M_b and $f_{\rm B}$ from non-perturbatively renormalized HQET with $N_{\rm f} = 2$ light quarks

Patrick Fritzsch

Institut für Physik, Humboldt-Universität zu Berlin, Germany

on behalf of the ALPHA Collaboration

Lattice 2011, 11 - 16 June
Squaw Valley, Lake Tahoe, USA

Problems in simulating heavy-light systems

Problem A

huge scale separations in relativistic treatment of b-quark systems on the lattice

m_{b} vs. a vs. L vs. predictability (#input par.)

Problems in simulating heavy-light systems

Problem A

huge scale separations in relativistic treatment of b-quark systems on the lattice

m_b vs. a vs. L vs. predictability (#input par.)

so far: solution through relaxation only

choose some way

. . . , RHQ, . . . , NRQCD, . . . , FermiLab action, . . . , HQET, . . .

and deal properly with its shortcomings, like

many input param.s, (non-)renormalizability, PT parameters, power divergences, . . .

Problems in simulating heavy-light systems

Problem A

huge scale separations in relativistic treatment of b-quark systems on the lattice

m_{b} vs. a vs. L vs. predictability (#input par.)

so far: solution through relaxation only

choose some way

. . . , RHQ, . . . , NRQCD, . . . , FermiLab action, . . . , HQET, . . .

and deal properly with its shortcomings, like

many input param.s, (non-)renormalizability, PT parameters, power divergences, . . .

Heavy Quark Effective Theory

effective theory of QCD, expansion in powers of $1/m_{\mathrm{b}}$

HQET on the lattice **Problem B:**

ops. of different dim. in $\mathcal{L}_{\mathrm{HQET}}$ mix and induce power divergences in a^{-1}

$\Rightarrow$ subtractions need to be performed NP'ly [Maiani,Martinelli,Sachrajda:'92]

Solution B:

NP matching of HQET and QCD in small physical volume [Heitger,Sommer:'01]

Problems in simulating heavy-light systems

Benefits of our approach

- power divergences subtracted non-perturbatively
  $\Rightarrow$ NP parameters of effective theory guarantee renormalizability
- applied step-scaling technique allows to reach large volumes to make contact to real physics
- matching requires only one input parameter, the B-meson mass $m_B^{\rm exp}$ (except for scale setting in large volume simulations)
- in HQET to order $1/m_{\rm b}$, truncation errors are $\mathrm{O}\left(\Lambda^2/m_{\rm b}^2\right)$
- approach can be extended to other quantities with modest effort

Heavy Quark Effective Theory

effective theory of QCD, expansion in powers of $1/m_{\rm b}$

HQET on the lattice **Problem B:**

ops. of different dim. in $\mathcal{L}_{\rm HQET}$ mix and induce power divergences in a^{-1}

$\Rightarrow$ subtractions need to be performed NP'ly [Maiani,Martinelli,Sachrajda:'92]

Solution B:

NP matching of HQET and QCD in small physical volume [Heitger,Sommer:'01]

Heavy Quark Effective Theory

to first order in $1/m_Q$

Lagrangian:

lowest (static) order | **1st order correction in $1/m_b$**

$$\mathcal{L}_{\rm HQET}(x) = \overline{\psi}_{\rm h}(x)\, D_0\, \psi_{\rm h}(x) \; - \omega_{\rm kin} \mathcal{O}_{\rm kin}(x) - \omega_{\rm spin} \mathcal{O}_{\rm spin}(x)$$

$$\mathcal{O}_{\rm kin}(x) = \overline{\psi}_{\rm h}(x) \mathbf{D}^2 \psi_{\rm h}(x)$$
$$\mathcal{O}_{\rm spin}(x) = \overline{\psi}_{\rm h}(x) \boldsymbol{\sigma}\cdot\mathbf{B} \psi_{\rm h}(x)$$

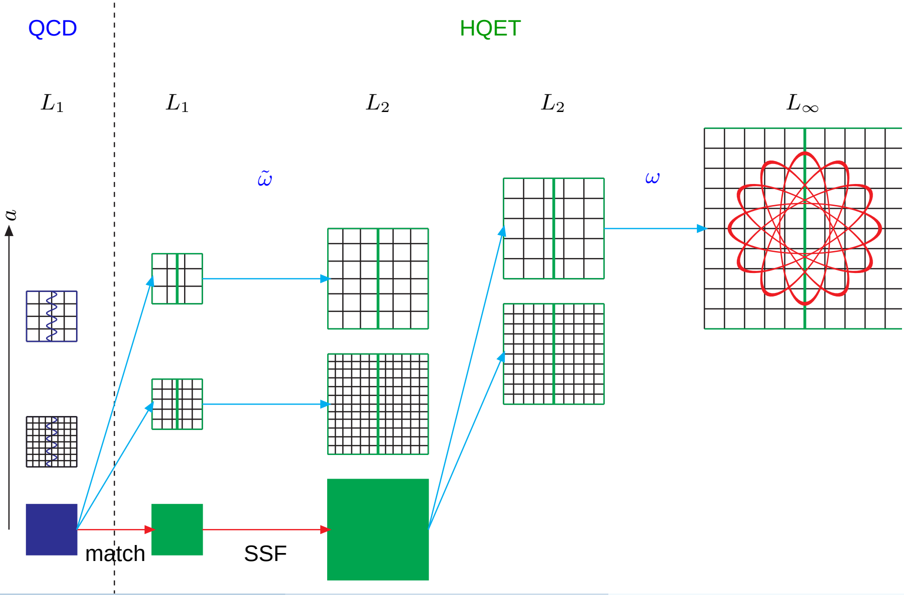
time component of the heavy-light axial current A_0 ($\mathbf{p} = 0$):

$$A_{0,\rm R}^{\rm HQET} = Z_{\rm A}^{\rm HQET}\,[A_0^{\rm stat} + c_{\rm A}^{(1)} A_0^{(1)}]\,,$$

$$A_0^{\rm stat} = \overline{\psi}_{\rm l}\,\gamma_0\gamma_5\,\psi_{\rm h}\,,$$
$$A_0^{(1)} = \overline{\psi}_{\rm l}\,\gamma_5\gamma_i \tfrac{1}{2}(\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S})\,\psi_{\rm h}$$

Parameters to match NP'ly: $\omega_i \in \left\{ m_{\rm bare}, Z_{\rm A}^{\rm HQET}, c_{\rm A}^{(1)}, \omega_{\rm kin}, \omega_{\rm spin} \right\}$

General Strategy



NP'ly determined HQET parameters

(std. matching conditions)

heavy (valence) quark masses NP'ly fixed to

$$z = L_1 M_Q = L_1 Z_M m_Q \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}$$

β	LM_Q	$am_{\rm bare}$	$\ln(Z_{\rm A}^{\rm HQET})$	$\frac{c_{\rm A}^{(1)}}{a}$	$\frac{\omega_{\rm kin}}{a}$	$\frac{\omega_{\rm spin}}{a}$
5.2	13	1.13(2)	−0.13(3)	−0.52(7)	0.470(6)	0.85(4)
	$z_{\rm b}$	⋆	⋆	⋆	⋆	⋆
	15	1.38(3)	−0.12(3)	−0.43(7)	0.429(5)	0.76(3)
5.3	13	0.90(2)	−0.13(3)	−0.54(8)	0.522(7)	0.94(4)
	$z_{\rm b}$	⋆	⋆	⋆	⋆	⋆
	15	1.13(2)	−0.12(3)	−0.45(8)	0.476(6)	0.84(4)
5.5	13	0.48(2)	−0.14(3)	−0.64(10)	0.656(8)	1.16(4)
	$z_{\rm b}$	⋆	⋆	⋆	⋆	⋆
	15	0.66(2)	−0.13(3)	−0.53(9)	0.599(8)	1.04(4)

CLS emsembles for large volume computations

subset used in this analysis

criterion on CLS based ensembles ($T = 2L$): $m_\pi L \gtrsim 2.1$ and $a \lesssim 0.08\text{fm}$

β	a (fm)	L/a	Lm_π	m_π (MeV)	no. of cnfg.s	separ. (MD u.)	label	code
5.2	0.075	32	2.4	386	800	8	A4 ○	DD
		32		331	200	4	A5 ○	MP
		48	3.6				B6	MP
5.3	0.065	32	2.1	438	1000	16	E5 □	DD
		48	3.1	312	500	8	F6 □	DD
		48		267	600	8	F7 □	DD
		64	4.2				G8	MP
5.5	0.048	48	2.3	442	400	8	N5 ◇	DD
		64	3.1	268	700	4	O7 ◇	MP

- full Jackknife analysis (100 bins) from small to large volume
- scale setting through f_K see M.Marinkovic's talk [today 6:30pm]

Large volume techniques

variance reduction through stochastic all-to-all props.

compute $N \times N$ correlator matrices

$$C_{ij}^{\text{stat}}(t) = \sum_{x,\mathbf{y}} \left\langle O_i(x_0+t,\mathbf{y})\, O_j^*(x) \right\rangle_{\text{stat}}$$

$$C_{ij}^{\text{kin/spin}}(t) = \sum_{x,\mathbf{y},z} \left\langle O_i(x_0+t,\mathbf{y})\, O_j^*(x)\, O_{\text{kin/spin}}(z) \right\rangle_{\text{stat}}$$

$$C_{A^{(1)},i}^{\text{stat}}(t) = \sum_{x,\mathbf{y}} \left\langle A_0^{(1)}(x_0+t,\mathbf{y}) O_i^*(x) \right\rangle_{\text{stat}}$$

using interpolating fields

$$O_k = \overline{\psi}_{\text{h}} \gamma_0 \gamma_5 \psi_{\text{l}}^{(k)}\,, \qquad \psi_{\text{h}}(x)\text{: static quark field}$$

$$O_k^* = \overline{\psi}_{\text{l}}^{(k)} \gamma_0 \gamma_5 \psi_{\text{h}}\,, \qquad \psi_{\text{l}}^{(k)}(x) = \left(1 + \kappa_{\text{G}}\, a^2\, \Delta\right)^{R_k} \psi_{\text{l}}(x)$$

$N = 3$ with APE-smeared links for different levels $R_k \in \{1,4,7\}$ of Gaussian smearing, $r_{\text{phys}} \approx 2a\sqrt{\kappa_{\text{G}} R_k} \sim 0.6\text{fm}$ kept fixed for each C^{stat}, $C^{\mathrm{kin/spin}}$, and $C^{\mathrm{stat}}_{A^{(1)}}$, we solve the GEVP

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)\,,$$

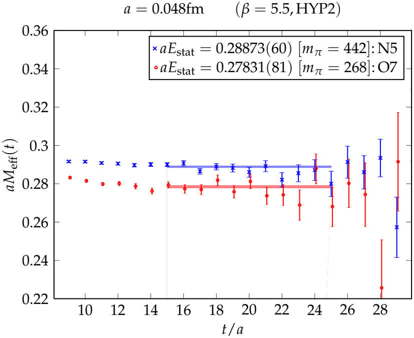
λ_n, v_n: eigenvalue & eigenvector of n^{th} state

$\Rightarrow$ energies E_n and operators Q_n with largest overlap to n^{th} state:

$$aE_n^{\mathrm{eff}}(t,t_0) = -\ln\left(\frac{\lambda_n(t+a,t_0)}{\lambda_n(t,t_0)}\right)$$

$$Q_n^{\mathrm{eff}}(t,t_0) = \frac{O^i(t)v_n^i(t,t_0)}{\sqrt{v_n^i(t,t_0)C_{ij}(t)v_n^j(t,t_0)}}\left(\frac{\lambda_n(t_0+a,t_0)}{\lambda_n(t_0+2a,t_0)}\right)^{t/2a}$$

Example: static energy aE_{stat}



with corrections

- for energies E_X:

$$\sim \mathrm{e}^{-t(E_4 - E_1)}$$

- and for matrix elements p^X:

$$\sim \mathrm{e}^{-t_0(E_4 - E_1)} \mathrm{e}^{-(t-t_0)(E_2 - E_1)}$$

The b-quark mass

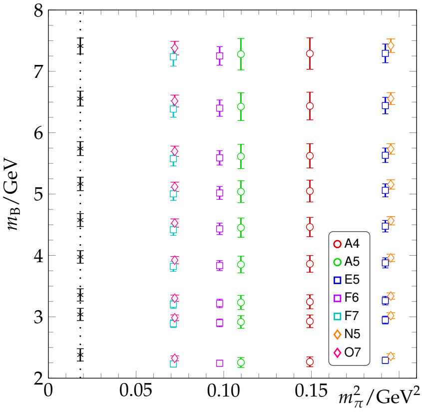
$$m_{\mathrm{B}} = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} \cdot E^{\mathrm{kin}} + \omega_{\mathrm{spin}} \cdot E^{\mathrm{spin}} \qquad = m_{\mathrm{B}}(z, m_\pi, a)$$

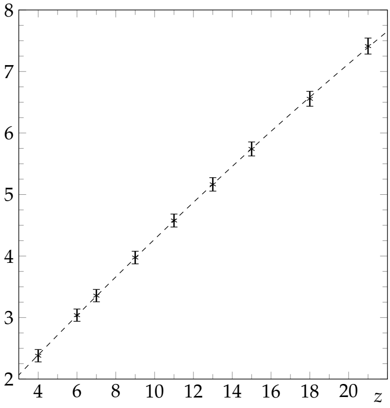
- parameters $\{m_{\mathrm{bare}},\, \omega_{\mathrm{kin}},\, \omega_{\mathrm{spin}}\}(z,a)$ & LV energies $\{E^{\mathrm{stat}},\, E^{\mathrm{kin}},\, E^{\mathrm{spin}}\}(m_\pi, a)$

The b-quark mass

$$m_\mathrm{B} = m_\mathrm{bare} + E^\mathrm{stat} + \omega_\mathrm{kin} \cdot E^\mathrm{kin} + \omega_\mathrm{spin} \cdot E^\mathrm{spin} \qquad = m_\mathrm{B}(z, m_\pi, a)$$

- parameters $\{m_\mathrm{bare}, \omega_\mathrm{kin}, \omega_\mathrm{spin}\}(z,a)$ & LV energies $\{E^\mathrm{stat}, E^\mathrm{kin}, E^\mathrm{spin}\}(m_\pi, a)$
- global fit: $m_\mathrm{B}\,(z, m_\pi, a) = B(z) + C \cdot m_\pi^2 + D \cdot a^2$ $\Rightarrow$ $m_\mathrm{B}(z \;, m_\pi^\mathrm{exp})$

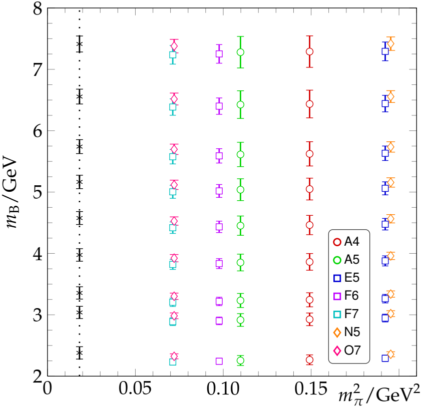


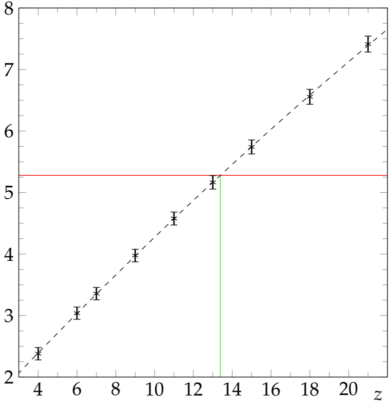


The b-quark mass

$$m_{\mathrm{B}} = m_{\mathrm{bare}} + E^{\mathrm{stat}} + \omega_{\mathrm{kin}} \cdot E^{\mathrm{kin}} + \omega_{\mathrm{spin}} \cdot E^{\mathrm{spin}} \qquad = m_{\mathrm{B}}(z, m_\pi, a)$$

- parameters $\{m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}\}(z,a)$ & LV energies $\{E^{\mathrm{stat}}, E^{\mathrm{kin}}, E^{\mathrm{spin}}\}(m_\pi, a)$
- global fit: $m_{\mathrm{B}}\,(z, m_\pi, a) = B(z) + C \cdot m_\pi^2 + D \cdot a^2 \;\Rightarrow\; m_{\mathrm{B}}\big(z_{\mathrm{b}}, m_\pi^{\mathrm{exp}}\big) = m_{\mathrm{B}}^{\mathrm{exp}}$





The b-quark mass & spin splitting

we invert

$$m_{\mathrm{B}}(z_{\mathrm{b}}, m_\pi^{\mathrm{exp}}) = m_{\mathrm{B}}^{\mathrm{exp}}$$

for $m_{\mathrm{b}}(m_{\mathrm{b}})$ in MSbar scheme

$$\Downarrow$$

$$m_{\mathrm{b}}(m_{\mathrm{b}}) = 4.234(76)_{\mathrm{stat}}(56)_z(14)_a \mathrm{GeV}$$

$$\Downarrow$$

parameters at physical b-quark mass

$$\omega_i \equiv \omega_i(m_{\mathrm{b}}, a)$$

from now on

PDG: $4.19^{+0.18}_{-0.06}$

The b-quark mass & spin splitting

we invert

$$m_B(z_b, m_\pi^{exp}) = m_B^{exp}$$

for $m_b(m_b)$ in MSbar scheme

$$\Downarrow$$

$$m_b(m_b) = 4.234(76)_{stat}(56)_z(14)_a \text{GeV}$$

$$\Downarrow$$

parameters at physical b-quark mass

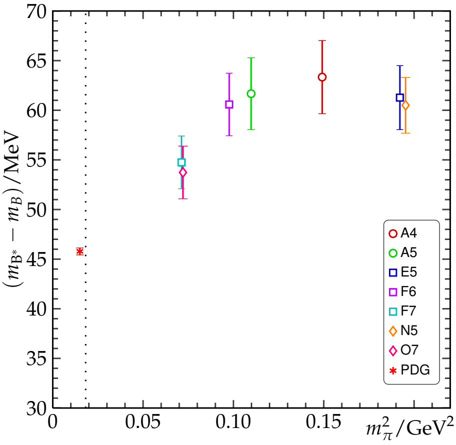
$$\omega_i \equiv \omega_i(m_b, a)$$

from now on

PDG: $4.19^{+0.18}_{-0.06}$

spin splitting at phys. b-quark mass

$$m_{B^*} - m_B = -\tfrac{3}{4}\omega_{spin} \cdot E^{spin}$$



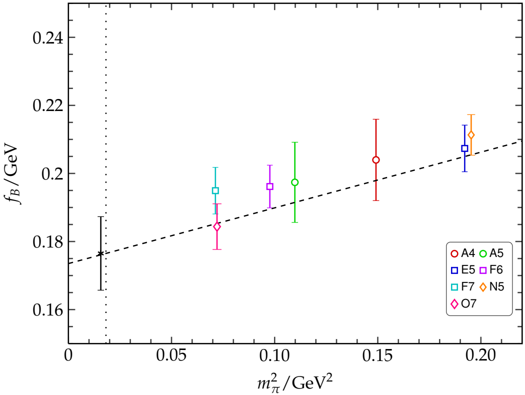
The B-meson decay constant

$$\ln(a^{3/2} f_{\rm B} \sqrt{m_{\rm B}/2}) = \ln(Z_{\rm A}^{\rm HQET}) + \ln(a^{3/2} p^{\rm stat}) + b_{\rm A}^{\rm stat} a m_{\rm q} + \omega_{\rm kin} p^{\rm kin} + \omega_{\rm spin} p^{\rm spin} + c_{\rm A}^{(1)} p^{{\rm A}^{(1)}}$$

The B-meson decay constant $f_{\mathrm{B}}(z)\big|_{z=z_{\mathrm{b}}}$

we extrapolate to physical point $f_{\mathrm{B}} \equiv \lim_{(m_\pi,a)\to(m_\pi^{\mathrm{exp}},0)} f_{\mathrm{B}}\,(m_\pi,a)$ using fit ansatz

$$f_{\mathrm{B}}\,(m_\pi,a) = b + cm_\pi^2 + da^2 \qquad \text{(LO)}$$



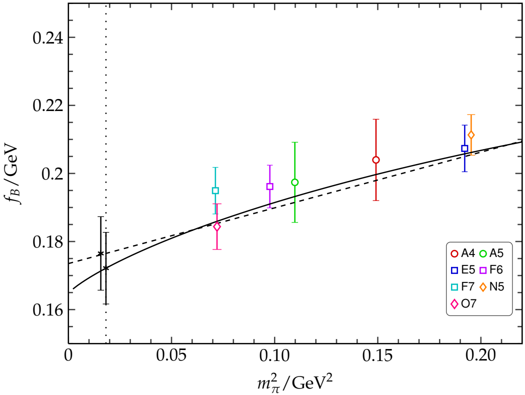
- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_{\mathrm{B}} = 176(11)(5)_a$ LO

The B-meson decay constant $f_B(z)|_{z=z_b}$

we extrapolate to physical point $f_B \equiv \lim_{(m_\pi,a)\to(m_\pi^{exp},0)} f_B(m_\pi, a)$ using fit ansatz

$$f_B(m_\pi, a) = b + cm_\pi^2 + da^2 \qquad \text{(LO)}$$

$$f_B(m_\pi, a) = b'\left[1 - \frac{3}{4}\frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + c' m_\pi^2 + d' a^2 \qquad \text{(HM}\chi\text{PT)}$$



- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_B = 176(11)(5)_a$ LO
- $f_B = 172(11)(5)_a$ HMχPT

$f_\pi = f_\pi^{exp}$, $\hat{g} = 0.51(2)$

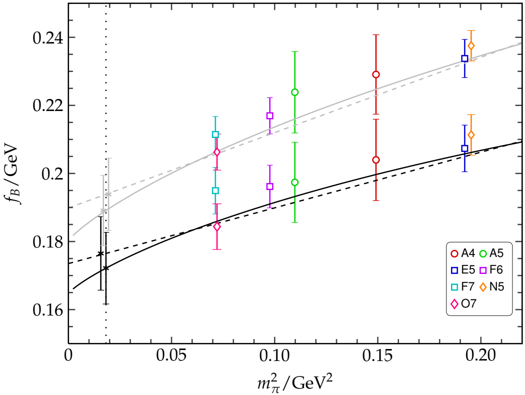
[PoS-Lat'10:BulavaETAL]

The B-meson decay constant $f_{\rm B}(z)|_{z=z_{\rm b}}$

we extrapolate to physical point $f_{\rm B} \equiv \lim_{(m_\pi,a)\to(m_\pi^{\rm exp},0)} f_{\rm B}(m_\pi, a)$ using fit ansatz

$$f_{\rm B}(m_\pi, a) = b + c m_\pi^2 + d a^2 \qquad \text{(LO)}$$

$$f_{\rm B}(m_\pi, a) = b' \left[1 - \frac{3}{4}\frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + c' m_\pi^2 + d' a^2 \qquad \text{(HM}\chi\text{PT)}$$



- no term in $(am_\pi)^2 \lesssim 0.02$
- $f_{\rm B} = 176(11)(5)_a$ LO
- $f_{\rm B} = 172(11)(5)_a$ HMχPT

$f_\pi = f_\pi^{\rm exp}, \ \hat{g} = 0.51(2)$

[PoS-Lat'10:BulavaETAL]

- static theory

$f_{\rm B} = 194(11)(5)_a$ LO

$f_{\rm B} = 189(11)(5)_a$ HMχPT

Summary & outlook

$$m_{\mathrm{b}} = 4.234(76)(56)_z(14)_a \mathrm{GeV}\,, \quad f_{\mathrm{B}} = 172(11)(5)_a \mathrm{MeV}$$

- HQET obs. to first order in $1/m_{\mathrm{b}}$ renormalized NP'ly ✓
- all systematic errors included ✓
- for the first time in $N_{\mathrm{f}} = 2$: power divergencies canceled NP'ly and continuum limit of certain observables has been taken in large volume ✓
- there are more observables to come
- only truncation error $O\big((\Lambda/m_{\mathrm{b}})^2\big)$ remain (but usually negligible)
- whole analysis still needs to be finalized
  - choose optimal set of kinematical parameters (in SF)
  - joint continuum extrapolation of HYP1 & HYP2 results (in LV)
  - ...

Thanks go to:
B.Blossier, J.Bulava, M.Della Morte, M.Donnellan, N.Garron, J.Heitger, G.von Hippel, B.Leder, M.Marinkovic, S.Schäfer, H.Simma, R.Sommer, F.Virotta