B physics from non-perturbative HQET with two dynamical light quarks

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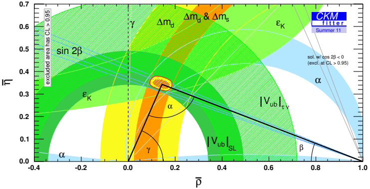
Outline

1. Obstacles of HQET on the lattice
2. Computational strategy of ALPHA
3. Overview of $N_\mathrm{f} = 2$ large volume (CLS) ensembles
4. Techniques used to compute LV matrix elements
5. Results in $N_\mathrm{f} = 2$

Motivation

Couplings of flavor-changing *weak interactions*:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



processes with $b \to u$ transitions

- Inclusive $B \to X_u \ell \nu$

  optical theorem and heavy quark expansion

- Exclusive $B \to \pi \ell \nu$

  hadronic formfactor $f_+(q^2)$

- Leptonic $B \to \tau \nu$

  hadronic decay constant f_B

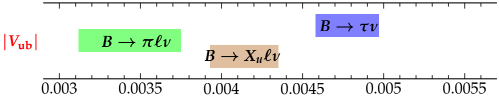
Lattice input

V_{ub} *'puzzle'*

$+$

$(\mathcal{B}(B \to \tau\nu), \sin(2\beta))$ discrepancy

PDG 2010:



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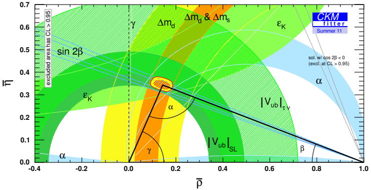

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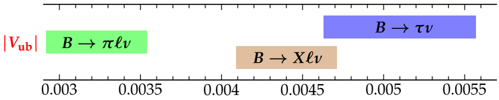

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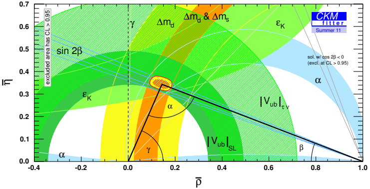

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precision??

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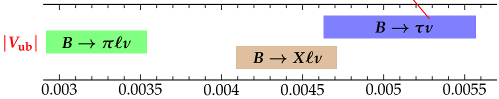

HQET on the lattice

Heavy Quark Effective Theory

Expansion in inverse heavy quark mass $1/m$ [Eichten; Isgur+Wise; Georgi]

$$\mathcal{L}_{\mathrm{HQET}} = \overline{\psi}_{\mathrm{h}} \Big[\underbrace{D_0 + \delta m}_{\text{static limit (LO)}} \underbrace{- \omega_{\mathrm{kin}} \boldsymbol{D}^2 - \omega_{\mathrm{spin}} \boldsymbol{\sigma B}}_{\text{NLO, O(1/m)}} \Big] \psi_{\mathrm{h}} + \ldots , \qquad \left. \begin{array}{c} \omega_{\mathrm{kin}} \\ \omega_{\mathrm{spin}} \end{array} \right\} \sim \frac{1}{2m}$$

operator $\mathcal{O}_{\mathrm{kin}} \equiv -\overline{\psi}_{\mathrm{h}} \boldsymbol{D}^2 \psi_{\mathrm{h}}$ kinetic energy from residual motion of heavy quark

operator $\mathcal{O}_{\mathrm{spin}} \equiv -\overline{\psi}_{\mathrm{h}} \boldsymbol{\sigma B} \psi_{\mathrm{h}}$ chromomagnetic interaction with gluon field

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With $\mathcal{L}_{\mathrm{HQET}} = \mathcal{L}_{\mathrm{stat}} + \sum_{n\geq 1} \mathcal{L}^{(n)}$, expand integrand in functional integral repres.

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[\phi]\, \mathcal{O}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{HQET}}} , \qquad \mathcal{Z} = \int \mathcal{D}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{HQET}}} ,$$

as a power series in $1/m$:

$$\mathrm{e}^{-\mathcal{S}_{\mathrm{HQET}}} = \exp\left\{ -a^4 \sum_x \mathcal{L}_{\mathrm{stat}}(x) \right\} \times \left\{ 1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \tfrac{1}{2}\left[a^4 \sum_x \mathcal{L}^{(1)}(x) \right]^2 - a^4 \sum_x \mathcal{L}^{(2)}(x) + \ldots \right\}$$

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This definition of HQET implies:

- $1/m$-terms appear as insertions of local operators only
  $\Rightarrow$ power counting: Renormalizability to each order in $1/m$
  $\Leftrightarrow$ $\exists$ continuum limit & universality (in contrast to NRQCD)
  (remark: **not** rigorously proven for *static theory to all orders in* g)
- Effective theory = (continuum) asymptotic expansion of QCD in $1/m$
- interaction with light d.o.f's still non-perturbative (in contrast to χPT)

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}[\phi]\, \mathrm{e}^{-\mathcal{S}_{\mathrm{rel}} - \mathcal{S}_{\mathrm{stat}}}\, \mathcal{O} \Big\{ 1 - a^4 \textstyle\sum_x \mathcal{L}^{(1)}(x) + \dots \Big\}$$

Heavy Quark Effective Theory on the lattice

- originally formulated by [Eichten+Hill '88-'90]: $D_0 + \delta m \to \nabla_0 + \delta m$
- again different discretisations: APE-,HYP-smeared actions
  mainly to cure bad $\frac{\text{signal}}{\text{noise}} \propto \exp[-E_{\text{stat}} x_0] \sim \exp[-(c g_0^2/a) x_0]$

Explicitly: EV in HQET to subleading order

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_{\text{stat}} + \omega_{\text{kin}}\; a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\text{kin}}(x) \rangle_{\text{stat}} + \omega_{\text{spin}}\; a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\text{spin}}(x) \rangle_{\text{stat}}$$

$$\equiv \langle \mathcal{O} \rangle_{\text{stat}} + \omega_{\text{kin}} \langle \mathcal{O} \rangle_{\text{kin}} + \omega_{\text{spin}} \langle \mathcal{O} \rangle_{\text{spin}}$$

with

$$\langle O \rangle_{\text{stat}} = \frac{1}{\mathcal{Z}} \int_{\text{fields}} O \; \exp\left\{ -a^4 \sum_x \left[\mathcal{L}_{\text{light}}(x) + \mathcal{L}_{\text{h}}^{\text{stat}}(x) \right] \right\}$$

mixing of operators of different dim. in $\mathcal{L}_{\rm HQET}$ induces power divergences

- **Example:** Mass renormalization pattern at static order of HQET

  mixing of $\overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ and $\overline{\psi}_{\rm h}\psi_{\rm h}$ $\rightsquigarrow$ linear divergence: $\delta m \propto a^{-1}$

$$\overline{m}_{\rm b}^{\overline{\rm MS}} = Z_{\rm pole}^{\overline{\rm MS}} \cdot m_{\rm pole} \,, \qquad m_{\rm pole} = m_{\rm b} - E_{\rm stat} - \delta m$$

$$\delta m = \frac{c(g_0)}{a} \sim {\rm e}^{+1/(2b_0 g_0^2)} \{ c_1 g_0^2 + c_2 g_0^4 + \ldots + O(g^{2n}) \}$$

- in PT: uncertainty = truncation error $\sim {\rm e}^{+1/(2b_0 g_0^2)} \cdot c_{n+1} \cdot g_0^{2n+2} \xrightarrow{g_0 \to 0} \infty$
  $\Rightarrow$ Non-perturbative $c(g_0)$ needed
  $\Rightarrow$ NP renormalization of HQET (resp. matching to QCD)
  required for continuum limit to exist

- power-law divergences even worse at higher orders in $1/m$:
  LO$\to$NLO: $a^{-1} \to a^{-2}$ in coeff.s of $\omega_{\rm kin}\mathcal{O}_{\rm kin}$, $\omega_{\rm spin}\mathcal{O}_{\rm spin}$ in $\mathcal{L}^{(1)}$ of $\mathcal{L}_{\rm HQET}$

Solution: NP'ly subtract power div. by exploiting finite volume

ALPHA's COMPUTATIONAL STRATEGY

General Strategy

[HeitgerSommer'01]

NP matching of QCD and HQET in small volume	$\Leftrightarrow$	relativistic b-quark feasible
+		
finite size scaling procedure	$\Leftrightarrow$	contact to large volumes

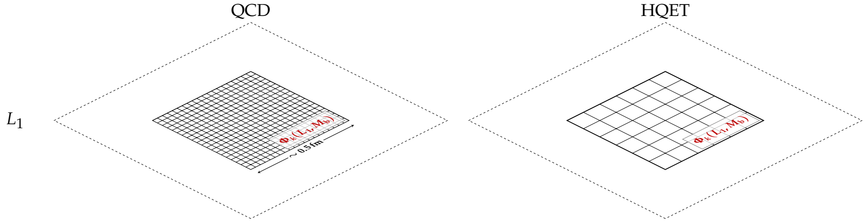
Framework:

- plaquette gauge action
- mass-degenerate doublet of non-perturbatively improved Wilson fermions
- two static quark actions (HYP discretization [HasenfratzKnechtli'01])

Ingredient: Schrödinger functional as intermediate renorm. scheme

- massless, finite volume renorm. scheme in the continuum
- Dirichlet b.c. in time $\Rightarrow$ 'IR save': $m = 0$ on the lattice
- NP definition of a running coupling $\Rightarrow \bar{g}^2(\mu)$, w/ box size $L = 1/\mu$
- $N_\mathrm{f} = 2$: QCD running coupling [ALPHA'04] and mass [ALPHA'05] known

General Strategy



Step 0: define line of constant physics

'light' sector:

$$\bar{g}^2(L_1/2) \equiv 2.989\,, \qquad L_1 m_l \equiv 0 \qquad (L_1 \approx 0.4\text{fm})$$ ⇒ tuning of $(\beta, \kappa_l, L_1/a)$

QCD: $L_1/a \in \{20, 24, 32, 40\}$ $\quad \Rightarrow a \leq 0.02\text{fm}$ $\quad \rightsquigarrow$ relativistic b-quark

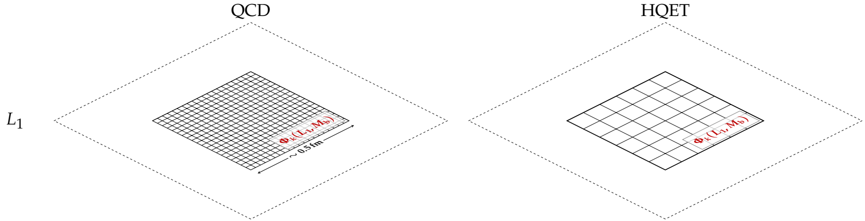
HQET: $L_1/a \in \{6, 8, 10, 12, 16\}$ $\quad$ coarser lattices sufficient

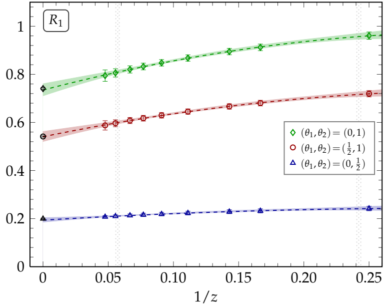
'heavy' sector in QCD: fix RGI heavy quark mass [PF,Heitger,Tantalo'11]

$$z = L_1 M = L_1 Z_M (1 + b_m\, a m_{q,h}) a m_{q,h} + O(a^2)\,, \qquad Z_M = \frac{Z(g_0) Z_A(g_0)}{Z_P(\mu, g_0)} h(L_1/2)$$

$$\in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$$

General Strategy





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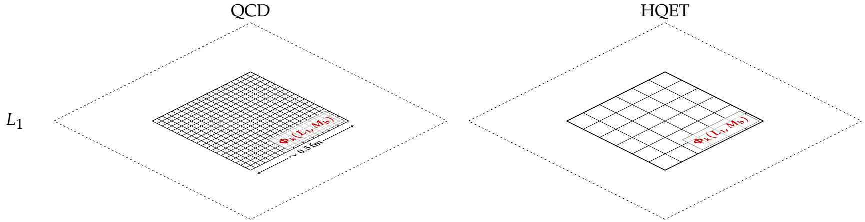


Step 1: choose proper observables for matching

$$\Phi^{\mathrm{QCD}} = \left(L_1\Gamma_{\mathrm{P}},\ \ln\left[-f_{\mathrm{A}}/f_1\right],\ R_{\mathrm{A}},\ R_1,\ \tfrac{3}{4}\ln[f_1/k_1]\right)^{\mathrm{t}}$$

with known expansion

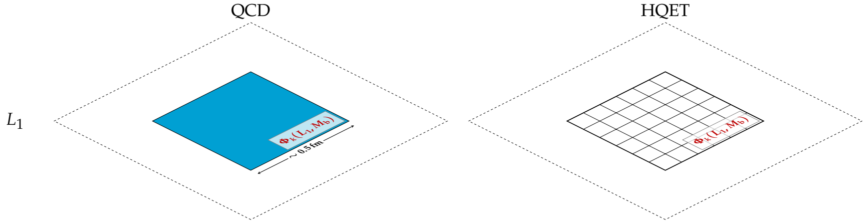
$$\Phi^{\mathrm{HQET}} = \eta + \varphi\cdot\omega$$

in order to extract HQET parameters at next-to-leading order

$$\omega = \left(m_{\mathrm{bare}},\ \ln Z_{\mathrm{A}}^{\mathrm{HQET}},\ c_{\mathrm{A}}^{(1)},\ \omega_{\mathrm{kin}},\ \omega_{\mathrm{spin}}\right)^{\mathrm{t}}$$

see [arXiv:1203.6516] for details

General Strategy

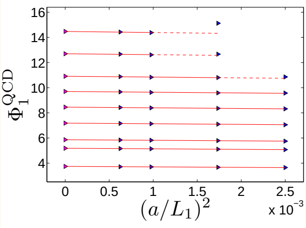


Step 2: take CL in QCD

Example: *PS eff. mass* $\Phi_1^{\mathrm{QCD}} = L_1 \Gamma_{\mathrm{P}}$

$$\Phi^{\mathrm{QCD}}(L_1, M) = \lim_{a \to 0} \Phi^{\mathrm{QCD}}(L_1, M, a)$$

for $L_1/a = 20, 24, 32, 40$



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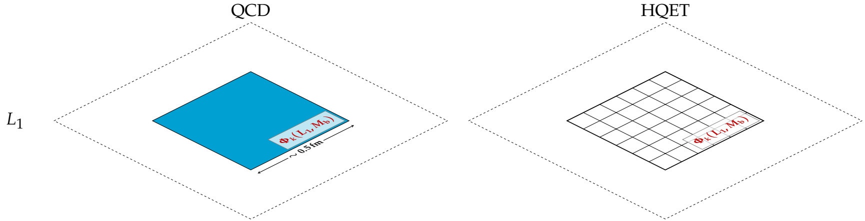

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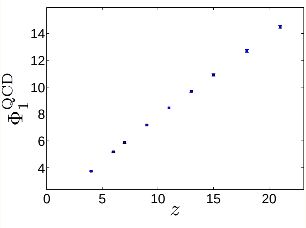

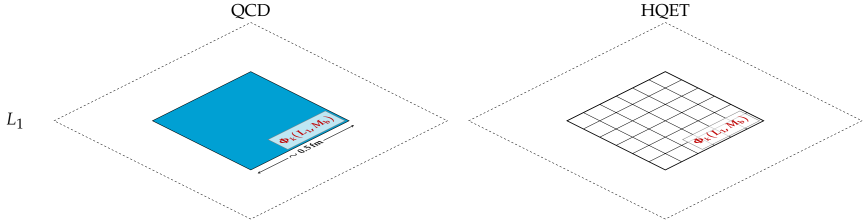

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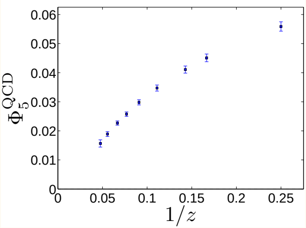

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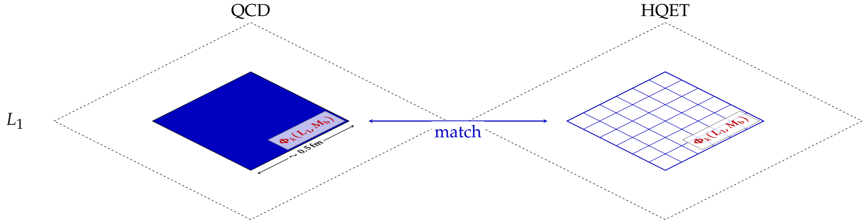


Step 3: match QCD and HQET in L_1

$$\Phi^{\mathrm{QCD}}(L_1, M) \equiv \Phi^{\mathrm{HQET}}(L_1, M, a)$$

$\forall z$ and $L_1/a \in \{6, 8, 10, 12, 16\}$

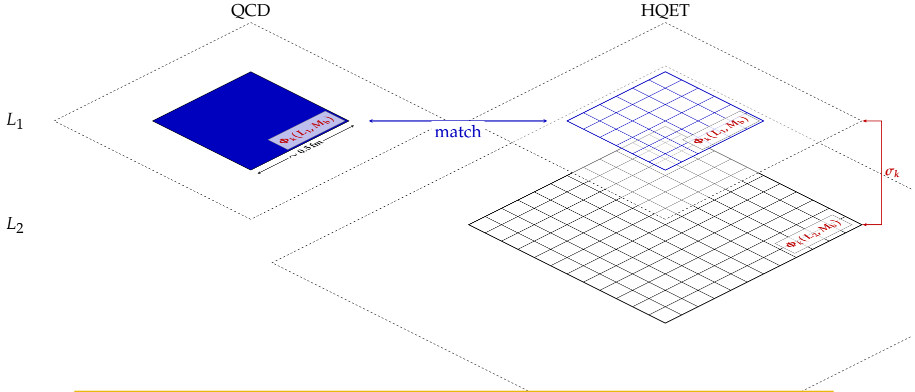
$$\Downarrow$$

$$\left(\Phi^{\mathrm{HQET}} = \eta + \varphi \cdot \omega\right)$$

$$\widetilde{\omega}(M, a) = \varphi^{-1}(L_1, a) \cdot \left(\Phi^{\mathrm{QCD}}(L_1, M) - \eta(L_1, a)\right)$$

$\widetilde{\omega}$ inherits 'mass-dependence' from yet arbitrary values of M

General Strategy

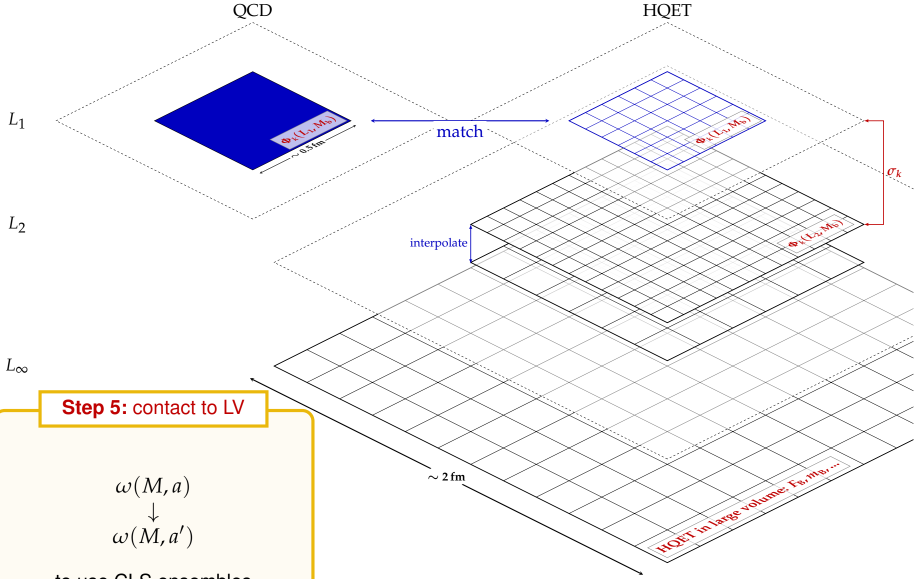


Step 4: finite size scaling

$L_1 \to L_2 = 2L_1$, while keeping bare parameters fixed

$$\begin{cases} \Phi^{\mathrm{HQET}}(L_2, M, 0) & = \lim_{a\to 0}\left[\eta(L_2,a) + \varphi(L_2,a)\cdot\tilde{\omega}(M,a)\right] \qquad \underline{\textit{CL exists!}} \\ \omega(M,a) & = \varphi^{-1}(L_2,a)\cdot\left(\Phi^{\mathrm{HQET}}(L_2,M,0) - \eta(L_2,a)\right) \end{cases}$$

General Strategy



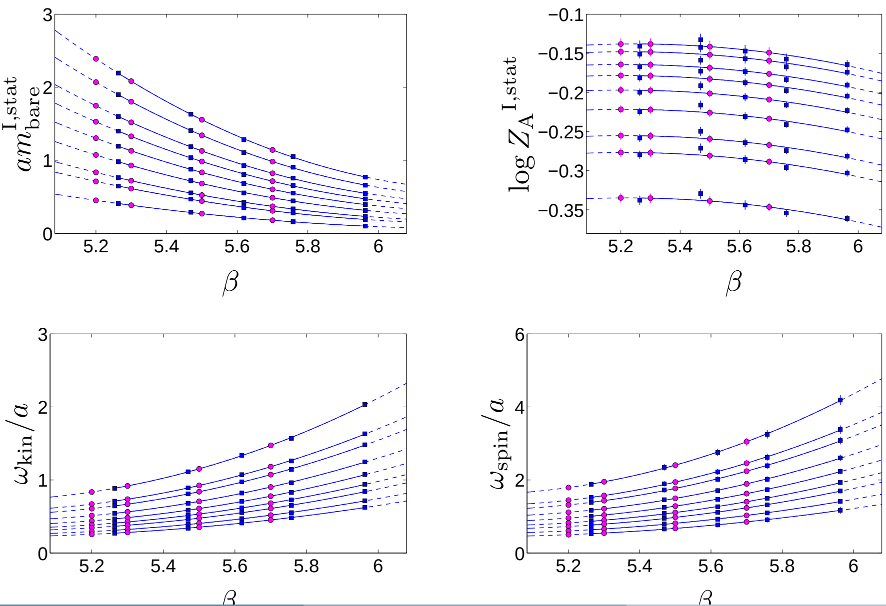
Step 5: contact to LV

$$\omega(M, a)$$
$$\downarrow$$
$$\omega(M, a')$$

to use CLS ensembles

NP'ly determined HQET parameters

[arXiv:1203.6516], $N_\mathrm{f} = 2$, HYP2, all z



NP'ly determined HQET parameters

[arXiv:1203.6516], $N_\mathrm{f} = 2$, $z = 13$, HYP1, HYP2

✓ expected absorption of power divergences

Example: bare quark mass

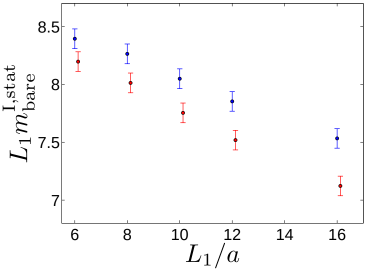
- static order:

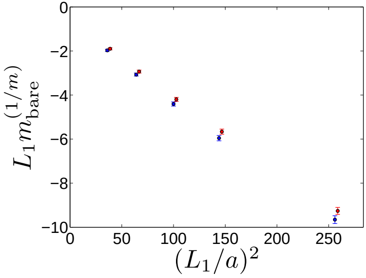
$$L_1 m_\mathrm{bare}^\mathrm{stat} \propto \frac{1}{a}$$

- $(1/m)$-correction:

$$L_1 m_\mathrm{bare}^{1/m} \propto \left(\frac{1}{a}\right)^2$$

✓ clear hierarchy in HQET expansion observed





NP'ly determined HQET parameters

(std. matching conditions)

parameters at β-values used in large volume simulations (HYP2)

β	LM_Q	$am_{\rm bare}$	$\ln(Z_{\rm A}^{\rm HQET})$	$\frac{c_{\rm A}^{(1)}}{a}$	$\frac{\omega_{\rm kin}}{a}$	$\frac{\omega_{\rm spin}}{a}$
5.2	13	1.207(18)	−0.139(31)	−0.54(9)	0.386(7)	0.825(30)
	$z_{\rm b}$	⋆	⋆	⋆	⋆	⋆
	15	1.459(20)	−0.119(31)	−0.50(9)	0.345(7)	0.727(28)
5.3	13	0.985(17)	−0.148(32)	−0.56(10)	0.425(8)	0.899(34)
	$z_{\rm b}$	⋆	⋆	⋆	⋆	⋆
	15	1.212(18)	−0.127(32)	−0.52(10)	0.380(8)	0.791(31)
5.5	13	0.582(14)	−0.166(36)	−0.68(12)	0.533(10)	1.109(42)
	$z_{\rm b}$	⋆	⋆	⋆	⋆	⋆
	15	0.769(15)	−0.142(36)	−0.63(12)	0.476(11)	0.976(39)

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$z_{\rm b} = L_1 M_{\rm b}$ to be determined through spectrum calculation in large volume HQET

CLS
LARGE VOLUME ENSEMBLES

CLS ensembles for large volume computations

subset used in this analysis

CLS based ensembles ($T = 2L$):

$m_\pi L \gtrsim 4.0$

β	a (fm)	L/a	Lm_π	m_π (MeV)	no. of cnfg.s	separ. (MD u.)	label	code
5.2	0.075	32	4.7	380	800	8	A4	DD
		32	4.0	330	200	4	A5	MP3
5.3	0.065	32	4.7	440	1000	16	E5	DD
		48	5.0	310	500	8	F6	DD
		48	4.3	270	600	8	F7	DD
		64	4.1	190	400	2	G8	MP5
5.5	0.048	48	5.2	440	400	8	N5	DD
		48	4.0	340	900	4	N6	MP3
		64	4.2	270	900	4	O7	MP2

- full Jackknife analysis (100 bins) from small to large volume
(but: new analysis software taking auto-correlations fully into account; Γ-Method)
- scale setting through f_K [arXiv:1205.5380]

Some details about our algorithm

MP-HMC implementation [MarinkovicSchaefer'10] supersedes domain decomposed HMC [Lüscher'05]

Idea: use efficient solver from DD-HMC package and get rid off inactive links (autocorr.$\searrow$)

$\Rightarrow$ allows to reach smaller pion masses

$\Rightarrow$ drawback: increased number of parameters to optimize

- mass preconditioning [Hasenbusch'10] for arbitrary N_{pf}
- SAP-GCR with switch for
  - deflation
  - chronological inversion

  for each pseudo-fermion $1, \ldots, N_{\mathrm{pf}}$
- Multiple time scale integrator [SextonWeingarten'92]
  - 2^{nd} order integrator [OmelyanEtAl'] for pseudo-fermions
  - leapfrog for gauge field on finest integration scale

Dynamical fermion simulations

criteria for subsequent data analysis:

- FV effects

$$Lm_\pi \geq 4.0$$

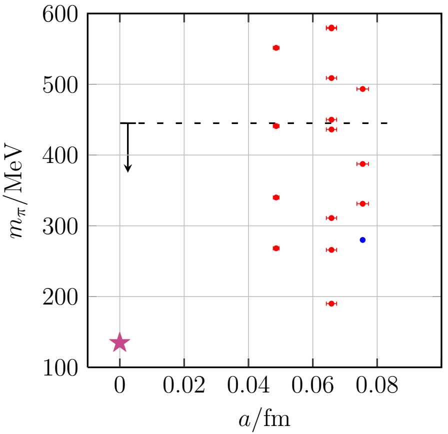
- chiral extrapolation

$$190 \lesssim m_\pi/\mathrm{MeV} \lesssim 450$$

- lattice spacings

$$a/\mathrm{fm} \in \{0.048, 0.065, 0.075\} < 0.1$$

CLS based



9 simulations fulfill our current criteria

+ 1 more this year

Large volume matrix elements

compute $N \times N$ correlator matrices

$$C_{ij}^{\text{stat}}(t) = \sum_{x,\mathbf{y}} \left\langle O_i(x_0+t,\mathbf{y})\, O_j^*(x) \right\rangle_{\text{stat}}$$

$$C_{ij}^{\text{kin/spin}}(t) = \sum_{x,\mathbf{y},z} \left\langle O_i(x_0+t,\mathbf{y})\, O_j^*(x)\, O_{\text{kin/spin}}(z) \right\rangle_{\text{stat}}$$

$$C_{A^{(1)},i}^{\text{stat}}(t) = \sum_{x,\mathbf{y}} \left\langle A_0^{(1)}(x_0+t,\mathbf{y}) O_i^*(x) \right\rangle_{\text{stat}}$$

using interpolating fields

$$O_k = \overline{\psi}_{\text{h}} \gamma_0 \gamma_5 \psi_{\text{l}}^{(k)}\,, \qquad \psi_{\text{h}}(x)\text{: static quark field}$$

$$O_k^* = \overline{\psi}_{\text{l}}^{(k)} \gamma_0 \gamma_5 \psi_{\text{h}}\,, \qquad \psi_{\text{l}}^{(k)}(x) = \left(1 + \kappa_{\text{G}}\, a^2\, \Delta\right)^{R_k} \psi_{\text{l}}(x)$$

$N = 3$ with APE-smeared links for different levels of Gaussian smearing such that $R_k \times (a/0.3\text{fm})^2 \in \{1, 4, 10\}$ kept fixed for each C^{stat}, $C^{\text{kin/spin}}$, and $C^{\text{stat}}_{A^{(1)}}$, we solve the GEVP

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)\,,$$

λ_n, v_n: eigenvalue & eigenvector of n^{th} state

$\Rightarrow$ energies E_n and operators Q_n with largest overlap to n^{th} state:

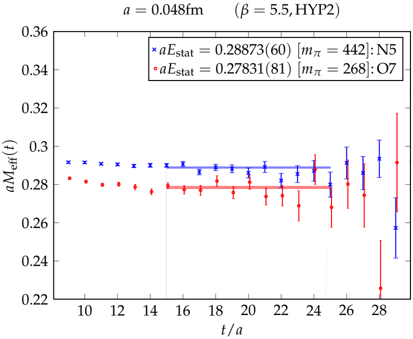
$$aE_n^{\text{eff}}(t,t_0) = -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right)$$

$$Q_n^{\text{eff}}(t,t_0) = \frac{O^i(t)v_n^i(t,t_0)}{\sqrt{v_n^i(t,t_0)C_{ij}(t)v_n^j(t,t_0)}}\left(\frac{\lambda_n(t_0+a,t_0)}{\lambda_n(t_0+2a,t_0)}\right)^{t/2a}$$

Large volume techniques, results

Results for aE_{stat} from GEVP at finest lattice spacing

Example: static energy aE_{stat}



with corrections (for $N = 3$)

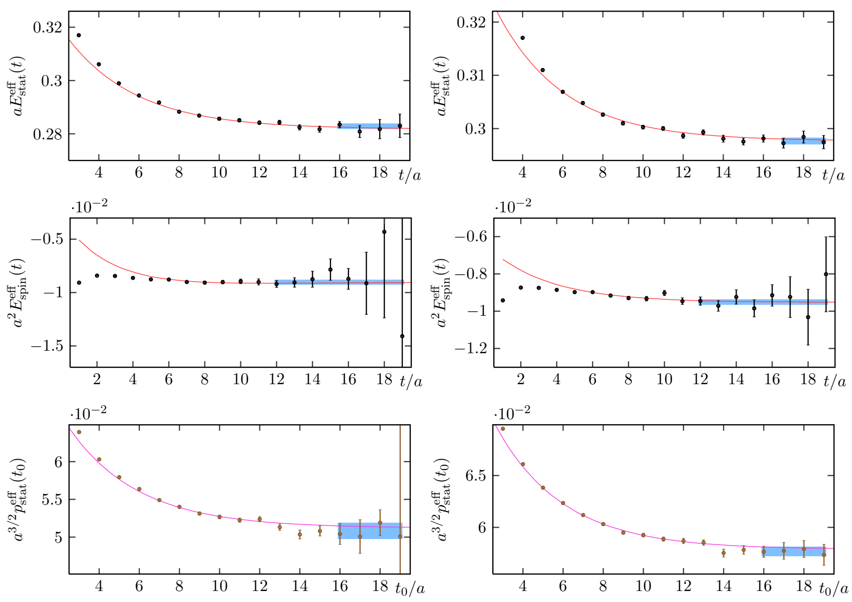
- for energies E_X:

$$\sim \mathrm{e}^{-t(E_4-E_1)}$$

- and for matrix elements p^X:

$$\sim \mathrm{e}^{-t_0(E_4-E_1)}\mathrm{e}^{-(t-t_0)(E_2-E_1)}$$

Large volume techniques, results



$N_{\mathrm{f}} = 2$ RESULTS

The b-quark mass

$$m_{\rm B} = m_{\rm bare} + E^{\rm stat} + \omega_{\rm kin} \cdot E^{\rm kin} + \omega_{\rm spin} \cdot E^{\rm spin} \qquad = m_{\rm B}(z, m_\pi, a)$$

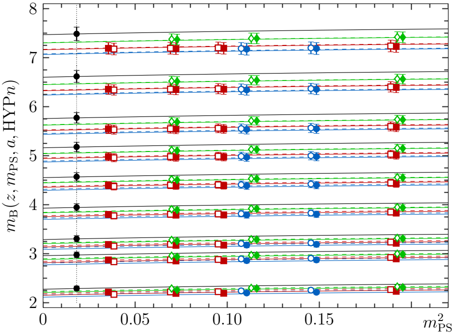
- parameters $\{m_{\rm bare},\ \omega_{\rm kin},\ \omega_{\rm spin}\}(z, a)$ & LV energies $\{E^{\rm stat},\ E^{\rm kin},\ E^{\rm spin}\}(m_\pi, a)$

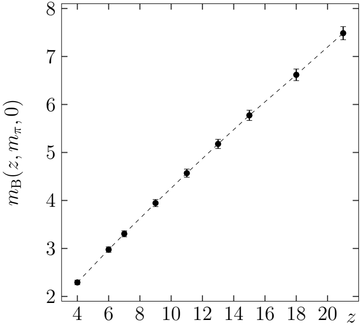
;

$$m_{\mathrm{B}} = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} \cdot E^{\mathrm{kin}} + \omega_{\mathrm{spin}} \cdot E^{\mathrm{spin}} \qquad = m_{\mathrm{B}}(z, m_\pi, a)$$

- parameters $\{m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}\}(z,a)$ & LV energies $\{E^{\mathrm{stat}}, E^{\mathrm{kin}}, E^{\mathrm{spin}}\}(m_\pi, a)$
- global fit ansatz from LO or NLO, i.e., using $B^*B\pi$ coupling $\widehat{g} = 0, 0.51(2)$

$$m_{\mathrm{B}}\,(z, m_\pi, a) = B(z) + C \cdot m_\pi^2 - \frac{3\widehat{g}^2}{16\pi f_\pi^2} \cdot m_\pi^3 + D \cdot a^2 \quad \Rightarrow \quad m_{\mathrm{B}}(z\ \ , m_\pi^{\mathrm{exp}})$$





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The b-quark mass

$$m_{\rm B} = m_{\rm bare} + E^{\rm stat} + \omega_{\rm kin} \cdot E^{\rm kin} + \omega_{\rm spin} \cdot E^{\rm spin} \qquad = m_{\rm B}(z, m_\pi, a)$$

- parameters $\{m_{\rm bare}, \omega_{\rm kin}, \omega_{\rm spin}\}(z,a)$ & LV energies $\{E^{\rm stat}, E^{\rm kin}, E^{\rm spin}\}(m_\pi, a)$
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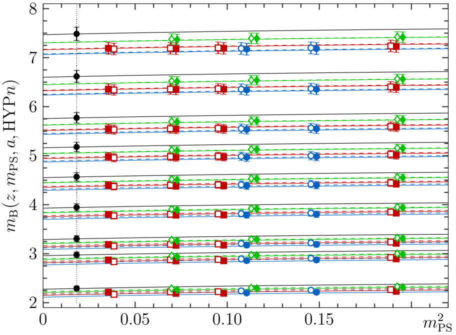

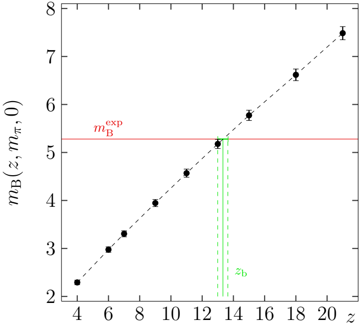

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The b-quark mass

inverting $m_{\mathrm{B}}(z_{\mathrm{b}}, m_{\pi}^{\mathrm{exp}}) = m_{\mathrm{B}}^{\mathrm{exp}}$ for RGI b-quark mass

$$z_{\mathrm{b}} \equiv L_1 M_{\mathrm{b}} = 13.34(33)(13)_z$$

$$\Downarrow$$

4-/3-loop running in the $\overline{\mathrm{MS}}$ scheme finally gives

$$\overline{m}_{\mathrm{b}}(\overline{m}_{\mathrm{b}}) = 4.22(10)(4)_z \mathrm{GeV}$$

$$\Downarrow$$

parameters at physical b-quark mass

$$\omega_i \equiv \omega_i(m_{\mathrm{b}}, a)$$

from now on

PDG (2010): $4.19^{+0.18}_{-0.06}\,\mathrm{GeV}$

PDG (2012): $4.18(3)\,\mathrm{GeV}$

The B-meson decay constant

$$\ln(a^{3/2} f_{\rm B} \sqrt{m_{\rm B}/2}) = \ln(Z_{\rm A}^{\rm HQET}) + \ln(a^{3/2} p^{\rm stat}) + b_{\rm A}^{\rm stat} a m_{\rm q} + \omega_{\rm kin} p^{\rm kin} + \omega_{\rm spin} p^{\rm spin} + c_{\rm A}^{(1)} p^{{\rm A}^{(1)}}$$

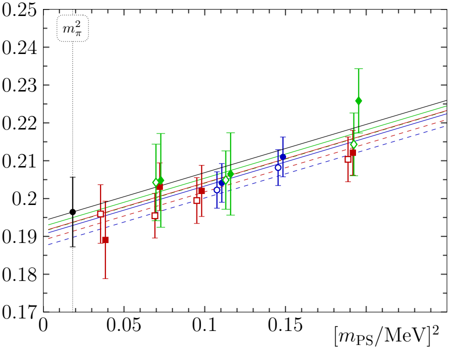
p^X: plateau values of eff. matrix elements from GEVP analysis

The B-meson decay constant $f_\mathrm{B}(z)|_{z=z_\mathrm{b}}$

we extrapolate to physical point $f_\mathrm{B} \equiv \lim_{(m_\pi,a)\to(m_\pi^{\mathrm{exp}},0)} f_\mathrm{B}(m_\pi, a)$ using fit ansatz

$$f_\mathrm{B}(m_\pi, a) = b + c m_\pi^2 + d a^2 \qquad \text{(LO)}$$

LO: $f_\mathrm{B}(z_\mathrm{b}, m_\mathrm{PS}, a)/\mathrm{GeV}$

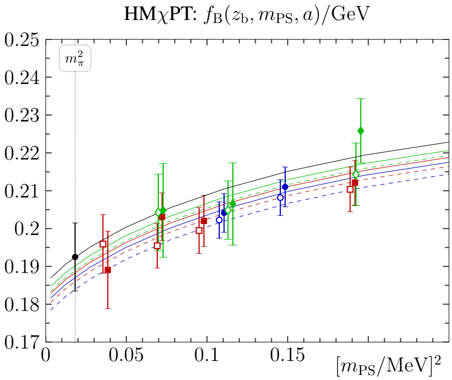


- for $\beta \in \{5.2, 5.3, 5.5\}$:
  HYP1 data: {●, ■, ◆}
  HYP2 data: {○, □, ◇}
- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_\mathrm{B} = 197(9)\mathrm{MeV}$ LO

we extrapolate to physical point $f_B \equiv \lim_{(m_\pi,a)\to(m_\pi^{exp},0)} f_B(m_\pi, a)$ using fit ansatz

$$f_B(m_\pi, a) = b + cm_\pi^2 + da^2 \qquad \text{(LO)}$$

$$f_B(m_\pi, a) = b'\left[1 - \frac{3}{4}\frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + c' m_\pi^2 + d' a^2 \qquad \text{(HM}\chi\text{PT)}$$



- for $\beta \in \{5.2, 5.3, 5.5\}$:
  HYP1 data: {●, ■, ◆}
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- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_B = 197(9)\text{MeV}$ LO
- $f_B = 193(9)\text{MeV}$ HMχPT

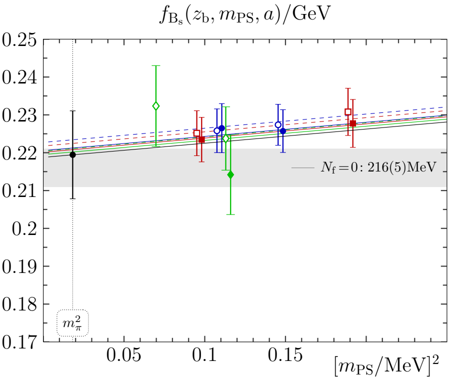
$f_\pi = f_\pi^{exp}$, $\hat{g} = 0.51(2)$

[PoS-Lat'10:BulavaETAL]

again: $f_{B_s} \equiv \lim_{(m_\pi,a)\to(m_\pi^{exp},0)} f_{B_s}(m_\pi, a)$ via

$$f_{B_s}(m_\pi, a) = b + cm_\pi^2 + da^2 \qquad \text{(LO)}$$

but no higher order formula available (to our knowledge)



- for $\beta \in \{5.2, 5.3, 5.5\}$:
  HYP1 data: {●, ■, ◆}
  HYP2 data: {○, □, ◇}
- not all ensembles analysis yet
- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_{B_s} = 219(12)\text{MeV}$ LO

Summary & outlook

- HQET obs. to next-to-leading order in $1/m_{\mathrm{b}}$ renormalized NP'ly ✓
- systematic errors included ✓
- $N_{\mathrm{f}} = 2$: power divergences canceled NP'ly $\rightsquigarrow$ continuum limit of certain observables taken in LV ✓

$$z_{\mathrm{b}} = 13.34(33)(13)_z\,, \qquad f_{\mathrm{B}} = 193(9)(4)_\chi \mathrm{MeV}$$
$$\overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}(\overline{m}_{\mathrm{b}}) = 4.22(10)(4)_z \mathrm{GeV}\,, \quad f_{\mathrm{B_s}} = 219(12)\mathrm{MeV}$$

still room to improve these results

- only truncation error $\mathrm{O}\big((\Lambda/m_{\mathrm{b}})^2\big)$ remains (but usually negligible)
- work in progress:
  - full analysis to be completed
  - measurements for $m_{B_{\mathrm{s}}}$, $f_{B_{\mathrm{s}}}/f_{\mathrm{B}}$, mass splittings, ...
  - $B \to \pi \ell \nu$ form factor $f_+(q^2)$
  - heavy baryons

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