2.2 Theoretical Ansatz (Reminder)

a) β-decay, parity violation => V-A structure of charged currents $\bar{\psi}(\gamma^\mu - \gamma^\mu\gamma^5)\psi$ $= \bar{\psi}\gamma^\mu(1-\gamma^5)\psi$ $\propto \bar{\psi}_L \gamma^\mu \psi_L$

b) Goldhaber experiment: H(ν)=-1 => ν left-handed (if m_ν=0)

c) Charged weak currents (CC): $\nu_\mu N \rightarrow \mu^- X$

Neutral weak currents (NC): $\nu_\mu N \rightarrow \nu_\mu X$ **(not mediated by photons)**

d) Neutral weak currents of charged fermions contain right-handed components

Photons couple to left- and right-handed charged fermions

ANSATZ: **gauge group $SU(2)_L \times U(1)_Y$**

$SU(2)_L$ doublets: $L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \ldots \begin{pmatrix} u \\ d \end{pmatrix}_L \ldots \begin{pmatrix} T_3 = +1/2 \\ T_3 = -1/2 \end{pmatrix} \rightarrow$ weak isospin $T = \frac{1}{2}$

$SU(2)_L$ singlets: $R = e_R, \ldots, u_R, d_R, \ldots \rightarrow \quad T_3 = 0, T = 0$

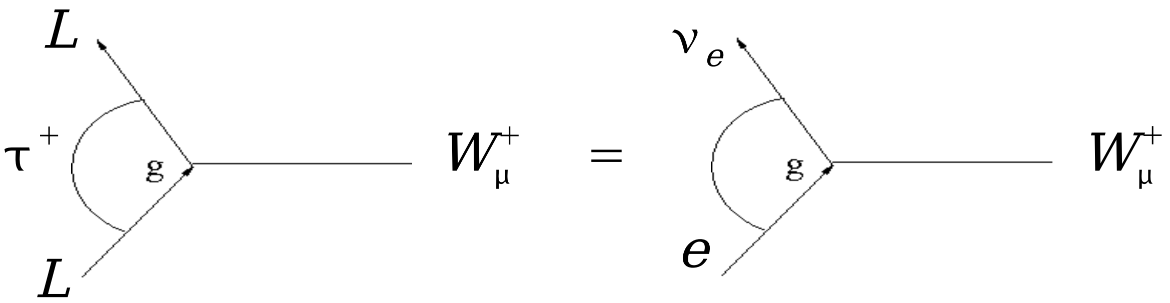
SU(2) generators: Pauli matrices $\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Interaction with gauge fields: $\mathscr{L}_{interaction} = \bar{L} i \gamma^\mu (\frac{ig}{2} W_\mu^a \tau^a) L$ (sum over $a = 1,2,3$)

2.2 Reminder: Theoretical Ansatz

$$\tau^1 W^1_\mu + \tau^2 W^2_\mu = \sqrt{2}\left[\tau^+ W^+_\mu + \tau^- W^-_\mu\right]$$

$$j^\pm_\mu = \bar{L}\gamma_\mu \frac{1}{2}\tau^\pm L$$ **with:** $$\tau^\pm = \frac{1}{2}(\tau^1 \pm i\tau^2) \quad \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \; \tau^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$$



Current coupling to W_μ^3: $$j^{\mu 3} = \bar{L}\gamma^\mu \frac{1}{2}\tau^3 L = \bar{L}\gamma^\mu \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} L = \frac{1}{2}\bar{\nu}_{e,L}\gamma^\mu \nu_{e,L} - \frac{1}{2}\bar{e}_L \gamma^\mu e_L$$

Can't be observed NC: no right-handed component !

Construct a NC being a singlet under $SU(2)_L$ trafo (orthogonal onto $j^{\mu 3}$) and containing right-handed components (as $j^{\mu\, em}$ does):

$$2 j^{\mu\, em} - 2 j^{\mu 3} = -2\bar{e}_R \gamma^\mu e_R - \bar{e}_L \gamma^\mu e_L - \bar{\nu}_{e,L}\gamma^\mu \nu_{e,L} =: j^{\mu\, Y}$$

Conserved "weak hypercharge": Y = 2 (Q – T_3) "Gell-Mann – Nishijima"

--> Gauge field of $U(1)_Y$: B^μ

Weak hypercharges of $e^-_R, e^+_L, \nu_{e,L}$ **?**

2.3 The weak mixing angle

From gauge fields W_μ^3 and B_μ to physical fields A_μ and Z_μ:

Consider only: $L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, R = e_R$

=> $\mathcal{L}_{lep} = \bar{R} i\gamma^\mu (\partial_\mu + \frac{ig'}{2} B_\mu Y) R + \bar{L} i\gamma^\mu (\partial_\mu + \frac{ig'}{2} B_\mu Y) L + \bar{L} i\gamma^\mu (\frac{ig}{2} W_\mu^a \tau^a) L$

$\mathcal{L}_{\nu\to\nu}$ **can only be mediated by Z exchange:**

$$\mathcal{L}_{\nu\to\nu} = \bar{\nu}_L i\gamma^\mu (\partial_\mu - \frac{ig'}{2} B_\mu) \nu_L + \bar{\nu}_L i\gamma^\mu (\frac{ig}{2} W_\mu^3) \nu_L = \bar{\nu}_L i\gamma^\mu (\partial_\mu \underbrace{- \frac{ig'}{2} B_\mu + \frac{ig}{2} W_\mu^3}_{\propto Z_\mu}) \nu_L$$

$$Z_\mu = \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g'^2 + g^2}} = -B_\mu \cdot \sin\theta_W + W_\mu^3 \cdot \cos\theta_W$$ **(normalized)**

$$A_\mu = \frac{g B_\mu + g' W_\mu^3}{\sqrt{g'^2 + g^2}} = B_\mu \cdot \cos\theta_W + W_\mu^3 \cdot \sin\theta_W$$ **(photon orthogonal)**

$$\cos\theta_W = \frac{g}{\sqrt{g'^2 + g^2}}, \ \sin\theta_W = \frac{g'}{\sqrt{g'^2 + g^2}} \qquad g' = g \cdot \tan\theta_W$$ **weak mixing/Weinberg angle**

2.3 The weak mixing angle

Replacement of $B_\mu = A_\mu \cdot \cos\theta_W - Z_\mu \cdot \sin\theta_W$

$$W_\mu^3 = A_\mu \cdot \sin\theta_W + Z_\mu \cdot \cos\theta_W$$

leads to

$$\mathscr{L}_{e \xrightarrow{\gamma} e} = \bar{e} i \gamma^\mu \left(\partial_\mu - \frac{i g g'}{\sqrt{g'^2 + g^2}} A_\mu\right) e \stackrel{\text{QED !}}{=} \bar{e} i \gamma^\mu (\partial_\mu + i q_e A_\mu) e \quad (q_e = -e)$$

$$\frac{g g'}{\sqrt{g'^2 + g^2}} = e = g \sin\theta_W = g' \cos\theta_W$$

“Unification”

2.4 Coupling of Z and W to fermions

a) Z-coupling to f_R with q_f e:

$$\mathscr{L}_{R \xrightarrow{Z} R} = -\frac{g}{\cos\theta_W} \underbrace{(-2 q_f \sin^2\theta_W)}_{g_R^f} (\bar{f}_R \gamma^\mu f_R) Z_\mu$$

b) Z-coupling to f_L with q_f e, T_3^f:

$$\mathscr{L}_{L \xrightarrow{Z} L} = -\frac{g}{\cos\theta_W} \underbrace{2(-q_f \sin^2\theta_W + T_3^f)}_{g_L^f} (\bar{f}_L \gamma^\mu f_L) Z_\mu$$

weak unit charge (wrt Z)

2.4 Coupling of Z and W to fermions

c) Vector and Axial-Vector coupling of Z boson:

$$\mathcal{L}_{Zf\bar{f}} \propto [g_R^f(\bar{f}_R\gamma^\mu f_R)+g_L^f(\bar{f}_L\gamma^\mu f_L)]Z_\mu = [g_R^f(\bar{f}\gamma^\mu \frac{1}{2}(1+\gamma^5)f)+g_L^f(\bar{f}\gamma^\mu \frac{1}{2}(1-\gamma^5)f)]Z_\mu$$

$$=[\frac{g_L^f+g_R^f}{2}(\bar{f}\gamma^\mu f)-\frac{g_L^f-g_R^f}{2}(\bar{f}\gamma^\mu\gamma^5 f)]Z_\mu=[\bar{f}\gamma^\mu(g_V^f-g_A^f\gamma^5)f]Z_\mu$$

g_V^f **vector current** g_A^f **axial-vector current**

$$g_V^f = T_3^f - 2q_f\sin^2\theta_W$$
$$g_A^f = T_3^f$$

=> No pure (V-A) structure as in charged currents

Example: $\mathcal{L}_{We\nu} = -\frac{g}{\sqrt{2}}[(\bar{\nu}_{eL}\gamma^\mu e_L)W_\mu^+ + (\bar{e}_L\gamma^\mu\nu_{eL})W_\mu^-]$

$$= -\frac{g}{2\sqrt{2}}[(\bar{\nu}_e\gamma^\mu(1-\gamma^5)e)W_\mu^+ + (\bar{e}\gamma^\mu(1-\gamma^5)\nu_e)W_\mu^-]$$

2.4 Coupling of Z and W to fermions

- **Determination of g and g':**

a) $g \sin\theta_W = e$ **known**

b) μ-decay --> Fermi constant G_F from muon lifetime: $\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}$

(How?)

Low-energy limit of charged current reaction: $\left(\frac{g}{2\sqrt{2}}\right)^2 = \frac{G_F M_W^2}{\sqrt{2}}$

g or sin(θ_W) known => M_W known

(How is the muon lifetime measured? Exp. Value?)

Measurement (What kind of reactions?):
$\sin^2\theta_W \sim 0.23$ (=> g ~ 0.6) => $M_W \sim 80$ GeV

Measure M_W directly => Consistency check