Strange quark mass and Lambda parameter by the ALPHA Collaboration

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for ALPHA Collaboration (project B2)

Outline

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Lattice parameters $\overset{\text{match}}{\longleftrightarrow}$ experimental data ?

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  and converted to: $\boxed{\overline{m}_s^{\overline{MS}}(2\mathrm{GeV}) \quad \text{and} \quad \Lambda^{\overline{MS}}}$

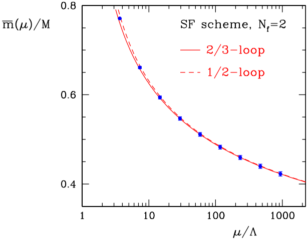
Setup:

- $\mathrm{O(a)}$ improved Wilson quarks, $\mathrm{N_f} = 2$
- Schrödinger functional scheme

  -running of α_s

  -quark mass renormalization

  by step scaling functions



Improvements:

- scale setting with r_0 (QCDSF, UKQCD)

  $\longrightarrow$ r_0-scale replaced by physical scale from f_K

  $\longrightarrow$ Λ, M_s

- continuum extrapolation of M_s , chiral extrapolation in $\mathrm{M}_{\mathrm{u,d}}$

- CLS / *ALPHA* simulations

	$\kappa_{\rm sea}$	m_π [MeV]	$m_\pi L$	MDU	$R_{act}\tau_{exp}$
$\beta = 5.2$	0.13565	632(20)	7.7	2950	
	0.13580	495(16)	6.0	2950	
	0.13590	385(13)	4.7	2986	25
$a \sim 0.08$fm	0.13594	331(11)	4.0	3094	
$\beta = 5.3$	0.13610	582(10)	6.2	927	
	0.13625	437(7)	4.7	5900	
	0.13635	312(5)	5.0	1769	50
$a \sim 0.07$fm	0.13638	267(5)	4.2	3473	
$\beta = 5.5$	0.13650	552(6)	6.5	1661	
	0.13660	441(5)	5.2	1686	200
$a \sim 0.05$fm	0.13671	268(3)	4.2	2796	

- τ_{exp} estimated from $\beta = 5.3$ and quenched scaling (Schaefer, Sommer, Virotta 2010)
- R_{act} - the fraction of active links (DD-HMC: ∼30%, MP-HMC 100%)

"setting the scale"

- calibration of the lattice spacing a in physical units:

  Q - dimensionful observable, $\frac{m_{q_i}}{m_{q_j}} = \mathrm{phys}$.

$$\boxed{a^{-1}[\mathrm{MeV}] = \frac{Q[\mathrm{MeV}]}{aQ}, \qquad Q = f_\pi, f_K, m_N, m_\rho, \ldots}$$

- different approximations of full QCD on the lattice (e.g. $N_f = 0, 2, 3, 4$)

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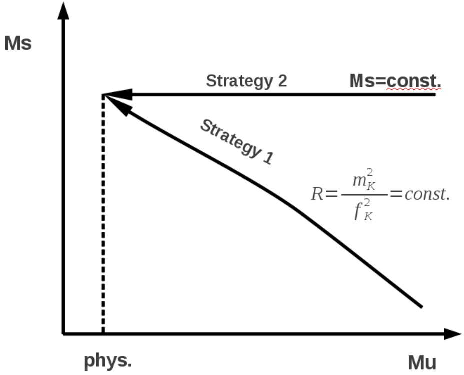
- We use f_K

  $\longrightarrow$ milder chiral extrapolation than from f_π

  $\longrightarrow$ better control over the systematic errors!

Setting the scale with f_K



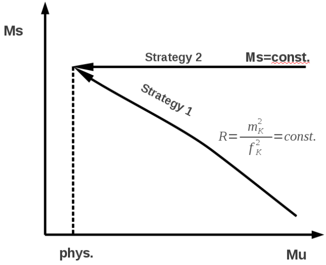
$\downarrow$

$M_u,\ M_d\ \equiv$ physical values

Strategy 1 to go to the physical point

- $$\boxed{\frac{m_K^2(\kappa_1,h(\kappa_1))}{f_K^2(\kappa_1,h(\kappa_1))} = \frac{m^2_{K,phys}}{f^2_{K,phys}} = R}$$
- $\mathrm{M_s} + \mathrm{M_u} \approx \mathrm{const.} + \mathrm{O}(\mathrm{M}^2)$
- interpolation determines $\kappa_3 = h(\kappa_1)$
- PQ-ChPT extrapolation:

  $f_K(\kappa_1, h(\kappa_1)) \rightarrow \mathrm{phys.\ point}$
- systematic expansion in m_π^2, $m_K^2 \leq m^2_{K,phys}$



Strategy 2 to go to the physical point

- fixed strange quark mass:

$$\boxed{m_{34}(\kappa_1, s(\kappa_1,\mu)) = \mu}$$

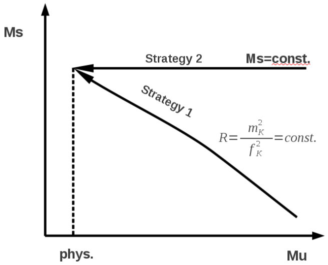
(*PCAC mass*)

- μ fixed, determine $s(\kappa_1,\mu)$ by interpolation in κ_3

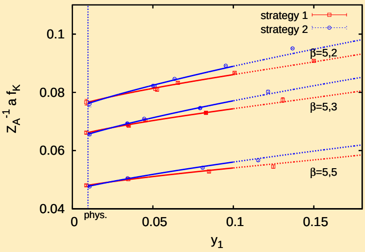
- heavy Kaon ChPT:

  extrapolate to $m_\pi^2/f_K^2 = \mathrm{phys.}$

- interpolate in μ to $m_K^2/f_K^2 = \mathrm{phys.}$



Two strategies for chiral extrapolation



- $y_1 = \frac{m_\pi^2(\kappa_1)}{8\pi^2 f_K^2(\kappa_1)}$;

  cut for $y_1 = 0.1 \quad (m_\pi < 500 MeV)$

- Constrained global fit:

  α_4, α_H independent of β

- $\text{a}\, f_K(\kappa_1, h(\kappa_1)) = \text{a}\, f_{K,phys}[1 + \bar{L}_K(y_1, y_K) + (\alpha_4 - \frac{1}{4})(y_1 - y_\pi) + O(y^2)]$ strategy 1

  $\bar{L}_K(y_1, y_K) = -\frac{1}{2} y_1 log(y_1) - \frac{1}{8} y_1 log(2y_K/y_1 - 1) + \frac{1}{2} y_\pi log(y_\pi) + \frac{1}{8} y_\pi log(y_\pi)$

- $\text{a}\, f_K(\kappa_1, s(\kappa_1, \mu)) = P(\mu)[1 - \frac{3}{8}[y_1 log(y_1) - y_\pi log(y_\pi)] + \alpha_H(y_1 - y_\pi) + O(M^2)]$ strategy 2

  $\text{a}^2 m_K^2(\kappa_1, s(\kappa_1, \mu)) = Q(\mu)[1 + \alpha'_H(y_1 - y_\pi) + O(M^2)]$

Lattice spacings

Strategy 1

β	5.2	5.3	5.5
$a[fm]$	0.0750	0.0655	0.04847
$\Delta_{stat.} a[fm]$	0.0024	0.0010	0.00048
$\Delta_{syst.} a[fm]$	0.0013	0.0011	0.00079

$\leftarrow$ dominated by error on Z_A

$y_1 < 0.1 : \quad \alpha_4 = 0.61(11)(8)$

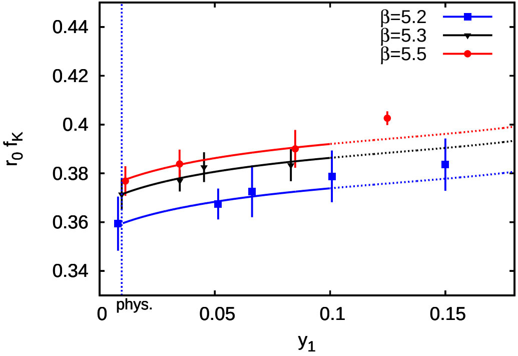
Strategy 2

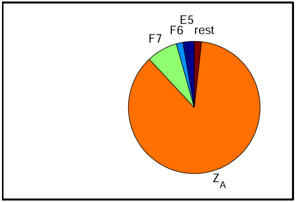
β	5.2	5.3	5.5
$a[fm]$	0.0745	0.0649	0.04808
$\Delta_{stat.} a[fm]$	0.0025	0.0010	0.00047
$\Delta_{syst.} a[fm]$	0.0014	0.0012	0.00090

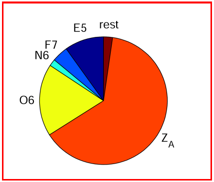
$\leftarrow$ dominated by error on Z_A

$y_1 < 0.1 : \quad \alpha_H = 1.17(8)(4)$

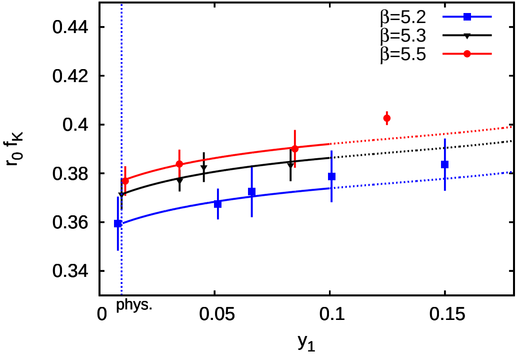
- systematics: chiral fit with chiral logs vs. linear fit (preliminary!)

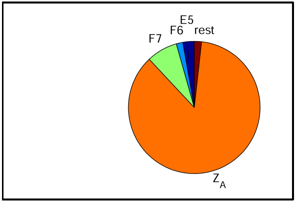


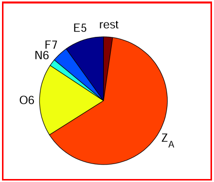




large component of the error is y_1 independent!







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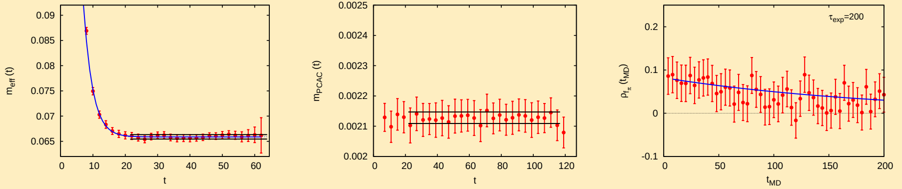
Z_A errors are being reduced!

Masses and decay constants

$128 \times 64^3 \text{lattice}, \quad a = 0.045\text{fm}, \quad m_\pi = 268\text{MeV}$



- Good control over the systematic errors
- Handle over the excited states $\rightarrow$ double exponential fit:

$$F(t) = A_0(e^{-mt} + e^{-m(T-t)}) + A_1(e^{-m't} + e^{-m'(T-t)}) \qquad [A_0, A_1, m]$$

- Criterion: statistical error $\geq 4 \times 1^{\text{st}}$ excited state contrib.
- Fit range determined $\quad \rightarrow \quad 2^{nd}$ step: single exponential ($A_1 = 0$)
- Errors computed conservatively!

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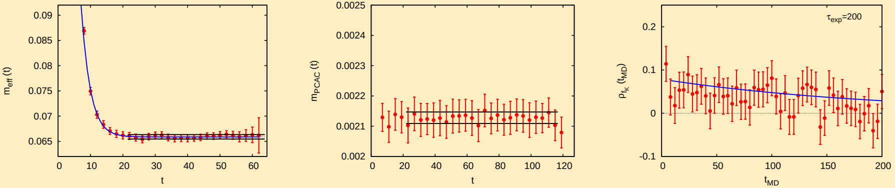

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Non-perturbative renormalization

- ALPHA Collaboration:

  - $N_f = 0$: Nucl.Phys. B413 (1994) 481-502 Nucl.Phys. B544 (1999) 669-698
  - N_f=2: Nucl.Phys. B713 (2005) 378-406 Nucl.Phys. B729 (2005) 117-134
  - $N_f = 4$: PoS Lattice2010:241,2010 10.1016/j.nuclphysb.2010.07.002

- Running coupling and quark mass

  $\rightarrow$ using recursive (step scaling) finite-size technique

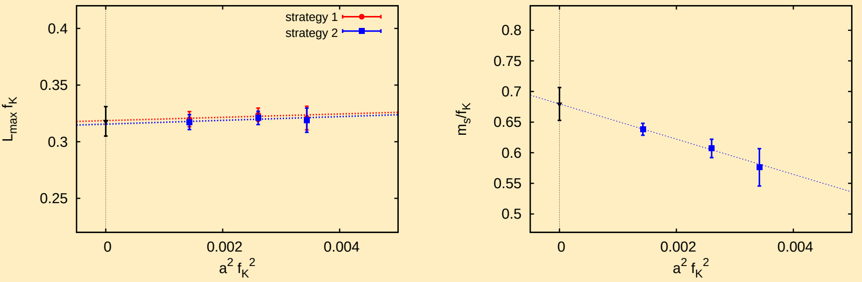
  $\rightarrow$ Fully non-perturbative procedure!

- Schrödinger functional coupling:

  $\rightarrow \overline{g}^2(L)$, $\mu = 1/L$

  $\rightarrow \boxed{\overline{g}^2(L_{max})}$, L_{max} - range of hadronic scale

Determination of Λ and $\overline{m}_s$



- $(f_K L_{max})_{cont} = 0.318(14)(6)$
- $\Lambda^{\overline{MS}} = \frac{\Lambda_{\overline{MS}}}{\Lambda} \frac{(\Lambda L_{max})_{cont}}{(f_K L_{max})_{cont}} f_K$

$$\Lambda^{\overline{MS}} = 316(26)(17)\,\mathrm{MeV}$$

- $\bar{g}^2(L_{max}) = 4.484$
- $(m_s/f_K)_{cont} = 0.680(27)(17)$
- $\overline{m}_s^{\overline{MS}}(\mu = 2\mathrm{GeV}) = \frac{\overline{m}_{\overline{MS}}(\mu)}{M} \frac{M}{\overline{m}(\tilde{L}_{max})} \frac{m_s(\tilde{L}_{max})}{f_K} f_K$

$$\overline{m}_s^{\overline{MS}}(\mu = 2\mathrm{GeV}) = 101.4(4.2)(2.5)\mathrm{MeV}$$

- $\bar{g}^2(\tilde{L}_{max}) = 4.61$

Conclusions

- Scale setting from f_K
  - two strategies for chiral extrapolation agree
  - simple linear extrapolation also within errors
- Result for m_s uses PT only through $\frac{\overline{m}_{\overline{MS}}}{M}$ $\leftarrow$ *4-loop*
- Comparing with previous ALPHA Collaboration computation for $N_f = 0, 2$:
  - $m_{s\ new}^{(2)} = 101.4(6.7) MeV \approx m_{s\ old}^{(2)} = 97(22) MeV$
  - $\underbrace{\Lambda_{\overline{MS}\ new}^{(2)} = 316(43) MeV > \Lambda_{\overline{MS}\ old}^{(2)} = 245(32) MeV}$

    - Λ changed significantly due to ALPHA Collaboration determination of r_0

      $\underline{\Lambda_{\overline{MS}\ new}\, r_0 = 0.78(6) > \Lambda_{\overline{MS}\ old}\, r_0 = 0.62(8)}$ ⟶Leder,Knechtli at Lattice'11

    - small shift in Λ due to $r_0 \to f_K$ ($r_0 = 0.48\mathrm{fm}$, ~~$r_0 = 0.5\mathrm{fm}$~~)
- In Progress: reducing errors coming from the determination of Z_A

Thank you!