

Instructions for the Z0 Praktikum

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This experiment is dedicated to the analysis of a subset of data recorded with the L3 experiment during 1992/1993, exploiting electron-positron collisions generated by LEP (Large Electron Positron Collider) at CERN. It allows to familiarise with the statistical methods employed by experimental physicists in the field of particle physics and to become acquainted with basic programming techniques useful inside and outside the field.

The experiment takes place on a computer. The data will be analysed by using a program written in Python. A basic version of the program will be passed to the students that will need to modify it in order to perform various measurements. Familiarity with Python is an advantage. Some knowledge of elementary particle physics is assumed.

Goal of the experiment

Measurement of the cross section of the Z-resonance, determination of the mass and decay time of the Z boson (M_Z, τ_Z), the electroweak mixing angle ($\sin\theta_W$), the number of quark colours. Improved knowledge in statistics, error handling and programming.

1 Physics of the Z-resonance

The following provides a short summary of the physics involving Z bosons and of the experimental apparatus used for the data taking. More details can be found in the literature suggested at: <https://www.physik.hu-berlin.de/de/eephys/teaching/lab/z0resonance>

The Z resonance in electron-positron collisions

The Standard Model of particle physics predicts that particles interact through the exchange of other particles, known as bosons. For example, the electromagnetic force is mediated by the exchange of photons, while the weak interactions, responsible for processes as the beta-decays, are mediated by the exchange of W and Z bosons. These two forces can be actually described by the same theory, known as the electroweak model. A very important parameter of such model is the electroweak mixing angle, or Weinberg angle $\sin\theta_W$, which gives information about how the two forces are mixed together. Unfortunately this angle cannot be predicted but must be measured experimentally. Similarly, parameters related to fundamental properties of the Z boson, as its mass, decay width, production cross section, cannot be predicted and need to be measured too. The Z boson can be easily produced by annihilating electrons and positrons at energies close to the mass of the boson itself, 91 GeV. Figure 1 shows the production cross section for the Z boson measured by the LEP experiments and some of the related parameters. The cross section increases rapidly as the energy gets closer to 91 GeV, and decreases shortly after. This phenomenon is known as resonance. The production and decay of the Z boson is represented by the Feynman diagram of Figure 2. The Z boson is unstable and decays into fermions, either leptons or quarks. Due to the properties of the strong interaction, quarks quickly convert into other particles, producing

cones of hadrons called jets. In principle, the electrons could convert to photons but the cross section of such a process is very small at these energies.

The decay width of the Z boson is inversely proportional to its lifetime and it can be determined by summing over the partial widths of the possible decays.

$$\Gamma_Z = \sum_f \Gamma_{Z \rightarrow ff} = \frac{1}{\tau_Z} \quad (\sim 2.5 \text{ GeV}) \quad (1)$$

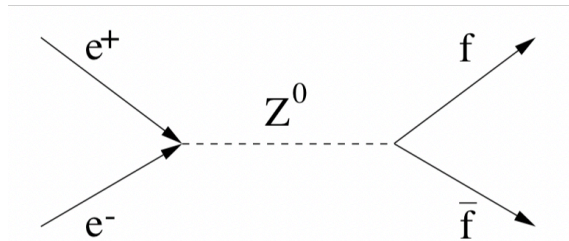
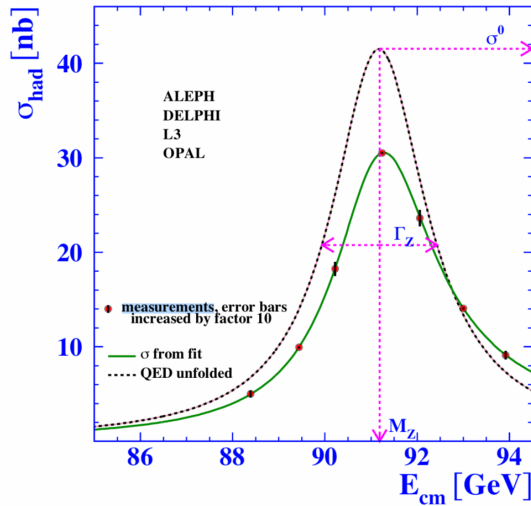


Figure 1: Combined cross section of the Z boson production from the LEP experiments.

Figure 2: Schematic representation of Z boson production and decay

The cross section can be described by using a probability density function known as Breit-Wigner distribution:

$$\sigma_f = \sigma_0 \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z} \quad (2)$$

where M_Z is the mass of the Z boson (the position of the maximum), s is the center of mass energy of the interaction, Γ_Z is the decay width and σ_0 is the cross section at the maximum¹. The cross section at the maximum is:

$$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{i=e}\Gamma_f}{\Gamma_Z^2} \quad (3)$$

where Γ_i and Γ_f are the initial and final state partial widths of the fermions. They are a measure of the coupling of the Z boson to the respective fermions and are a function of the electroweak mixing angle:

$$\Gamma_f = \frac{G_F M_Z^3}{24\sqrt{2}\pi} [1 + (1 - 4|Q_f| \sin^2 \theta_W)^2] \quad (4)$$

where Q_f is the charge of the fermion under consideration and $G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant. The charge is 0 for the three neutrinos, ± 1 for the charged leptons, $+2/3$ for the u, c quarks, and $-1/3$ for the d, s and b quarks². The total width for the hadronic decays can be obtained by multiplying the widths of the quark decays by a colour factor:

$$\Gamma_{had} = N_C(\Gamma_u + \Gamma_d + \Gamma_s + \Gamma_c + \Gamma_b) = N_C(2\Gamma_u + 3\Gamma_d) \quad (5)$$

The top quark is excluded by the equation because it is heavier than the Z boson and therefore it cannot be produced from its decay. Finally, the total width of the Z resonance is obtained by summing the hadronic width with the ones of the neutrinos and leptons:

$$\Gamma_Z = \Gamma_{had} + 3\Gamma_\nu + 3\Gamma_l \quad (6)$$

¹ This PDF is derived from the matrix element that regulates the process and in particular from the propagator of the Z boson. The real shape of the peak is actually a little bit flatter due to QED effects, as the radiation of photons by fermions.

² It is therefore obvious that there are only four distinct widths: the one of neutrinos, the one of leptons, the one of up-type quarks and the one of down-type quarks

LEP and the L3 experiment

The LEP accelerator produced electron-positron collisions which were then detected by one of the four detectors (OPAL, DELPHI, ALEPH and L3). The data analysed in this experiment correspond to the ones collected by the L3 detector. A schematic representation of the detector is shown in Figure 3.

The detector is composed by multiple layers, able to detect various features of the particles produced in the collision:

- The vertex chamber (to measure the trajectory of the particles)
- The electromagnetic and hadronic calorimeters (to measure the energy of leptons and hadrons)

- The muon chamber (to gather further information about muons, that are minimum ionising particles and able to escape the previous layers of the detector)
- Magnet (to bend the trajectory of the charged particles and measure their momentum)

A typical event in which a Z resonance is produced is shown in Figure 4. The plot on the left correspond to a decay into muons, the one on the right to a decay into quarks. In the first case, two muons (green lines) are produced and detected by the muon chambers. Their trajectory is also bent by the magnetic field but their momentum is very large therefore the effect is not visible by eye. A small amount of energy is also deposited in the calorimeters. In the case of a decay into jets, two large groups of hadrons are produced in opposite directions. Their trajectory is measured by the inner part of L3, while their energy is measured by the calorimeters.

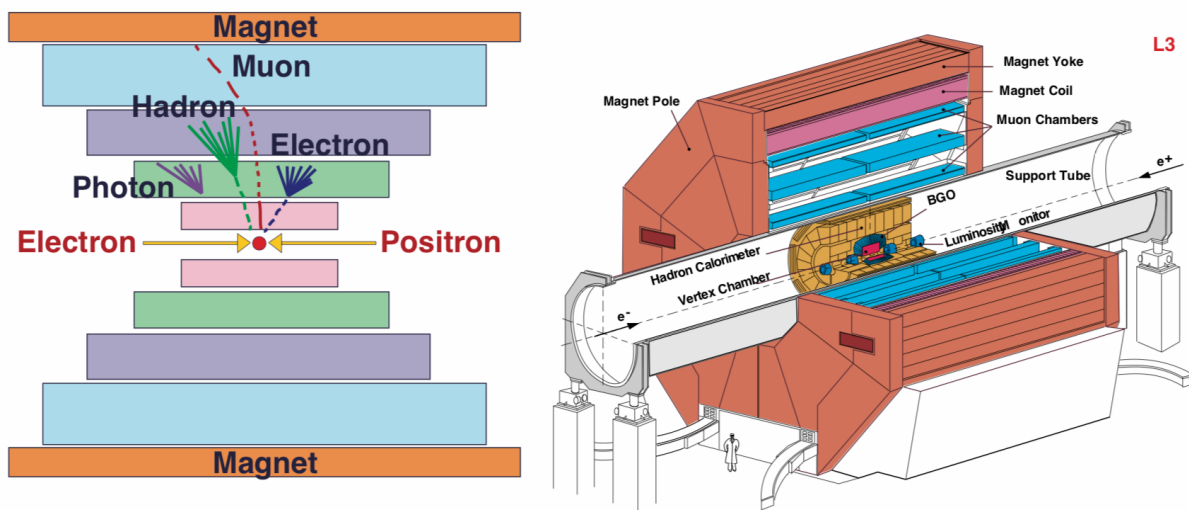


Figure 3: Schematic of the L3 detector

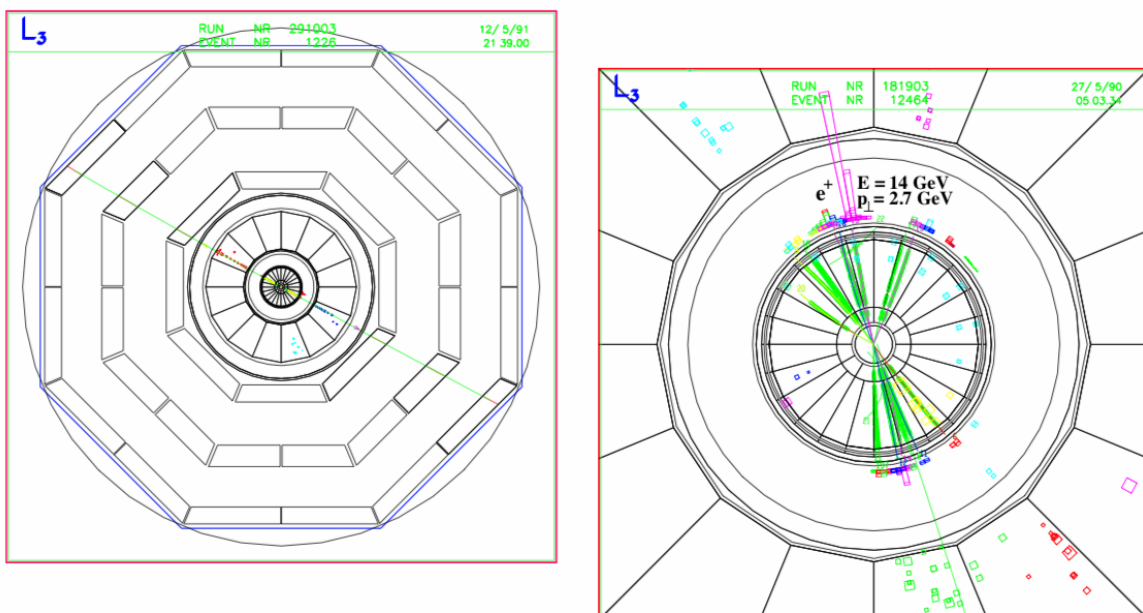


Figure 4: Production of a Z resonance decaying into muons (left) or jets (right)

2. Technical details

The test will be executed on a computer of the Experimental Particle Physics group. The account name to be used to access such computer is “fprakt”. The password will be given by the supervisor. It is also possible to run the program on a personal computer. In this case it is necessary to have a working installation Python and Jupyter. Full lists of the python modules required will be provided by the supervisor. Usually when installing python and its associated libraries it is easiest to use the anaconda package (<https://www.anaconda.com>).

Setting up the software

After logging in, open the terminal and create a directory to house your code and enter the folder:

```
- mkdir z0  
- cd z0
```

Download the needed code from <https://www.physik.hu-berlin.de/de/eephys/teaching/lab/z0resonance> and untar it with

```
- tar xfv z0.tgz
```

The resulting folder contains 2 files of simulated data at 91 GeV

“**hadrons.dat**”, “**muons.dat**”, 3 real datasets collected at different energies “**89GeV.dat**”, “**91GeV.dat**”, “**93GeV.dat**” and a Python Jupyter notebook “**praktikum.ipynb**” which contains the code used for the test. An introduction on how to use it will be given by the supervisor. The code is however heavily commented and it should be mostly self-explanatory.

The datasets are organised as follows: one line with the number of particles produced for each event followed by one line for each of the particles in the event. Such lines contain the x,y,z components of the particle momenta and their mass.

The program is organised as follows: the first parts initialise some useful functions. In particular the “**read_events**” function is the one used to loop through all the input files and retrieve the information about the particles. The information inside the input files is only the px,py,pz,m of the particles, therefore another couple of variables are added. The information about the particles is then stored within the program to be used later.

The second part of the program uses such functions to loop through the particle information, create some histograms and compare them. You can create any kind of histogram, from distributions of particle properties to global event properties, and apply selection on the variables you want to save so as to see how the distributions change before and after such selection.

By counting the number of events in the histogram without selection and in the ones on which the selection is applied you can obtain the efficiency of the selection and from there the cross section of the process.

The following part allows you to fit the obtained cross sections with a Breit-Wigner PDF. The result of the fit can be used to compute the various properties of the Z boson, the Weinberg angle and the number of colours using the information and formulae mentioned in this document.

3 Execution of the test

The first part of the test is dedicated to the determination of the parameters of the Z-resonance: its mass, cross section at peak and decay width. The available datasets correspond to three distinct center-of-mass energies (89.48, 91.33 and 93.02 GeV) and include all possible decay processes for the Z boson. The Monte-Carlo samples have been generated at 91 GeV and include the decay into hadrons or the one into muons. By comparing MC and data events it is possible to identify the cuts which are most adapt to separate hadronic and muonic decays from the others. It is then possible to compute the cross section of such decays at the three available energies and fit the Breit-Wigner PDF on them, extrapolating the parameters. By using this result it is possible to compute the hadronic and leptonic partial widths and finally the Weinberg angle and the number of colours parameter.

Extrapolation of the parameters from the hadronic decays

The first thing to do is to identify the best criteria to distinguish between hadronic and non-hadronic decays. A set of useful cuts is included in the literature at <https://www.physik.hu-berlin.de/de/eephys/teaching/lab/z0resonance>. You can apply those and compare the data and MC distributions to see if they are sufficient to exclude all other backgrounds or if you need to apply a stricter selection. It is important to select the maximum amount possible of good (hadronic) events, while excluding the maximum amount possible of non-hadronic events.

The selection of the events is done in the “read_events” part of the Python notebook. Modify it accordingly to your preference. Then read in the datasets, apply your chosen events selection and calculate how many events of the hadronic MC are filled in the histograms passing the hadronic selection. The ratio between this number and the total number of events in the MC file gives you the efficiency of the selection.

$$\epsilon = \frac{N_{MC}^{cut}}{N_{MC}} \quad (7)$$

Now compare the distribution you obtain for some kinematic variable produced by MC events and by data events, for the three centre of mass energies³. Do you see any difference? Do you think part of this difference is coming from additional background events in the datasets? If yes, estimate how many background events are passing the cut. Finally the total number of hadronic events in data can be computed with the following formula:

$$N_{had} = \frac{N_{tot}^{cut} - N_{bkg}^{cut}}{\epsilon} \quad (8)$$

where N_{tot}^{cut} is the number of data events passing the cut, N_{bkg}^{cut} is the estimated number of background events and ϵ is the efficiency computed on the hadronic MC events. We are however interested in converting the number of events into a cross section. This is generally done with the following formula:

$$\sigma = \frac{N_{obs}}{\epsilon \cdot A \cdot L} \quad (9)$$

where L is the “integrated luminosity”, a measure of the total number of events delivered by the accelerator and recorded by L3 during the data acquisition period. Details regarding its measurement can be found in literature. A is the acceptance of the detector. The integrated luminosity for the three datasets is:

89.48 GeV : 179.3 nb⁻¹

91.33 GeV : 135.9 nb⁻¹

93.02 GeV : 151.1 nb⁻¹

All values have a relative uncertainty of 1%.

However, we compute the efficiency and acceptance at the same time by extrapolating it from MC events, therefore the formula can be simplified as:

$$\sigma = \frac{N_{tot}^{cut} - N_{bkg}^{cut}}{\epsilon \cdot L} \quad (11)$$

This allows you to compute three distinct values for the hadronic cross section, for each of the centre of mass energies. The next step is to fit these values to extrapolate the Z-resonance parameters. As mentioned above, the Breit-Wigner PDF doesn't take into account QED effects. The function used for the fit is therefore a convolution between the Breit-Wigner and a term that estimates the probability of a photon emission that reduces the centre of mass energy from s to s'.

$$\sigma(s)^{QED} = \int_0^s \sigma(s) \cdot r(s, s') ds' \quad (12)$$

The “r” PDF is defined in the program as “radcorr” and it depends on the fine structure constant, on the mass of the electron and other QED properties. Figure 1 shows the effect of the QED corrections on the Breit-Wigner. The cross section at the maximum is reduced by approximately 30%.

The parameters returned by the fit are the cross section at the maximum of the Breit-Wigner (Born), the decay width of the Z boson and its mass. You can set within the code the initial values used in the fit. How does the fit behave? What is the Chi2 of the result and the number of degrees of freedom? Together with the cross sections at the three energies you also have to provide the errors. Compute the error propagation on the cross section formula in order to estimate the statistical error. What about systematical errors?

You can obtain those by varying the cuts performed to select the hadronic decays. Estimate them and include them in the fit.

Extrapolation of the parameters from the muonic decays

Repeat the experiment with the muonic decays. Again the appropriate cuts can be found in literature but you can add more if you like. Compute the cross section, mass and width of the Z resonance with the muon selection. How do the values compare to the hadronic ones? Try to fix the gamma and mass parameters of the fit to the ones obtained from the hadronic decay and fit only the cross section. Does the result change? Why?

Determination of the partial width of the electron decay and of the Weinberg angle

By using the equations above compute the partial width of the electron decay in two ways:

1. By using the parameters extrapolated with the muonic decay and Equation 3
2. By using the parameters extrapolated with the hadronic decay and Equations 3, 4 and 6

Which result is better? Proceed computing the Weinberg angle from equation 4. Any problem here?

Determination of the partial width of the hadronic decay and the number of colours

Compute the partial width of the hadronic decay and then the number of colours from:

$$\Gamma_{had}^{SM} = N_C k_{QCD} (2\Gamma_u + 3\Gamma_d) \quad (13)$$

The factor $k_{qcd} = 1 + \alpha_s = 1.04$ takes into account gluon emissions from quarks.

4 Additional notes

The events generated by simulation do not represent an exact description of the measured data. This is partly due to the not perfect calibration of data, to defective detector components and to imperfections in the simulation.

For each result, give statistical and systematic uncertainties estimate separately. Which error is dominant? How is the statistical error defined? Please make reasonable simplifications and approximations in the error propagation. Correlations can be neglected in many cases. You can estimate the contribution to the systematic uncertainty that results from your selection cuts by varying them in a reasonable range, and recalculating the cross-section. Also take into account the uncertainty of the luminosity measurement. Is this a systematic or statistical error? Compare your results with published results.

5 Presenting the results

Return the results not later than 2 weeks to the supervisor. The discussion of the work has to be presented in a PDF file accompanied by a zip file including the code. It is

important to comment the code properly so that other people (and the supervisor) are able to reproduce your result. The PDF should be organised as follows:

1. Theory: briefly explain the reason of the experiment, what you are measuring, and the physics behind it
2. Detector: summarise the features of the L3 experiment
3. Analysis: explain how you implemented your cuts, which efficiencies and cross section you obtained, the results of the fits and all the parameters you extrapolated. Always include statistical and systematical errors and eventually plots to explain the reason behind a given choice.
4. Comparison with literature: justify deviations from literature. The most recent measurements of the fundamental constants can be found in <http://pdg.lbl.gov/>