On-shell methods for NLO calculations

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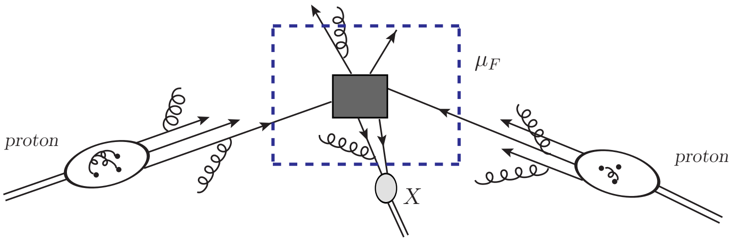
the modell of Hadron scattering

Factorisation of the (high energy) scattering process into Partonevolution und -scattering [COLLINS,SOPER,STERMAN]



subject of the following slides: Parton cross section σ_{Parton}

pertubative expansion and corrections

$$\sigma_{Parton} = \left[\hat{\sigma}_0 \left(\alpha_s^z\right) + \hat{\sigma}_1 \left(\alpha_s^{z+1}\right) + \ldots\right] \quad \alpha_s = \alpha_s(\mu_R^2)$$

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1. virtual corrections
   $\Rightarrow$ reduction and determination of the scale dependency
2. final state Bremsstrahlung
   $\Rightarrow$ infrared savety of observables

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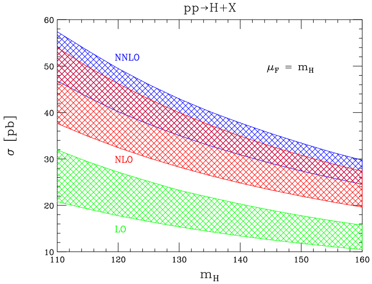
example: Higgs production ($\mu_F = \mu_R = m_H$)



[Anastasiou, Melnikov, Petriello]

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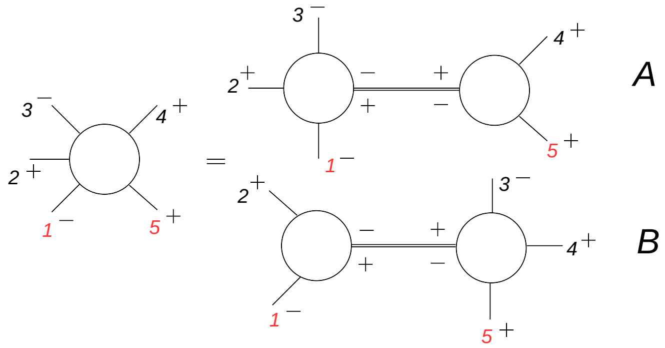
   - use Cauchys theorem to solve integrals
   - use treelevel amplitudes instead of vertices as building blocks

a treelevel example: 5 pt. gluon function



$$\hat{p}_1(z) = \lambda_1 \left(\widetilde{\lambda}_1 - z\,\widetilde{\lambda}_5\right)$$

$$\hat{p}_5(z) = (\lambda_1 + z\,\lambda_5)\,\widetilde{\lambda}_5$$

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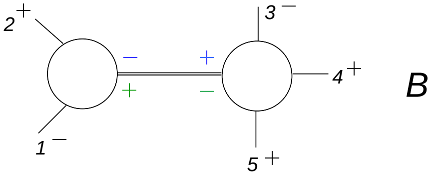

pole of contribution B: $z_{1,2} = \frac{[1\,2]}{[5\,2]}$

denominator: $\langle 3|1|2] - z_{1,2}\,\langle 3\,1\rangle[5\,2] = 0$

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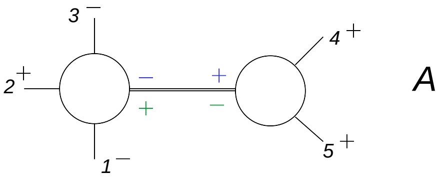

pole of contribution A: $z_{4,5} = -\frac{\langle 5\,4\rangle}{\langle 1\,4\rangle}$

$$A_5^{2-} = -\mathbb{i}\frac{\langle 1\,3\rangle^4\,[4\,5]^3}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 1|5|4]\,\langle 3|4|5]\langle 4|5|4]} = \mathbb{i}\frac{\langle 1\,3\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

massive fermions and on shell methodes

they suffer from:

- a more difficult *inner* colour ordering of fermions
- less symmetries → no vanishing helicity amplitudes
- less compact representation in terms of Weyl spinors

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remaining properties:

- parity condition for involved gluons (factor 2)
- spin axis quantization freedom of heavy quarks (factor 4)

the situation at one loop

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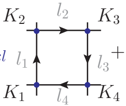
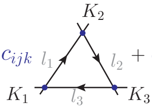
2. the scalar integralbase at one loop [f.e. ELLIS, ZANDERIGHI]

$A^{1-loop} \sim d_{ijkl}$



$+ c_{ijk}$



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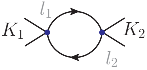
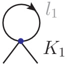


$+ a_i$



(a result of 30 years hard work in pertubative QFT)
⇒ remaining task: compute the coefficients

the situation at one loop

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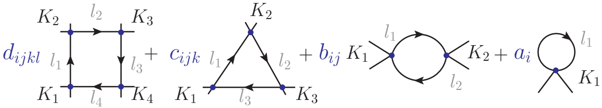

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$\Rightarrow$ remaining task: compute the coefficients

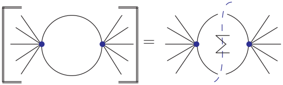
3. additional *inner* colour ordering due to loop contributions
$\rightarrow$ primitive amplitudes

unitarity cuts I - foundations

consequences of S-matrix theory:

1. optical theorem

$$SS^\dagger = \mathbb{1} \quad \overset{\text{1 loop}}{\Longrightarrow} \quad 2\mathbb{i}\,\mathrm{Im}$$



2. discontinuities of analytical continued matrix elements

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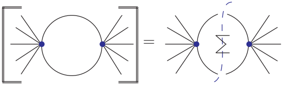

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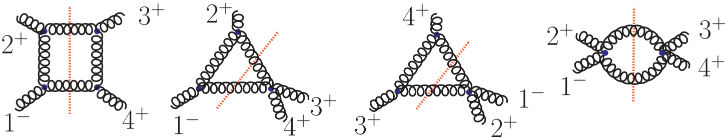
Sewing of treelevel amplitudes to 1-loop amplitudes allows for the calculation of the logarithmic parts of $\mathcal{F}$.

example: a $gg \to gg$ 1loop configuration



unitarity cuts II - recent methodes

- the basic step of the calculation:

$$\frac{1}{l^2 + \mathbb{i}\epsilon} \rightarrow \pi\delta^{(+)}(l^2) \qquad \text{(Cutkosky rules)}$$

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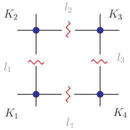
- implementation of multiple cuts (generalized unitarity)

[BRITTO,FENG]; [OPP]; [FORDE]



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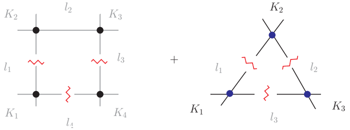

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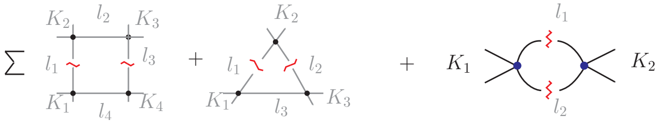

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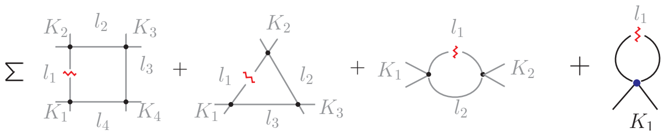

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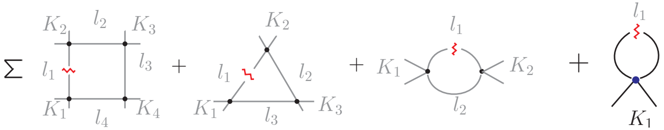

- a step further: generalizing to *d* dimensional loop momenta

[BERN, MORGAN]; [ANASTASIOU, BRITTO, FENG, KUNSZT, MASTROLIA]; [BADGER]

parametrisation by Forde

- loop moemtum is an object with *four* degrees of freedom
  $\rightarrow$ choose a fixed base with complex momenta

$$2\, l^{\mu} = \alpha_1 < k_1|\gamma^{\mu}|k_1] + \alpha_2 < k_2|\gamma^{\mu}|k_2] + \alpha_3 < k_1|\gamma^{\mu}|k_2] + \alpha_4 < k_2|\gamma^{\mu}|k_1]$$

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- unfixed coefficients $\rightarrow$ complex parameters
- analytically cumbersome for double and single cut
- numerically instabilities in a naive implementation

conclusions

1. physics @ LHC $\Rightarrow$ cross sections @ NLO
2. on-shell methods
    - are promising tools for this task
    - have high potential for further progress

backup slides

factorization

task:

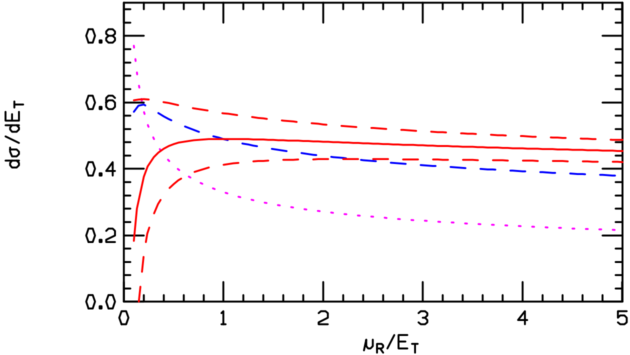
- separation of intrisic processes from scattering events
- description for the time evolution of the intrisic processes

$$\sigma_{pp\to X} = \sum_{r,s,t} \int dx_r\, dx_s\, f(x_r,\mu_F^2)\, f(x_s,\mu_F^2) \times \hat{\sigma}_{rs\to t} \times D(x_t,\mu_F^2)_{t\to X}$$

where

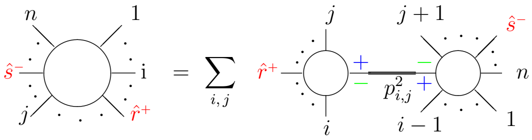
$$\hat{\sigma}_{rs\to t} = \left[\hat{\sigma}_0 + \alpha_s(\mu_R^2)\hat{\sigma}_1 + \ldots\right]_{rs\to t}$$

scale dependence



Scale dependence for single jet production (inkl.) at $E_T = 100$ GeV. Theoretical predictions at LO (dottet curve), NLO (blue dashed) and NNLO (different modells in red). [GLOVER 02]

general form of the recursion



$$A(z=0) = \sum_{i,j,h} A(i,\ldots,\hat{r}^{+},\ldots,j,-\hat{P}_{i,j}^{h})\ \frac{-i}{p_{i,j}^{2}}\ A(\hat{P}_{i,j}^{-h},\text{j+1},\ldots,\hat{s}^{-},\ldots,\text{i-1})$$

$$P_{i,j} = p_i + p_{i+1} + \cdots + \hat{p}_r\,(z_{i,j}) + \cdots + p_{j-1} + p_j$$