

HQET-QCD conversion functions $C_X(z)$

mass dependence in QCD

$$\Lambda = m_\star \exp\left\{ -\int^{g_\star} \frac{\mathrm{d}g}{\beta(g)} \right\} , \qquad M = m_\star \exp\left\{ -\int^{g_\star} \mathrm{d}g \frac{\tau(g)}{\beta(g)} \right\} ,$$

thus

$$\frac{M}{\Phi}\frac{\partial\Phi}{\partial M} \equiv \gamma_{\mathrm{PS}}^{(M)}(M/\Lambda) + \mathrm{O}(\Lambda/M)$$

$$\gamma_{\mathrm{PS}}^{(M)}(M/\Lambda) \equiv \frac{\gamma_{\mathrm{match}}(M/\Lambda)}{1-\tau(M/\Lambda)}$$

gives

$$\Phi(M,\Lambda) = C_{\mathrm{PS}}\left(\frac{M}{\Lambda}\right)\Phi_{\mathrm{RGI}} + \mathrm{O}\left(\frac{\Lambda}{M}\right) , \quad C_{\mathrm{PS}} = \exp\left\{ \int^{g_\star(M/\Lambda)} \mathrm{d}g \frac{\gamma_{\mathrm{PS}}^{(M)}(g)}{\beta(g)} \right\}$$

matrix element Φ_{RGI} unambiguous, computable in effective theory,
mass independent

ALPHA Collaboration B-physics project

- $B^*B\pi$ coupling

[CLS based ,preliminary]