Towards a new determination of the QCD Λ-parameter from running couplings in the three-flavour theory

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IN COLLABORATION WITH
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$\alpha_s(\mu)$

$\alpha_s^{(5)}(M_Z)$ world averages

PARTICLE DATA GROUP 2013

[Beringer et al.]

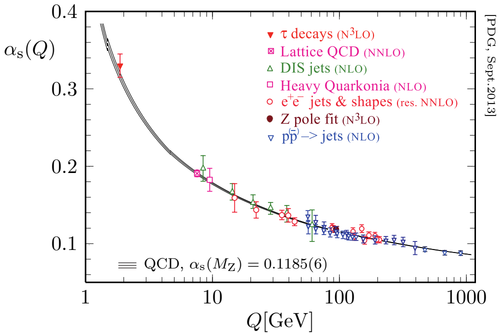
w/ lattice: $0.1185(6)$

w/o lattice: $0.1183(12)$

FLAVOR LATTICE AVERAGING GROUP 2014

[1310.8555v2]

lattice only: $0.1184(12)$



The QCD Λ–parameter

RENORMALIZATION GROUP: $\mu\frac{\partial\bar{g}}{\partial\mu} = \beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -\bar{g}^3(b_0 + b_1\bar{g}^2 + b_2\bar{g}^4 + \ldots)$

Aim:

$$\Lambda \equiv \mu\left[b_0\bar{g}^2(\mu)\right]^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}^2(\mu))} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}g\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

- *exact equation* (for any N_f) — $\forall\mu$ in *mass-independent scheme*
- *trivial scheme dependence* — $\Lambda_X/\Lambda_Y = \text{const.}$
- use a suitable physical coupling (scheme) to compute Λ (defined $\forall\mu$, regularisation independent, ...) — $\bar{g}^2_{\mathrm{qq}}, \bar{g}^2_{\mathrm{SF}}, \bar{g}^2_{\mathrm{GF}}, \ldots$
- Requires: non-perturbative $\beta(\bar{g})$ to cover wide range of couplings over intermediate energies $\mu \in [\mu_{\mathrm{min}}, \mu_{\mathrm{max}}]$ — $\mu_{\mathrm{min}} \leftrightarrow \mu_{\mathrm{max}}$

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intermediate, massless finite-volume renorm. scheme
+
continuum finite-size scaling ($\mu = 1/L$)

Here:

- Schrödinger functional scheme (SF)
- $N_\mathrm{f} = 3$

The traditional strategy

SCHRÖDINGER FUNCTIONAL (SF) as intermediate FV renorm. scheme, $\mu \equiv 1/L$

for any physical coupling scheme:

Pattern:

$$\frac{\Lambda}{f_{\rm K}} \equiv \frac{1}{[f_{\rm K} L_{\max}]} \cdot \frac{L_{\max}}{L_n} \cdot [L_n \Lambda]$$

CONTRIBUTION TO TOTAL ERROR BUDGET:

- scale setting observable $f_{\rm K}$ (input) $\Delta f_{\rm K} \simeq 0$
- hadronic low energy scale $L_{\max} = 1/\mu_{\min}$ $\Delta[(a f_{\rm K})(L_{\max}/a)]_{a\to 0} \sim 1\%$ [Bruno,Tue-P3B]
- safe use of PT at high energies $L_n = L_{\min} \sim (64\,{\rm GeV})^{-1}$ $\Delta_{\rm PT}[L_n \Lambda] \simeq 0$

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- continuum step-scaling function $\sigma(u)$:
  $L_{\rm max} \to L_{\rm max}/2 \to \cdots \to L_{\rm max}/2^n = L_n$

  $\rightsquigarrow$ error accumulation per step: $\Delta\sigma_k\ ,\ k \in \{1, 2, \cdots, n\}$

  $$\sigma_k = \lim_{a\to 0} \Sigma(u_k, a/L)\ , \quad u_k = \bar{g}^2(L_k)$$

  - cutoff effects
  - statistical accuracy
  - RG scaling
- safe use of PT at high energies $L_n = L_{\rm min} \sim (64\,{\rm GeV})^{-1}$ — $\Delta_{\rm PT}[L_n\Lambda] \simeq 0$

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so far: $\bar{g}^2_{\rm SF}(L)$ — but there are advantages of the gradient flow ...

A comparison

Step-scaling error budget:

$$\left(\frac{\Delta\Lambda}{\Lambda}\right)^2 \propto \sum_k \left(\frac{\Delta L}{L}\right)^2\Bigg|_{\sigma_k}, \quad \text{with} \quad \left|\frac{\Delta L}{L}\right| = \frac{\Delta\bar{g}^2}{2\bar{g}\beta(\bar{g})} \sim \frac{\Delta\bar{g}^2}{2b_0\bar{g}^4} \sim \frac{1}{\bar{g}^2}\left[\frac{\Delta\bar{g}^2}{\bar{g}^2}\right] \quad \text{in the continuum}$$

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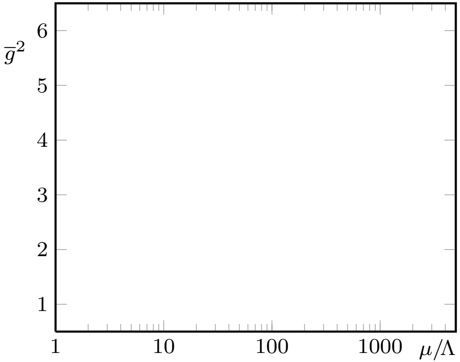
$\Rightarrow \Delta L/L$	const.	$\sim$ const$/\bar{g}^2$	
Summary	✓ small volume ✗ large volume	✗ small volume ✓ large volume	

+ min(COMPUTING COST) + max(CONTROL SYSTEMATICS) $\Rightarrow$ our new strategy

Combine SF and GF running couplings



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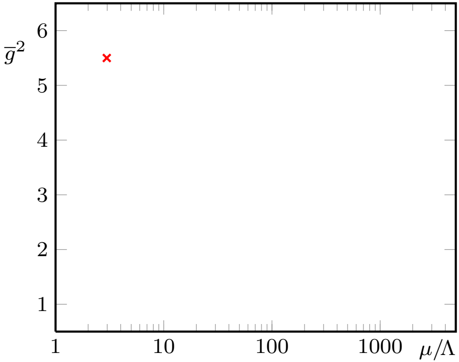

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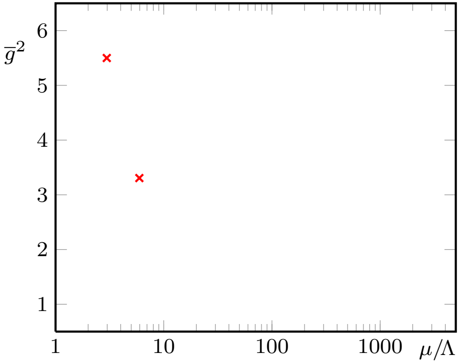

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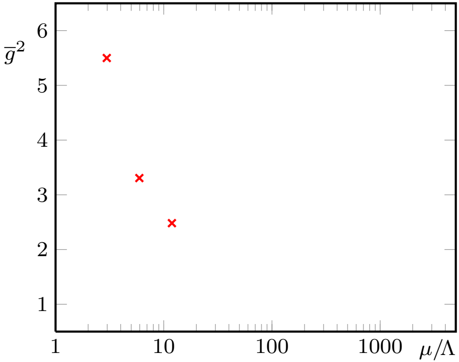

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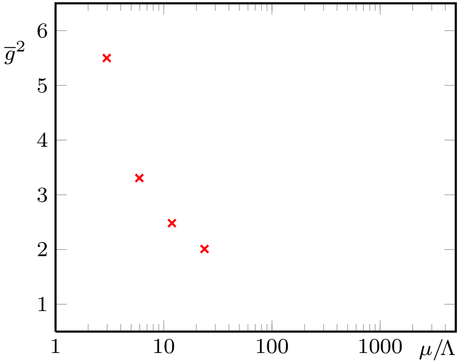

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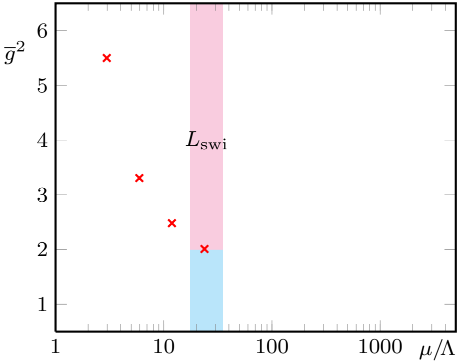

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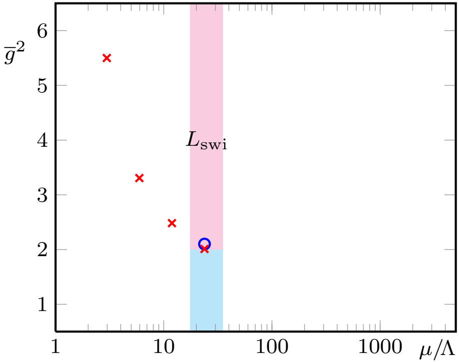

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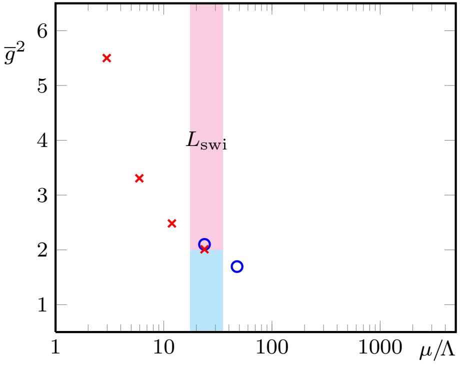

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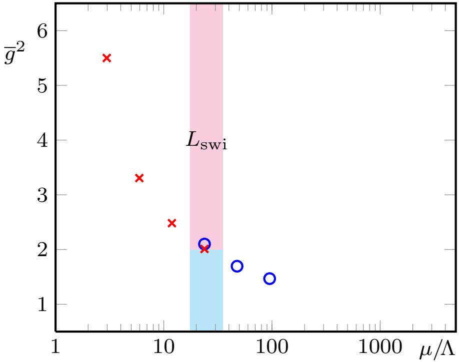

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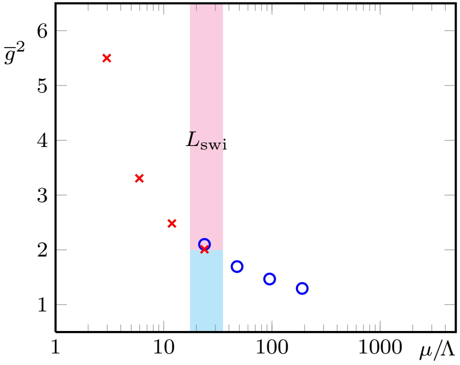

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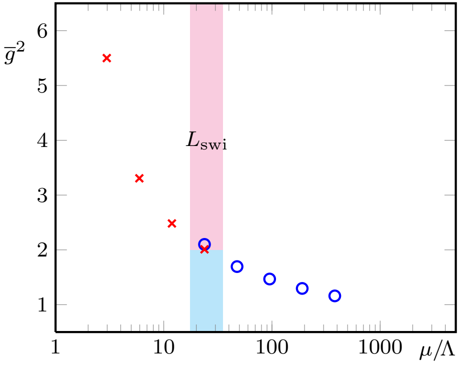

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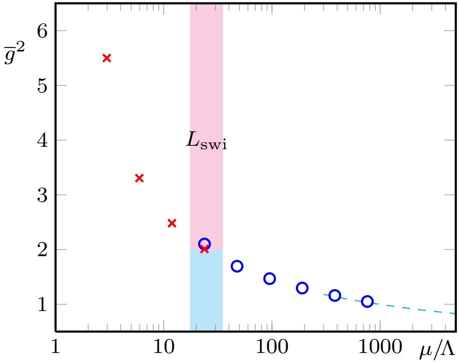

point 6 + 7 to be finished soon

preliminary results follow

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5. match $\bar{g}^2_{\rm SF}$ & $\bar{g}^2_{\rm GF}$ non-perturbatively

$$u = \bar{g}^2_{\rm GF}(L_{\rm swi}) \text{ fixed}$$

$$\bar{g}^2_{\rm SF}(L_{\rm swi}) = \lim_{a\to 0} \Psi(u, a/L)$$

   or vice versa

6. step scaling with SF coupling
7. connect $\bar{g}^2_{\rm SF}$ to PT at high energies

$N_\mathrm{f} = 3$ degenerate flavours of massless quarks

Tuning

tune to vanishing mass $Lm = 0$ $\Leftrightarrow$ 'critical' quark mass $m_0 = m_c$

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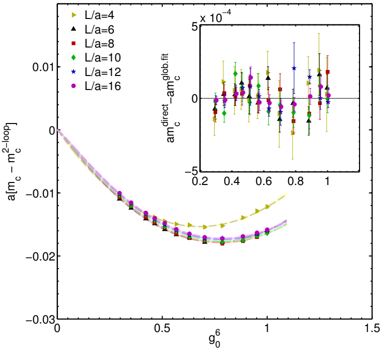


60 simulations $\forall L/a$ $(6 \leq \beta \leq 9, |Lm| < 0.005)$

$\Downarrow$

$\rightsquigarrow$ smooth function $\forall\, g_0^2 \leq 1$

$$am_c(g_0^2, a/L) = [am_c(g_0^2, a/L)]_{\text{2-lp}} + k_1(a/L)\cdot g_0^6 + k_2(a/L)\cdot g_0^8 + k_3(a/L)\cdot g_0^{10}$$

$\Downarrow$

$|Lm_c| < 0.001$

additional uncertainty from LCP tuning *negligible*

Step-scaling function for $\bar{g}^2_{\mathrm{SF}}$

preliminary

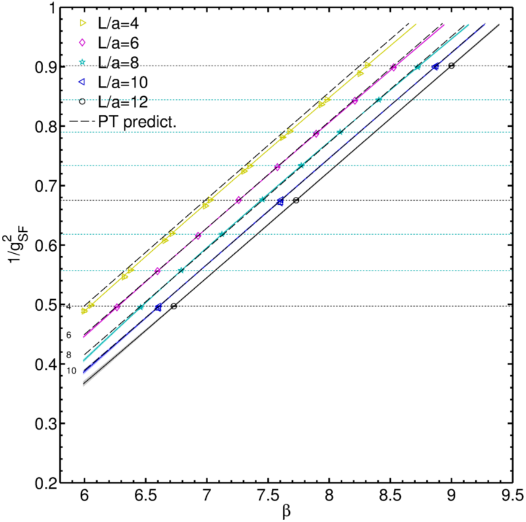
Fix $\bar{g}^2_{\mathrm{SF}}(L_k) = u_k$

- $1.1 \leq u_k \leq 2$
- equidistant in $1/\bar{g}^2_{\mathrm{SF}}$
- $L/a = 4, 6, 8, 10, 12$

and simulate at $2L/a$

$\Downarrow$

$\Sigma(u_k, a/L)$



Step-scaling function for $\bar{g}^2_{\mathrm{SF}}$

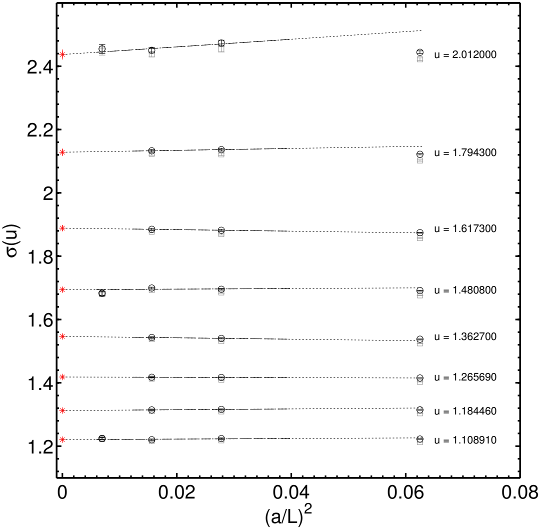
preliminary

$L \to 2L$:

$$\sigma(u) = \lim_{a\to 0} \Sigma^{(2)}(u, a/L)$$

- ignore $2L/a = 8$ data
- $2L/a = 20$ still missing
- PT improvement: $\Sigma \to \Sigma^{(2)}$
- CL: linear fit to $2L/a = 12, 16, 24$ data
- global & local fit ansatz compatible

local continuum limit extrapolations:



Step-scaling function for $\bar{g}^2_{\mathrm{SF}}$

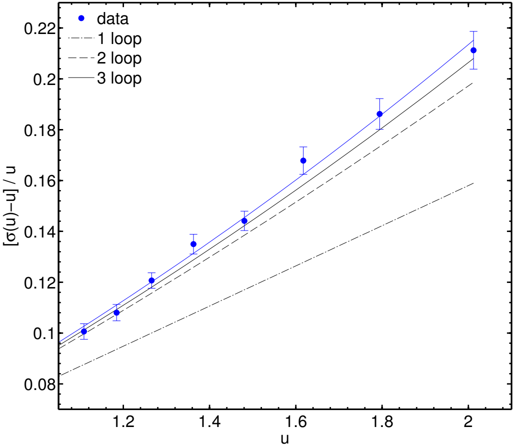
preliminary

continuum step-scaling function $\sigma(u)$:

- data: $\sigma(u_k) \pm \Delta\sigma(u_k)$
- fit ansatz

$$\sigma(u) = u + s_0 u^2 + s_1 u^3 + s_2^{\mathrm{fit}} u^4 + s_3^{\mathrm{fit}} u^5$$

- s_0, s_1 fixed (scheme indep.)
- fitted parameters $s_2^{\mathrm{fit}}, s_3^{\mathrm{fit}}$ + $\mathrm{cov}(s_i^{\mathrm{fit}}, s_j^{\mathrm{fit}})$



Summary

need improvement of present knowledge over $\alpha_s(\mu) \leftrightarrow \Lambda$

in general

- systematic uncertainties well under control using lattice simulations + finite-size scaling + physical running couplings
- PT invoked at very high energies only $\gtrsim 100\,\mathrm{GeV}$
- hadronic scale set through large volume simulations

- there are particular (dis-)advantages for each running coupling scheme
  - GF-coupling: advantageous to reach even larger physical volumes
  - SF-coupling: advantageous for running in small volumes (femto universe)
- new, combined strategy employs both $\bar{g}^2_{\mathrm{SF}}$ and $\bar{g}^2_{\mathrm{GF}}$ in order to
  - increase accuracy in Λ-parameter
  - increase range of couplings covered & controlled by finite-size scaling
  - be cost efficient
- high-energy running of $\bar{g}^2_{\mathrm{SF}}(L)$ in good shape
- exact definition of $\bar{g}^2_{\mathrm{GF}}(L)$ still to be fixed

we are in a good position to achieve our goal: $\Delta\Lambda/\Lambda \lesssim 5\% \quad \leftrightarrow \quad \Delta\alpha/\alpha \lesssim 1\%$