Simulated random surfaces and effective string models in 3d Z(2) gauge theory

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Background

Z(2) Wilson LGT on a $L_0 \times L \times L$ lattice:

$$Z = \sum_{\{\sigma_\mu(x)=\pm 1\}} \mathrm{e}^{\beta \sum_{x,\mu<\nu} \sigma_{\mu\nu}(x)}$$

Polyakov line correlation:

$$G(\vec{x}) = \langle \pi(\vec{x}) \pi(\vec{0}) \rangle, \quad \pi(\vec{x}) = \prod_{x_0=0}^{L_0-1} \sigma_0(x_0, \vec{x})$$

Random surface simulation, line defects at $\vec{u}$ and $\vec{v}$, wandering over the lattice (→previous talk by Tomasz Korzec)

$$G(\vec{x}) = \rho(\vec{x}) \frac{\langle\langle \delta_{\vec{x},\vec{u}-\vec{v}} \rangle\rangle}{\langle\langle \delta_{\vec{u},\vec{v}} \rangle\rangle}$$

- confined phase, build suitable ρ such that $\langle\langle \delta_{\vec{x},\vec{u}-\vec{v}} \rangle\rangle$ varies little
- signal/noise ratio ≈ independent of separation $\vec{x}$
- critical slowing down $z \approx 2$ [unfortunately]

Potential and string states

Polyakov correlation $\sim$ partition function with static charges

$$G(\vec{x}\,) = \sum_{n \geqslant 0} w_n \mathrm{e}^{-V_n(\vec{x}\,)\,L_0} \simeq w_0\,\mathrm{e}^{-V_0(\vec{x}\,)\,L_0} \qquad (L_0 \longrightarrow \infty)$$

$\mathrm{e}^{-V_n(\vec{x}\,)} \leftrightarrow$ transfer matrix in 0-direction with static charges, distance $\vec{x}\,$, $w_n \in \mathbb{N}$

alternative interpretation:

$$\sum_{x_2=0}^{L-1} G(x_1, x_2) = \sum_{n \geqslant 0} |v_n|^2\,\mathrm{e}^{-\tilde{E}_n x_1} \simeq |v_0|^2\,\mathrm{e}^{-\tilde{E}_0 x_1} \qquad (x_1, L \longrightarrow \infty)$$

$\mathrm{e}^{-\tilde{E}_n} \leftrightarrow$ transfer matrix in 1-direction, flux-state of length L_0, at zero momentum p_2

Lüscher, Weisz, 2004, have shown:

$$G(\vec{x}\,) = \sum_{n \geqslant 0} |v_n|^2\, 2r \left(\frac{\tilde{E}_n}{2\pi r} \right)^{\frac{1}{2}(D-1)} K_{\frac{1}{2}(D-3)}\big(\tilde{E}_n r\big) \qquad (r = |\vec{x}\,|)$$

arbitrary D, continuum, $K_{\dots}$=Bessel function, key: Radon transform

Effective string theory

- effective degrees of freedom: ‘vibrating’ surface bounded by the two Polyakov lines $\rightarrow$displacement field
- terms in effective action organized by dimension, powers of ∂_μ (non-renorm.)
- constrained by symmetry and open/closed string duality

$\longrightarrow$ leads to asymptotic large L_0-expansion:

$$\tilde{E}_0 = \sigma L_0 - \frac{\pi}{6L_0} - \frac{\pi^2}{72\sigma L_0^3} - \frac{\pi^3}{432\sigma^2 L_0^5} + \mathrm{O}(L_0^{-7})$$

- no free constants to this order ($D=3$, Lüscher, Weisz, Aharony, Karzbrun)
- dual to (equivalent) expansion of $V_0(r)$ for $r \rightarrow \infty$

all order formula from Nambu Goto action (= area of the surface):

$$z^2 = s^2\left(1 - \frac{1}{3s}\right), \quad s = \frac{\sigma L_0^2}{\pi}, \quad z = \frac{\tilde{E}_0 L_0}{\pi}, \quad s, z \rightarrow \infty$$

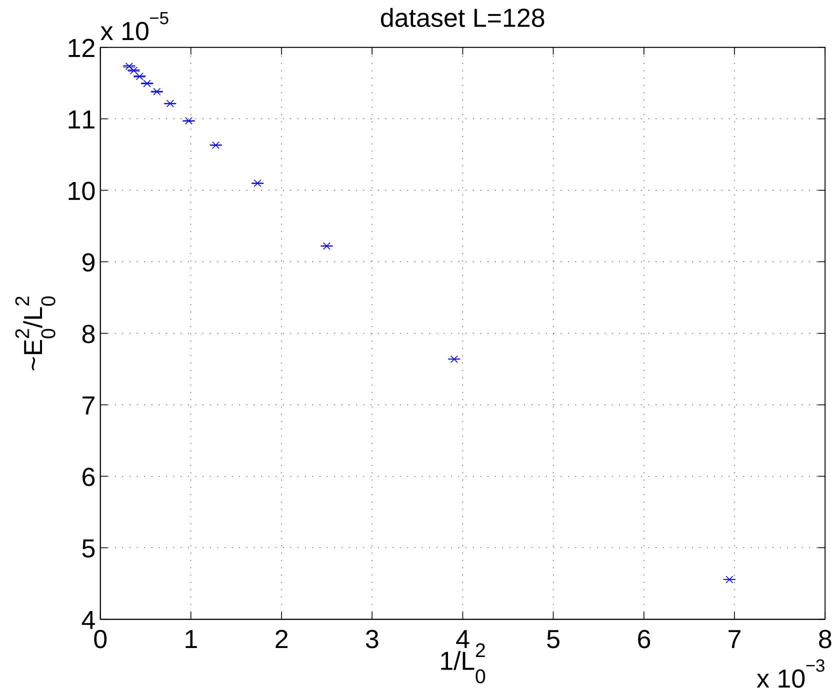
- NG leads to above terms and beyond; ultimately deviations expected!
- We have precise data for $\tilde{E}_0$ at many L_0 and $L/a = 64, 128$

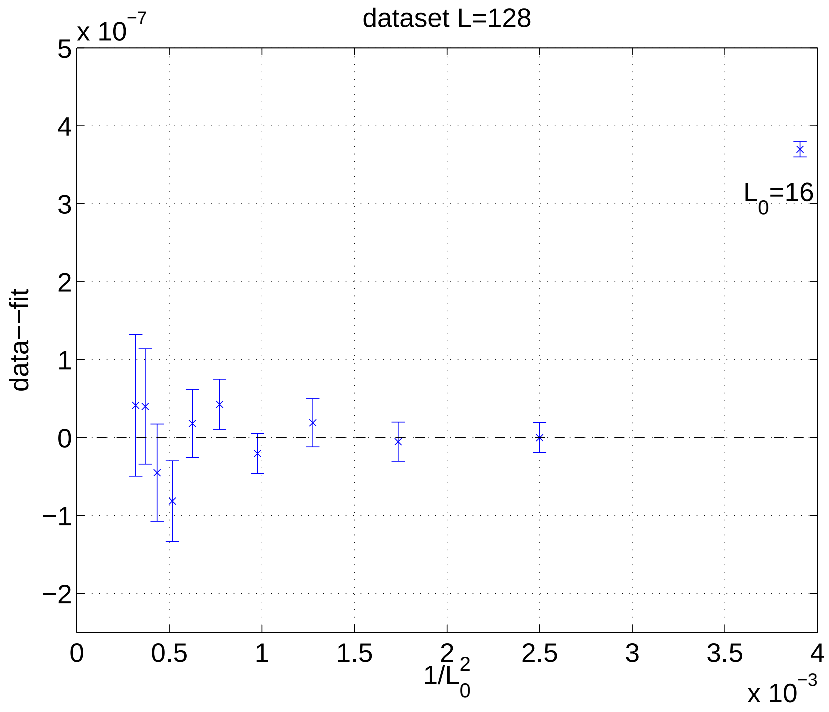
Data ... glance only ...

$\beta = 0.73107$			$\beta = 0.75146$		
$L = 64$	$\sigma \approx 0.044$		$L = 128$	$\sigma \approx 0.011$	
L_0	$\tilde{E}_0$	$\frac{\text{stat}}{10^6 \text{its}}$	L_0	$\tilde{E}_0$	$\frac{\text{stat}}{10^6 \text{its}}$
6	0.160286(15)	12	12	0.080998(6)	85
8	0.279241(21)	12	16	0.139831(9)	74
10	0.384096(28)	12	20	0.192035(20)	33
12	0.482528(36)	12	24	0.241174(30)	33
14	0.577660(47)	12	28	0.288694(43)	32
16	0.670782(60)	12	32	0.335173(39)	72
18	0.762774(77)	12	36	0.381209(55)	69
20	0.853968(98)	12	40	0.426695(82)	58
22	0.94450(12)	12	44	0.47173(11)	57
24	1.03480(14)	12	48	0.51682(14)	54
26	1.12416(16)	12	52	0.56187(18)	52
28	1.21429(19)	12	56	0.60661(24)	48

Data analysis

$$\text{Nambu Goto fit}: \quad \frac{\tilde{E}_0^2}{L_0^2} = \sigma^2 + c_1 \frac{1}{L_0^2}, \qquad \text{NG} \Longleftrightarrow r = \left(\frac{3c_1}{\pi\sigma}\right)^2 = 1$$





$L = 64$

$L_{0,\mathrm{min}}$	χ^2/dgf	σ	r
12	7.9/7	0.0440334(32)	1.0114(23)
10	8.0/8	0.0440330(25)	1.0109(13)
8	99/9	–	–

$L = 128$

$L_{0,\mathrm{min}}$	χ^2/dgf	σ	r
24	6.5/7	0.0109986(15)	0.9986(41)
20	6.5/8	0.0109986(10)	0.9987(20)
16	110/9		

- NG behavior compatible with our data
- Are we also testing the (presumably) non-universal term $\propto 1/L_0^7$ here??

$$\frac{\text{error}}{\text{term}} = \frac{\delta \tilde{E}_0}{5\pi^4/(10368\sigma^3 L_0^7)} = \frac{2\times 10^{-5}}{10^{-6}} \qquad (\text{at } L_0 = 20)$$

while $\pi^3/(432\sigma^2 L_0^5) = 1.8\times 10^{-5}$, $\pi^2/(72\sigma L_0^3) = 3.8\times 10^{-4}$

$$\tilde{E}_0 = \sigma L_0 - \frac{\pi}{6L_0} - \frac{\pi^2}{72\sigma L_0^3} - \frac{\pi^3}{432\sigma^2 L_0^5} + \mathrm{O}(L_0^{-7})$$

Conclusions

- all order strong coupling simulation of Z(2) gauge theory in $D=3$
- precise estimation of ground state energy of string state possible
  - easy isolation of ground state at large separation
- excellent agreement with effective string theory
  - earlier small mismatch = lattice artefact, gone as $a \to a/2$
- consistent with Nambu Goto action up to (incl.) L_0^{-5} term
  - these term are expected for all acceptable string actions
- terms L_0^{-7} and beyond still cannot be disentangled