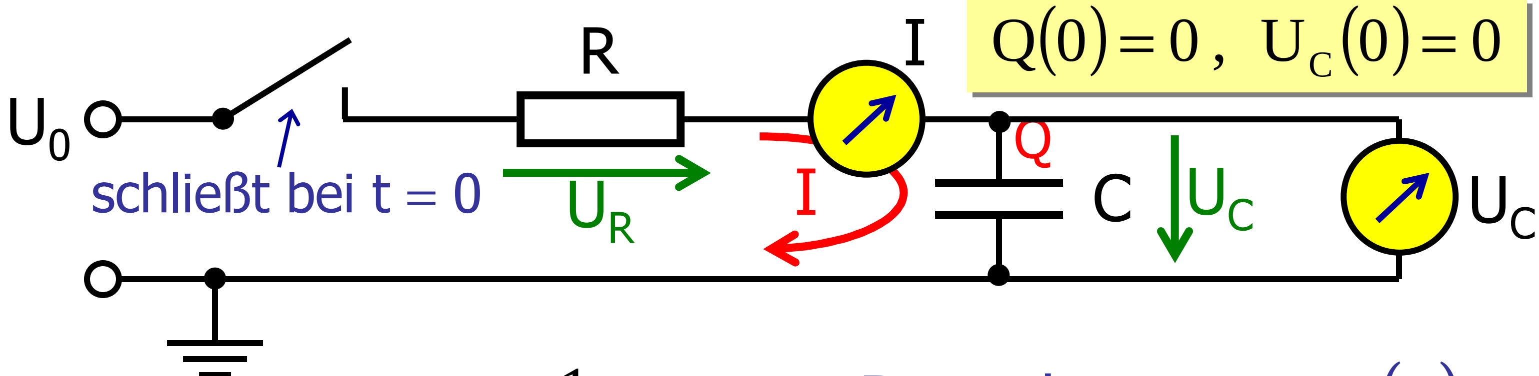
Auf- / Entladen eines Kondensators

quasistatisch ≈ Folge statischer Situationen

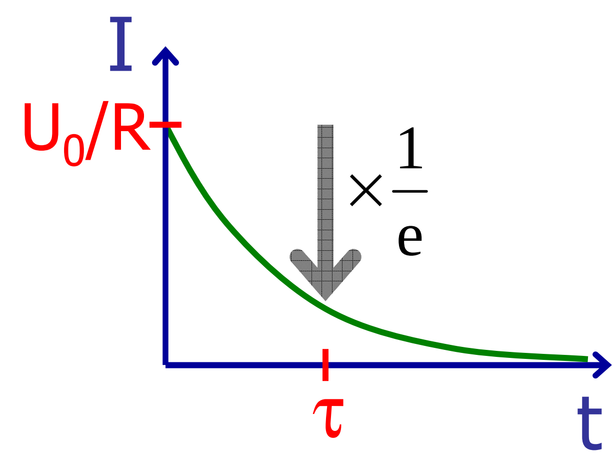
$$Q(0)=0\,,\;\; U_C(0)=0$$



$$U_0 = U_R + U_C = R\,I(t) + \frac{1}{C}Q(t)$$

<u>Bemerkung:</u> $\Rightarrow\; I(0) = \frac{U_0}{R}$

$$0 = R\dot{I} + \frac{1}{C}\dot{Q} = R\dot{I} + \frac{1}{C}I \quad \Rightarrow \quad \dot{I} = -\frac{1}{RC}I$$



<u>Lösung:</u> $$I(t) = \frac{U_0}{R}\exp\left(-\frac{t}{\tau}\right) \;, \qquad \tau = RC$$

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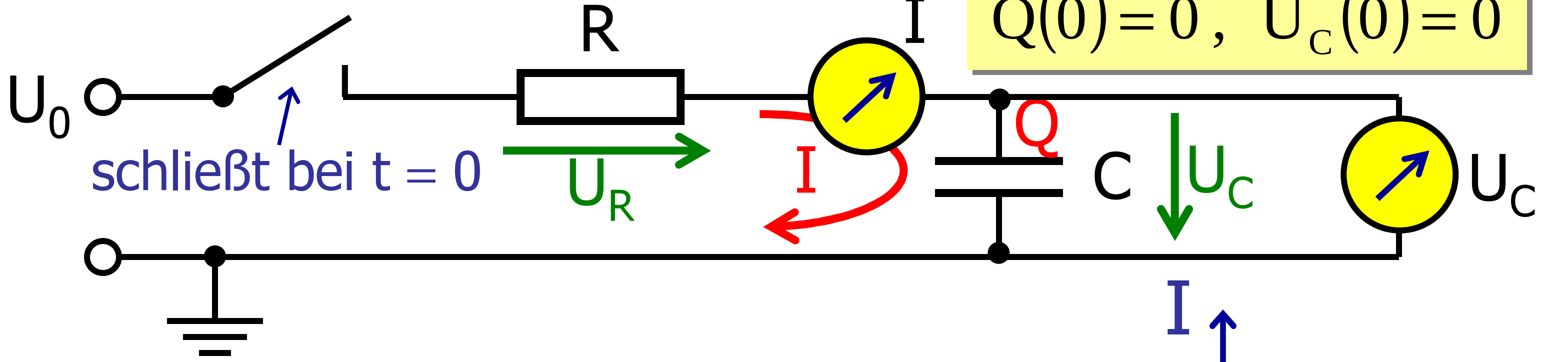
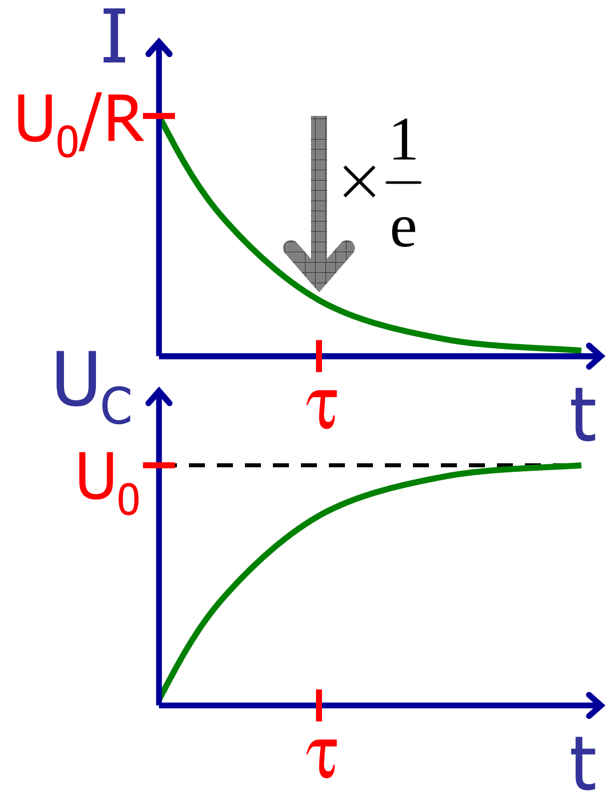


$$I(t)=\frac{U_0}{R}\exp\left(-\frac{t}{\tau}\right) \quad , \qquad \tau = RC$$



Kondensatorspannung:

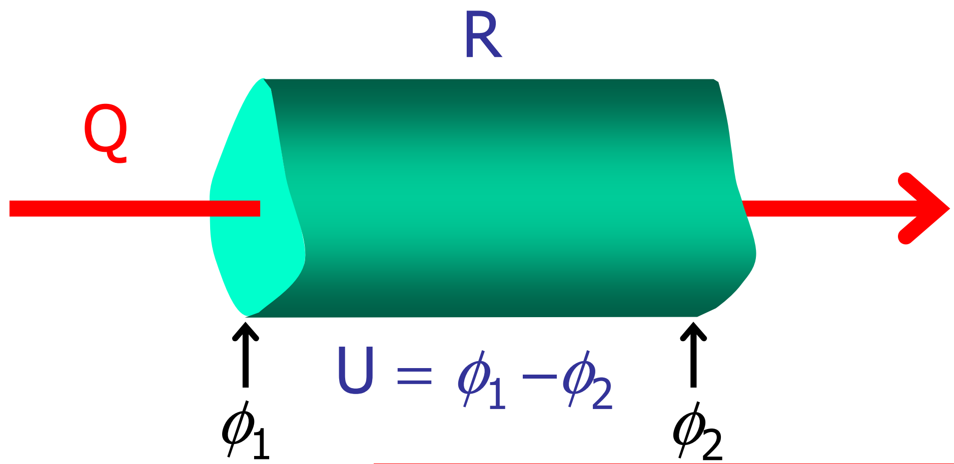
$$U_C(t)=\frac{Q(t)}{C}=$$

$$=\frac{1}{C}\cdot\int_0^t I(\tilde{t})\,d\tilde{t} = U_0\cdot\left(1-\exp\left(-\frac{t}{\tau}\right)\right)$$

2.3. Joulsche Wärme

Arbeit des E-Feldes:

$$W = Q(\phi_1 - \phi_2) = QU$$



Elektrische Leistung:

$$P = \frac{dW}{dt} = U\frac{dQ}{dt} = U\,I$$

↑ U = const.

Einheiten:

$$[P] = VA = W = Watt$$

$$[W] = Ws \;,\quad 1Ws = 1J$$

Ohmsches Gesetz $\Rightarrow$

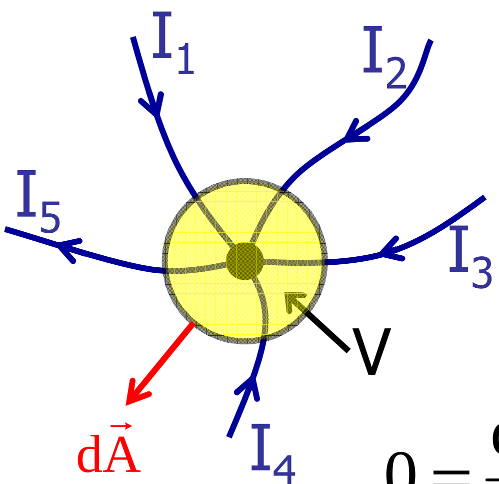
$$P = U\,I = R\,I^2 = \frac{U^2}{R}$$

$$I = const. \;\Rightarrow\; P \propto R \quad , \quad U = const. \;\Rightarrow\; P \propto 1/R$$

2.4. Kirchhoffsche Regeln

Analyse von Netzwerken von Leitern, (allgemeinen) Widerständen, Spannungs- / Stromquellen, …

a) Knotenregel: Knoten = punktförmige Leiterverbindung



$$I = \int \vec{j}\, d\vec{A} \quad \Rightarrow$$

auslaufend: $I > 0$

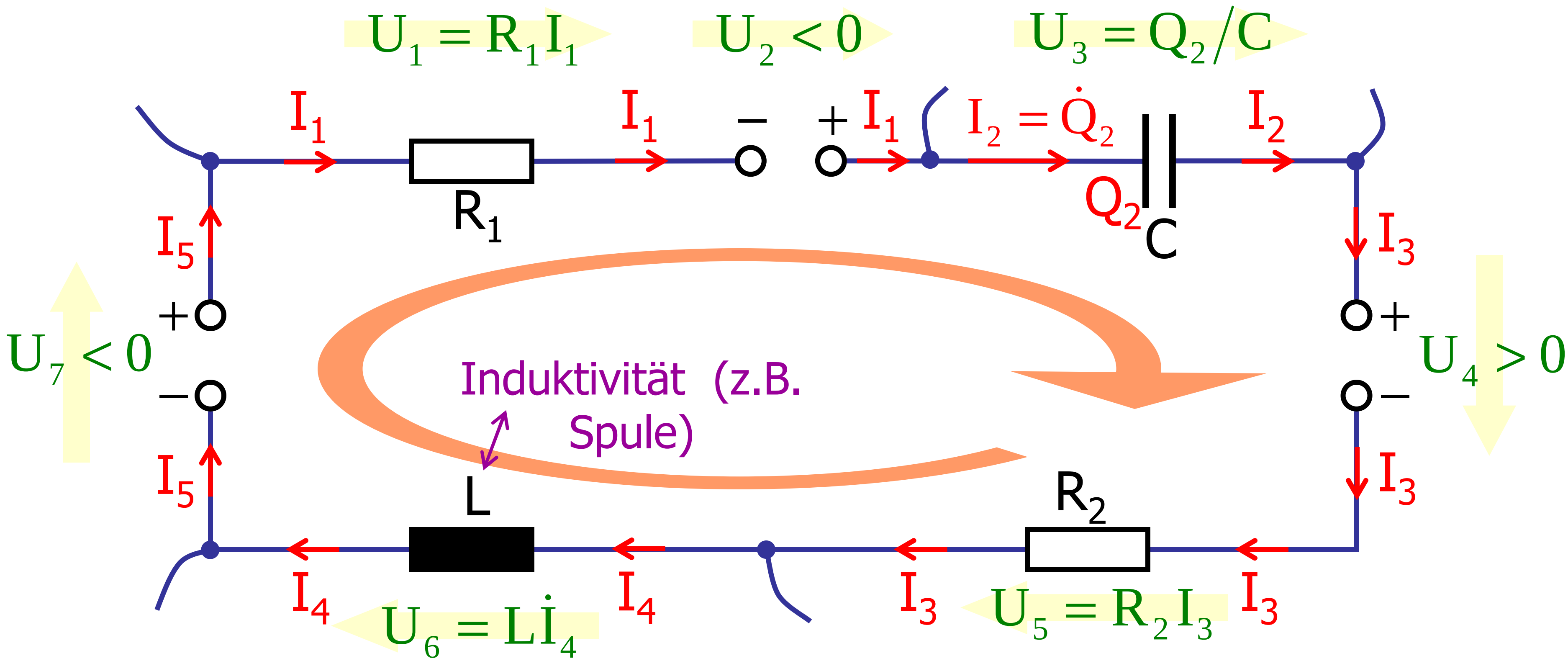
einlaufend: $I < 0$

$$0 = \frac{dQ}{dt} = \frac{d}{dt}\int_V \rho\, dV = -\int_V \operatorname{div}\vec{j}\, dV = -\oint_{Ob(V)} \vec{j}\, d\vec{A} = -\sum_i I_i$$

$$\Rightarrow \quad \sum_{i \in \text{Knoten}} I_i = 0$$

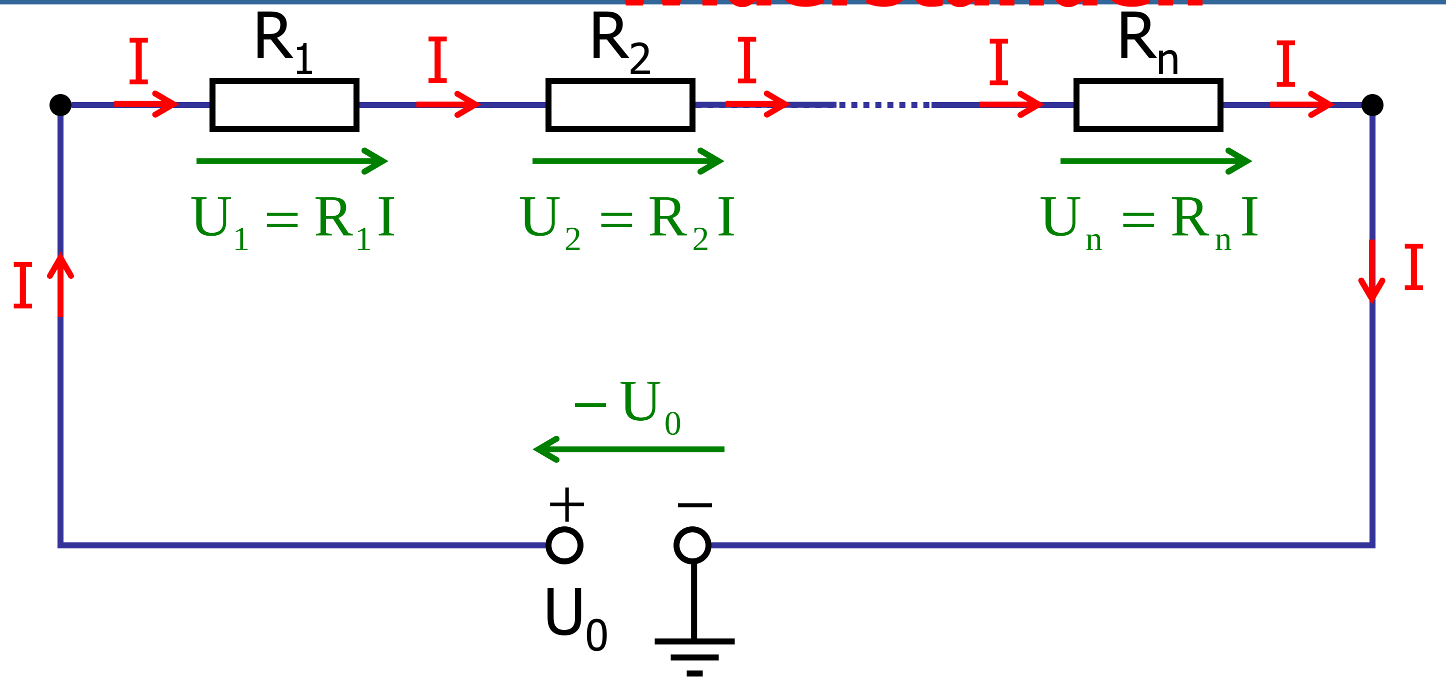
Kirschoffsche Regeln

b) Maschenregel: Masche = Schleife in der Schaltung



$$\oint_{\text{Masche}} \vec{E}\, d\vec{s} = 0 \quad \Rightarrow \quad \sum_{i \in \text{Masche}} U_i = 0$$

Reihenschaltung von Widerständen

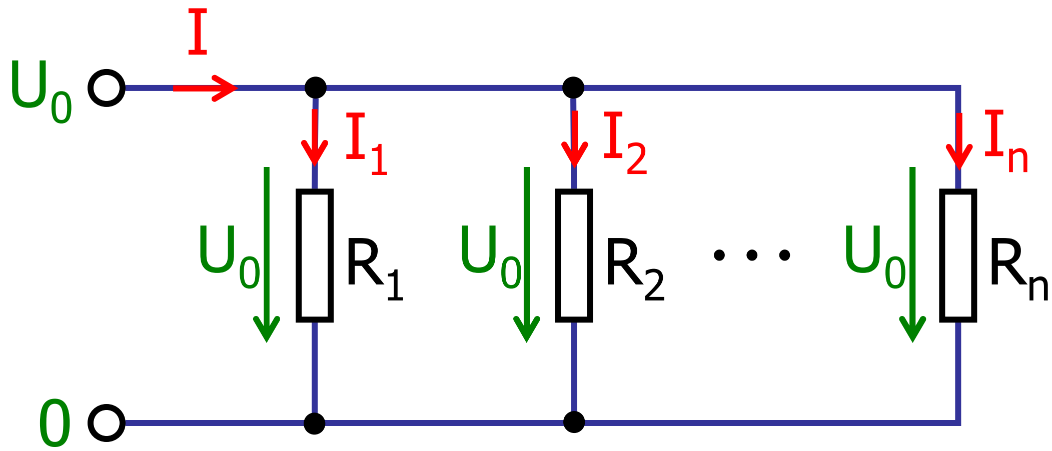


Maschenregel:

$$-U_0 + R_1 I + R_2 I + \ldots + R_n I = 0 \quad \Rightarrow \quad U_0 = \underbrace{\left(\sum_{i=1}^{n} R_i\right)}_{R_{tot}} \cdot I$$

$$R_{tot} = \sum_i R_i$$

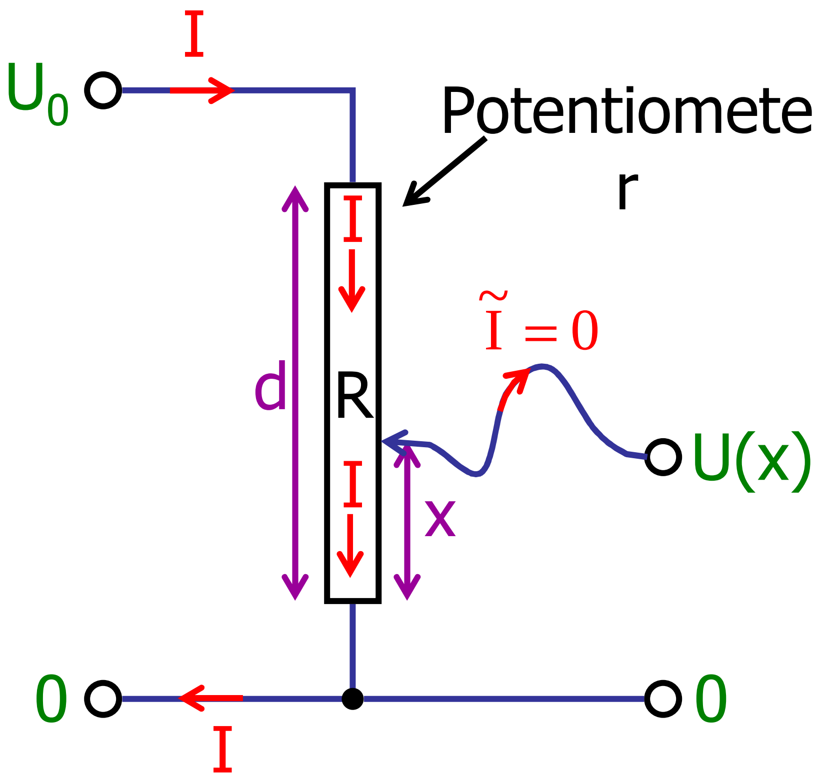
Paralellschaltung von Widerständen

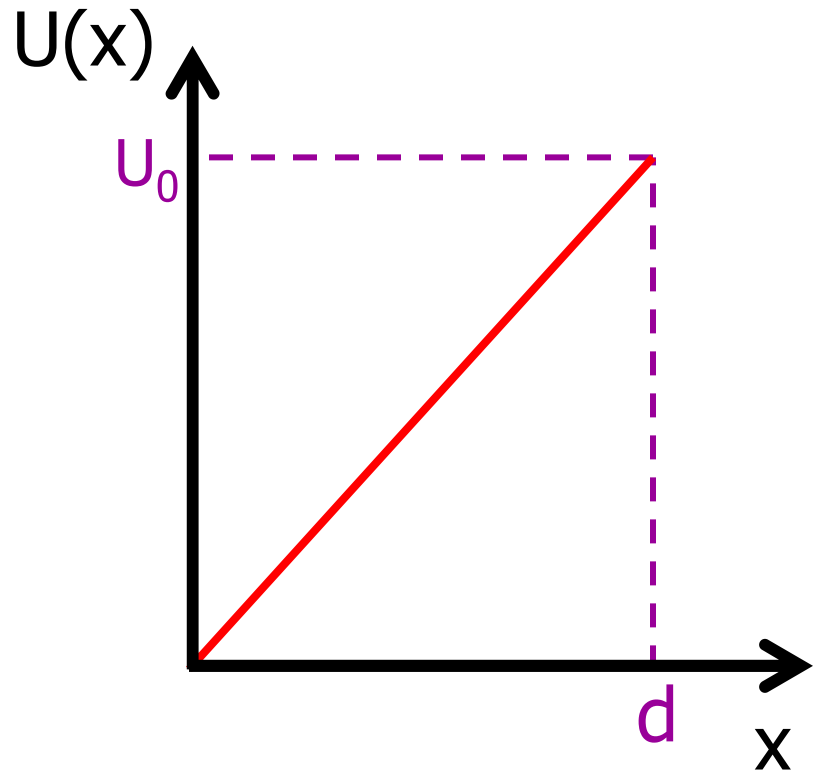


<u>Knotenregel:</u> $-I + I_1 + I_2 + \ldots + I_n = 0 \quad \Rightarrow \quad \frac{U_0}{R_{tot}} = \sum_{i=1}^{n} \frac{U_0}{R_i}$

$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

Spannungsteiler

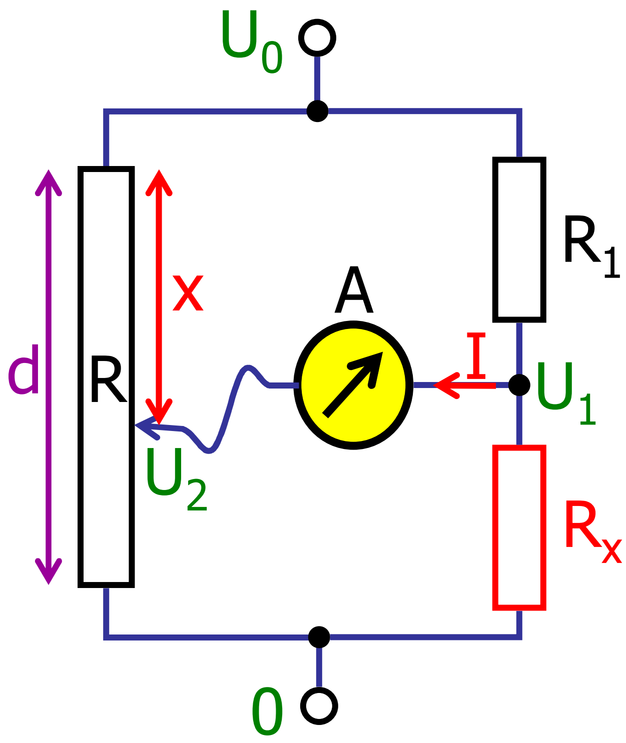




$$U(x) = I \cdot \left(\frac{x}{d} R \right) = \frac{x}{d} \cdot U_0$$

Wheatstonesche Brückenschaltung

Nullabgleich: $I = 0 \quad \Leftrightarrow \quad U_1 = U_2$



$$\Leftrightarrow \quad \frac{R_1}{R_x} = \frac{x}{d - x}$$

$$R_x = \frac{d - x}{x} \cdot R_1$$