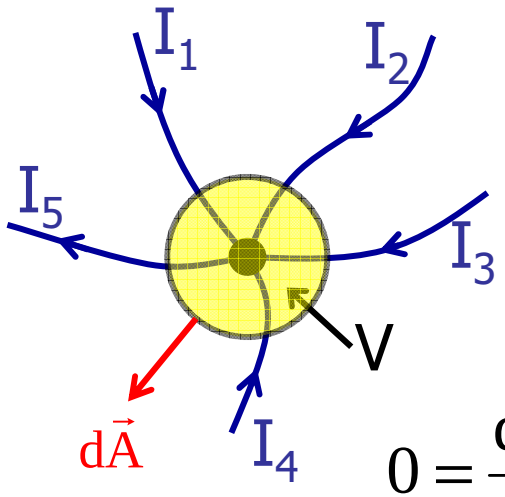
2.4. Kirchhoffsche Regeln

Analyse von Netzwerken von Leitern, (allgemeinen) Widerständen, Spannungs- / Stromquellen, ...

a) Knotenregel: Knoten = punktförmige Leiterverbindung



$$I = \int \vec{j}\, d\vec{A} \quad \Rightarrow$$

auslaufend: $I > 0$

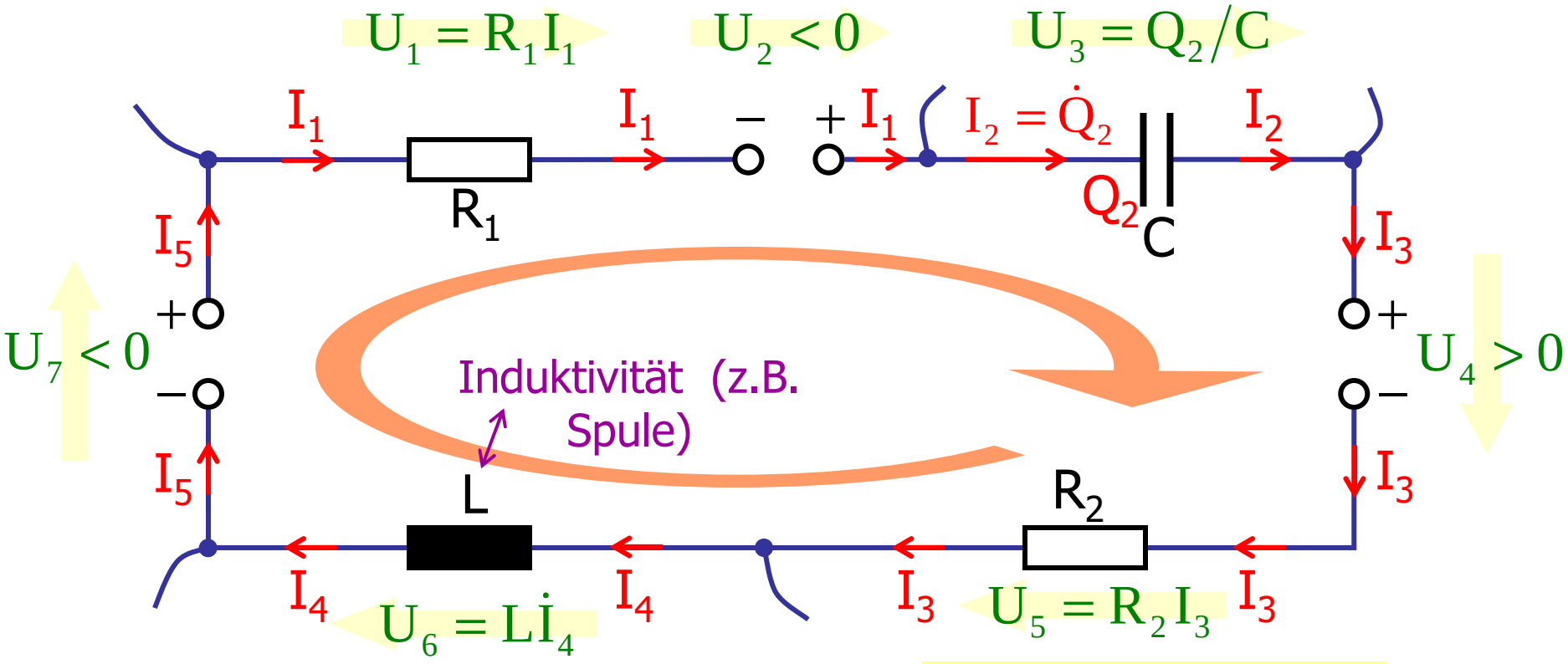
einlaufend: $I < 0$

$$0 = \frac{dQ}{dt} = \frac{d}{dt}\int_V \rho\, dV = -\int_V \mathrm{div}\,\vec{j}\, dV = -\oint_{Ob(V)} \vec{j}\, d\vec{A} = -\sum_i I_i$$

$$\Rightarrow \quad \sum_{i \in \text{Knoten}} I_i = 0$$

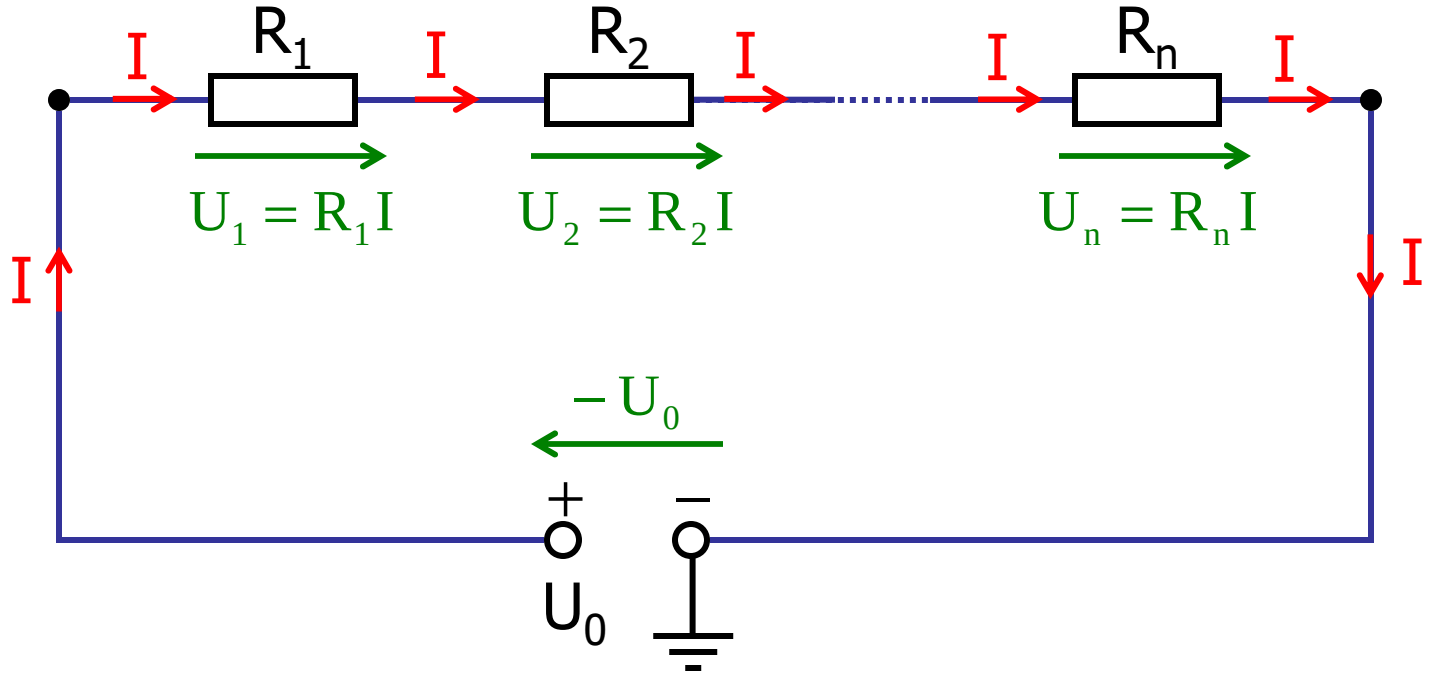
Kirschoffsche Regeln

b) Maschenregel: Masche = Schleife in der Schaltung



$$\oint_{\text{Masche}} \vec{E}\, d\vec{s} = 0 \quad \Rightarrow \quad \sum_{i \in \text{Masche}} U_i = 0$$

Reihenschaltung von Widerständen

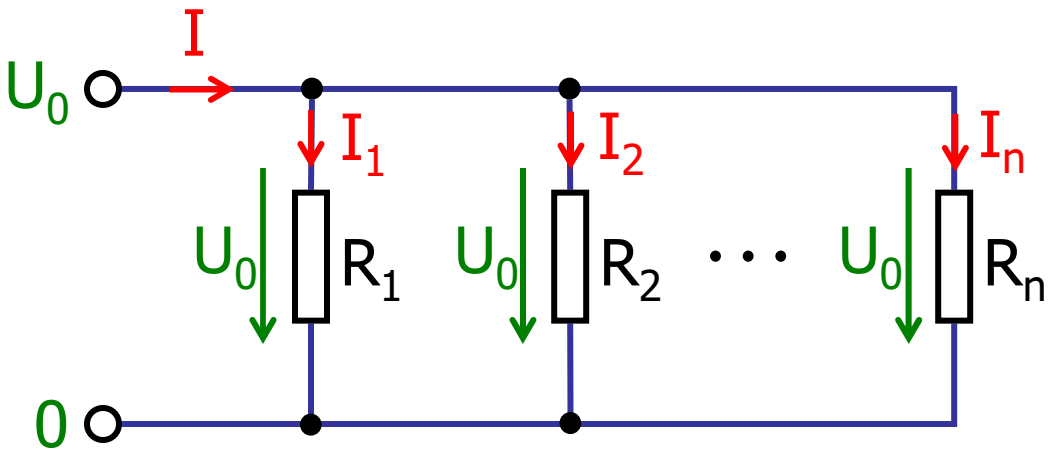


Maschenregel:

$$-U_0 + R_1 I + R_2 I + \ldots + R_n I = 0 \quad \Rightarrow \quad U_0 = \underbrace{\left(\sum_{i=1}^{n} R_i \right)}_{R_{tot}} \cdot I$$

$$R_{tot} = \sum_i R_i$$

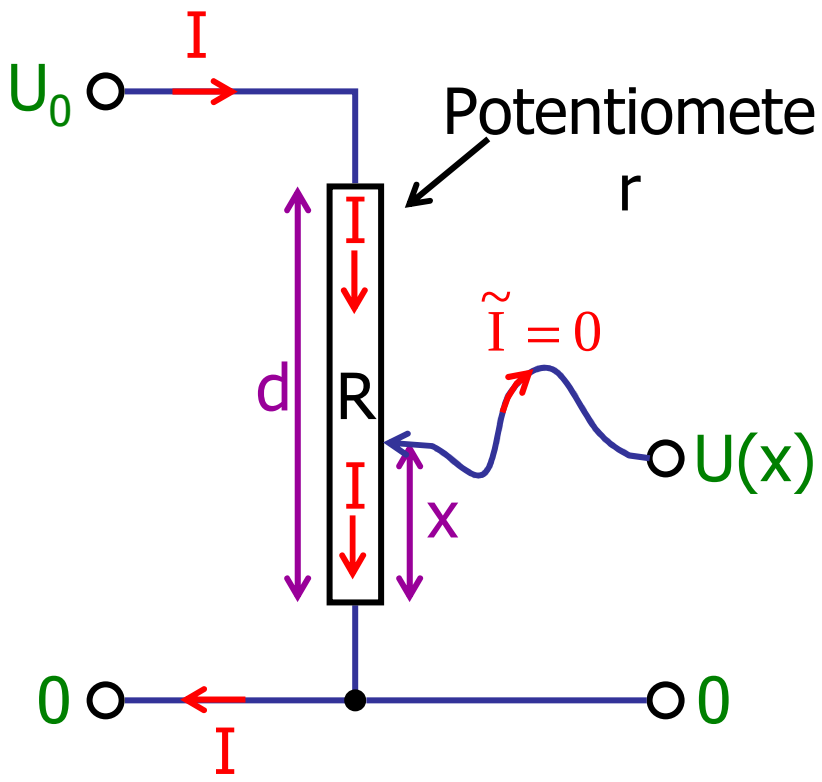
Paralellschaltung von Widerständen

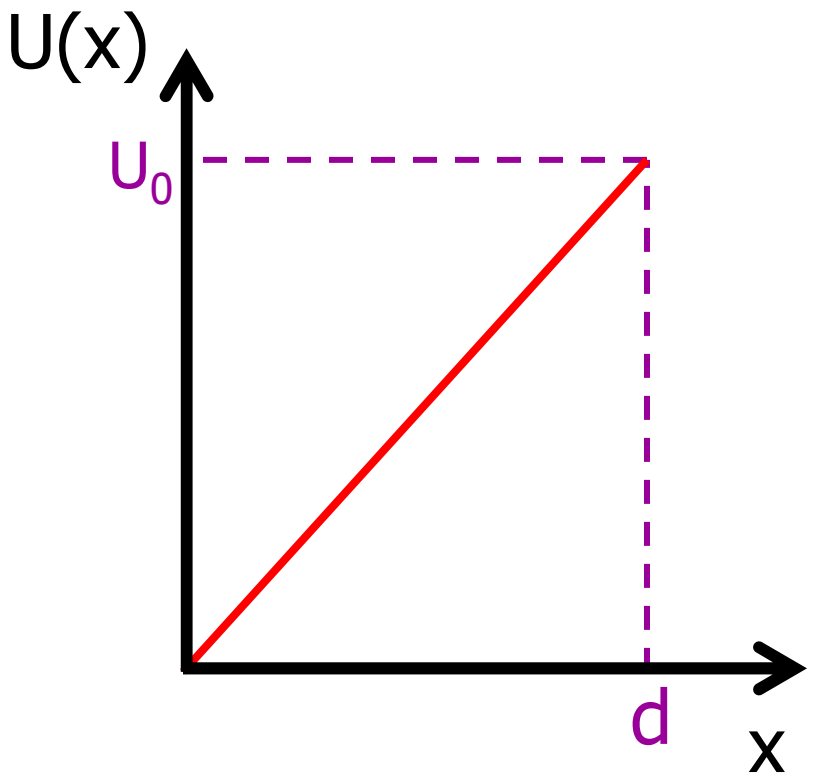


Knotenregel: $-I + I_1 + I_2 + \ldots + I_n = 0 \quad \Rightarrow \quad \frac{U_0}{R_{tot}} = \sum_{i=1}^{n} \frac{U_0}{R_i}$

$$\frac{1}{R_{tot}} = \sum_i \frac{1}{R_i}$$

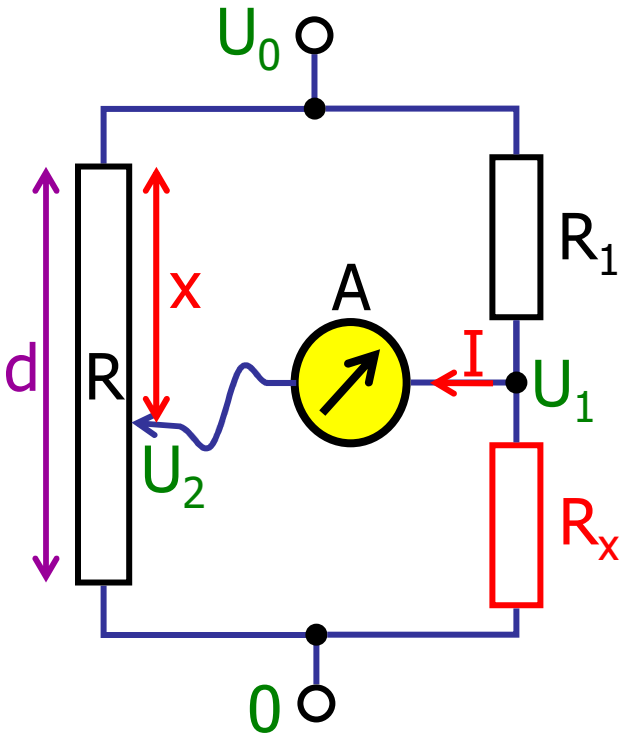
Spannungsteiler





$$U(x) = I \cdot \left(\frac{x}{d} R \right) = \frac{x}{d} \cdot U_0$$

Wheatstonesche Brückenschaltung



Nullabgleich: $I = 0 \quad \Leftrightarrow \quad U_1 = U_2$

$$\Leftrightarrow \quad \frac{R_1}{R_x} = \frac{x}{d-x}$$

$$R_x = \frac{d-x}{x} \cdot R_1$$