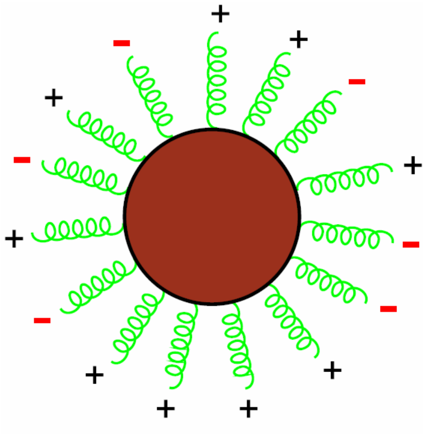
QCD at Colliders

Lecture 1



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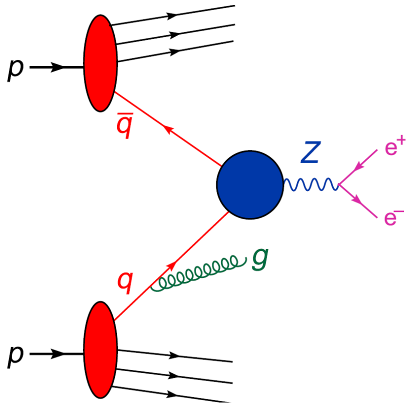
Goals

1. Qualitative understanding of QCD-improved parton model – where can we expect perturbative QCD to work at all?
2. Levels of approximation
3. A little about modern color and helicity organization of amplitudes
4. Soft and collinear behavior
5. Anatomy of higher order QCD calculations
6. Why are some QCD corrections so large? Can we resum the large corrections?

QCD factorization & parton model

- Asymptotic freedom guarantees that at short distances (large transverse momenta), partons in the proton are almost free.
- They are sampled “one at a time” in hard collisions.
- Leads to QCD-improved parton model:



“suitable” final state

Parton distribution function: prob. of finding parton a in proton 1, carrying fraction x_1 of its momentum

factorization scale (“arbitrary”)

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1\int_0^1 dx_2\; f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Partonic cross section, computable in perturbative QCD

partonic CM energy^2

renormalization scale (“arbitrary”)

What are suitable final states?

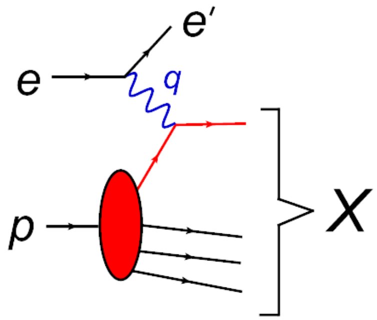
- They should not be sensitive to long-distances (soft physics)

→“Infrared-safe” observables,

smooth under soft and collinear limits (return to later)

- They should be sufficiently inclusive:

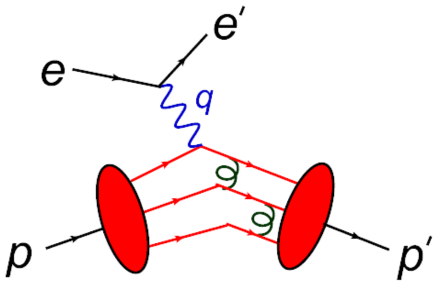
DIS ep → eX (OK) vs. ep → ep (very small) vs. diffraction (HERA)



$q^2 = -Q^2$ large

$x = \frac{Q^2}{2P\bullet q}$ fixed

$M_{\mathrm{had}}^2 = (q+P)^2 = \frac{1-x}{x}Q^2$ large

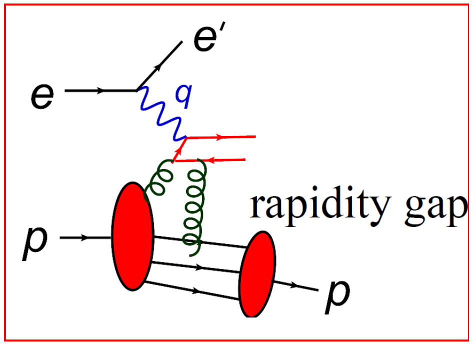


$A \sim 1/Q^4$

$q^2 = -Q^2$ large

$x \to 1$

$M_{\mathrm{had}}^2 = m_{\mathrm{p}}^2$ small

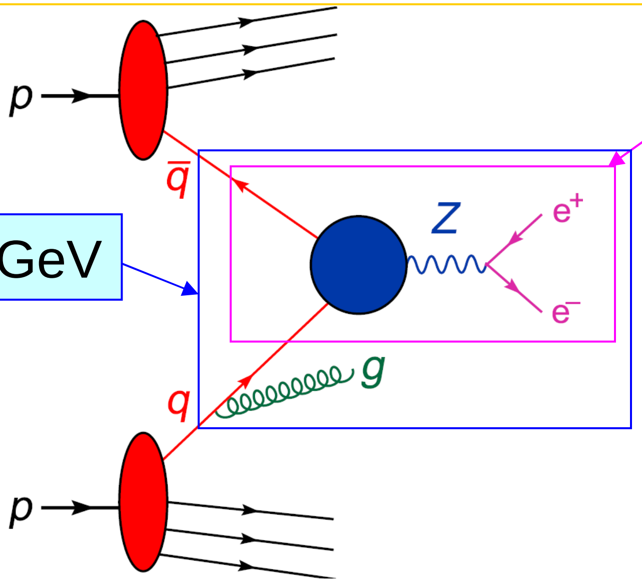


Parton evolution

- partons in the proton are not quite free
- distributions $f_a(x,\mu_F)$ evolve as scale μ_F at which they are resolved varies

$$\sigma^{pp\to X}(s;\alpha_s,\mu_R,\mu_F) = \sum_{a,b}\int_0^1 dx_1\int_0^1 dx_2\, f_a(x_1,\alpha_s,\mu_F)\, f_b(x_2,\alpha_s,\mu_F) \times \hat{\sigma}^{ab\to X}(sx_1x_2;\alpha_s,\mu_R,\mu_F)$$

Example:



large μ_F, ~ m_Z

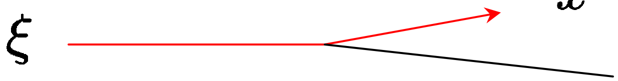
small μ_F, ~ few GeV

$p_T(g)$ ~ 10 GeV, say

Parton evolution (cont.)

- parton distributions are nonperturbative
- must be measured experimentally
- experimental data at much lower μ_F^2 than (100-1000 GeV)^2
- fortunately, evolution at μ_F > 1-2 GeV is perturbative
- DGLAP equation (return to later)

$$\mu^2 \frac{\partial}{\partial \mu^2} f_a(x,\mu) = \frac{\alpha_s(\mu)}{2\pi} \sum_b \int_x^1 \frac{d\xi}{\xi} P_{ab}(x/\xi, \alpha_s(\mu))\, f_b(\xi,\mu)$$



$$x = \frac{x}{\xi} \times \xi$$

$$P_{ab}(x,\alpha_s) = P_{ab}^{(0)}(x) + \frac{\alpha_s}{2\pi} P_{ab}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ab}^{(2)}(x) + \cdots$$

LO (1974) NLO (1980) NNLO (2004)

Levels of Approximation

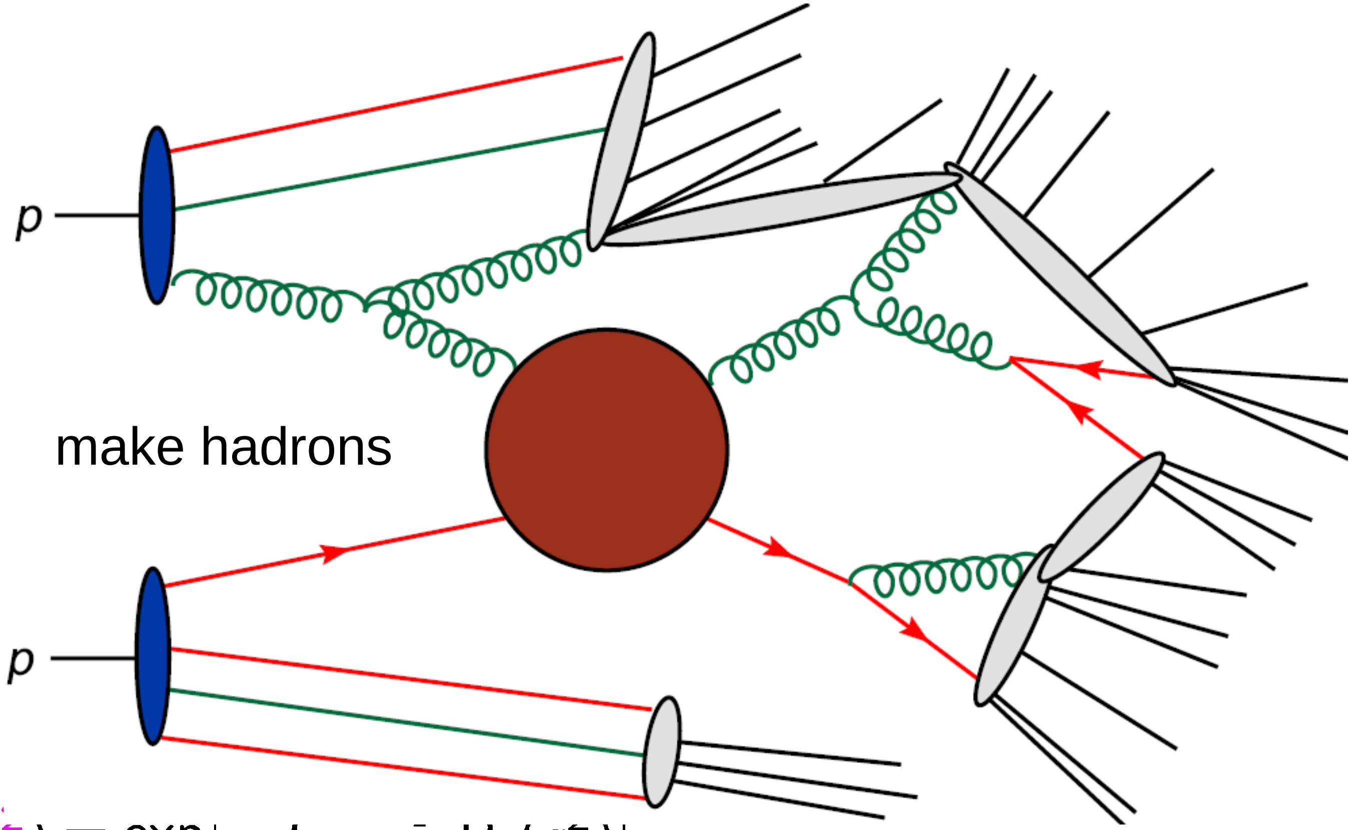
- Monte Carlos (PYTHIA, HERWIG,…)
- LO, fixed-order matrix elements (MEs)
- LO MEs matched to parton showers (ALPGEN, SHERPA, MADGRAPH/EVENT, …)
- NLO MEs (parton level) (MCFM, BLACKHAT, …)
- NLO MEs matched to showers (MC@NLO, POWHEG, more to come?)
- NNLO MEs (FeWZ, HNNLO, …)
- MC@NNLO? [We wish!]

Monte Carlos

- Based on properties of soft and collinear radiation in QCD
- Partons surrounded by “cloud” of soft and collinear partons
- Leading double logs of Q_{hard}/Q_{soft} exponentiate, can be generated probabilistically
- Shower starts with basic 2 → 2 parton scattering
  -- or basic production process for *W, Z, tt*, etc.
- Further radiation approximate, requires infrared cutoff
- Shower can be evolved down to very low Q_{soft} , where models for hadronization and spectator interactions can be applied
- Complete hadron-level event description attained
- Normalization of event rates unreliable
- Event “shapes” sometimes unreliable

Monte Carlos in pictures

Splitting probability: $$P_g(q^2) = \int_0^1 dz \frac{\alpha_s(q^2)}{2\pi} \hat{P}_{gg}(z) \Theta(q^2 - q_0^2)$$

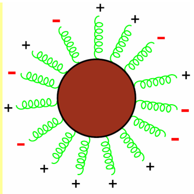


Exercise: Find my mistake!

Sudakov

$$\Delta_g(Q^2, q^2) = \exp\left[-\int_{q^2} \frac{\;}{\tilde{q}^2} P_g(q^2)\right]$$

Leading order matrix elements



- Based on sum of all tree-level Feynman diagrams in QCD
- Generates correct hard radiation pattern (at tree level)
- Event "shapes" often fairly reliable
- Event rates (normalization) still fairly unreliable, especially if:
  - more jets → more powers of $\alpha_s(\mu_{R,F})$
  - gluons in the initial state (lots of extra soft radiation)
  - cases where new subprocesses appear at NLO ($q\bar{q} \rightarrow \gamma\gamma$)
- Description is only at parton level
- Sophisticated programs can now rapidly produce tree-level cross sections for very high multiplicity
- Some use Feynman diagrams MadGraph; GRACE; CompHEP,…
- Other use recursive or iterative organization

Berends, Giele, VECBOS, NJETS; HELAC; ALPHA → ALPGEN

- Recent techniques spun off from "twistor string theory":

- on-shell recursion relations, …

Britto, Cachazo, Feng (2004)

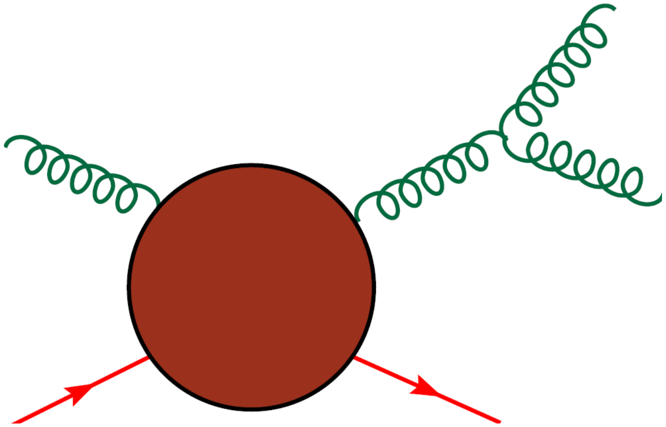
Matching MEs to showers

- Would like to have both:
  - accurate hard radiation pattern of MEs
  - hadron-level event description of parton-shower MCs
- Why not just use 2 → 3,4,… parton processes as starting point for the shower?
- Problem of double-counting:

  When does radiation “belong”

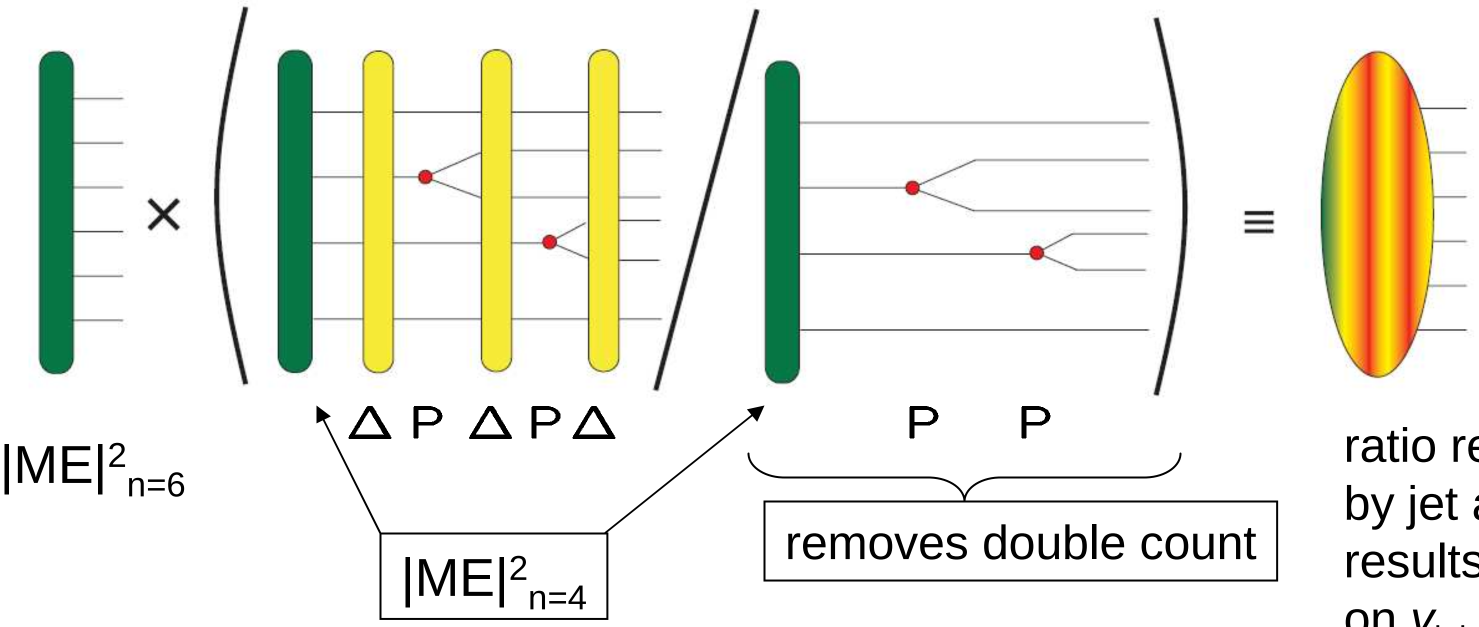
  to the shower, and

  when to the hard matrix element?



ME/shower matching (or merging)

- CKKW matching: Catani, Kuhn, Krauss, Webber, hep-ph/0109231
  - separate ME and shower domains using a common jet cluster algorithm variable (k_T algorithm with $y = y_{\mathrm{ini}}$)
- an example in pictures: Nagy, Soper, hep-ph/0607046



$|\mathrm{ME}|^2_{n=6}$ × (Δ P Δ P Δ / P P) ≡

$|\mathrm{ME}|^2_{n=4}$

removes double count

ratio regulated by jet algorithm; results depend on y_{ini}

ME/shower matching (cont.)

Several other general matching schemes available or in the works, e.g,:

MLM scheme (ALPGEN)

Lonnblad, hep-ph/0112284 (Ariadne)

CKKW (Sherpa)

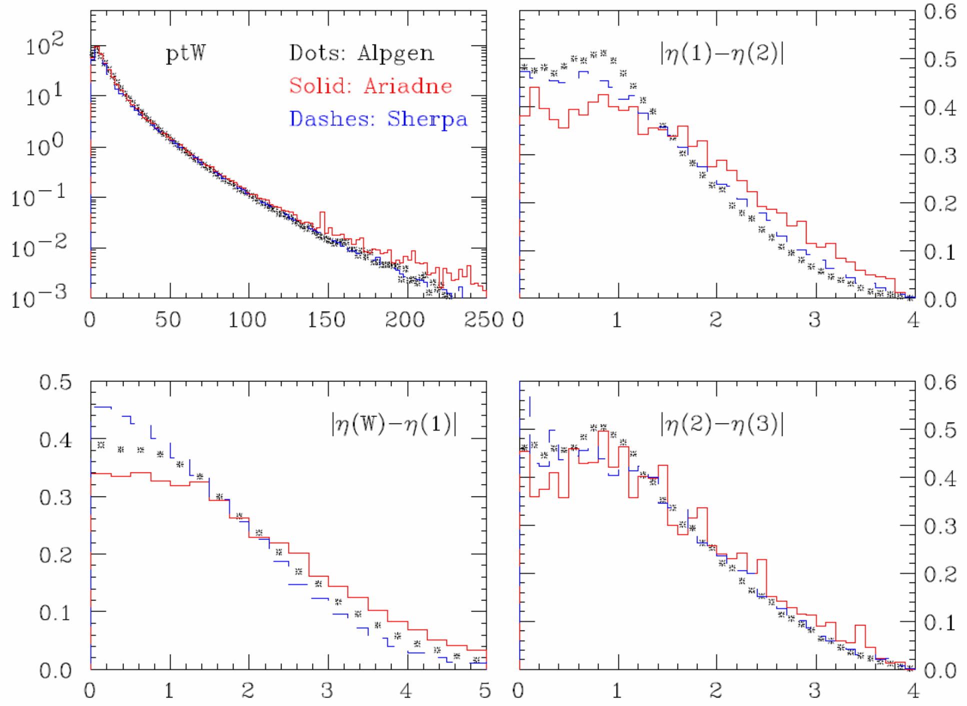
Mrenna, Richardson, hep-ph/0312274

Nagy, Soper, hep-ph/0601021

Skands, Giele, Kosower

ALPGEN, Ariadne, Sherpa compared in Hoche et al., hep-ph/0602031

$p\bar{p} \to W + 4$ jets at Tevatron



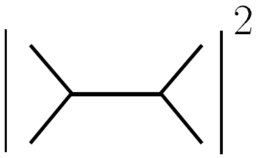
reasonable agreement between different schemes

NLO ME calculations

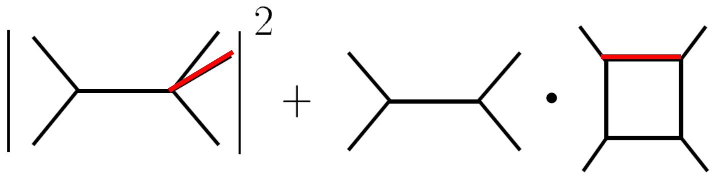
- Based on sum of all one-loop QCD Feynman diagrams for a given *n*-parton process (plus any “electroweak” particles)
- Also need to square tree amplitudes for *(n+1)*-parton process
  - these contribute at same order in α_s
  - infrared singularities cancel between virtual and real terms
- Event “shapes” usually quite reliable
  - except near kinematic boundaries (e.g. $p_T(W) \to 0$)
- Normalization of event rates usually pretty reliable (10% level)
- Description is only at parton level
- One-loop amplitudes were until recently hand-crafted
  - often with agonizing care taken over the finished product!
- NLO programs scattered about
  - many at http://www.cedar.ac.uk/hepcode/ , http://mcfm.fnal.gov/
- Feynman diagrams very often used
- Unitarity techniques now becoming competitive, especially for many final-state gluons

Infrared cancellations at NLO

LO



NLO



real virtual

Use dimensional regularization,
$D = 4 - 2\epsilon$

$d^4k \;\to\; d^{4-2\epsilon}k$
in all phase-space and loop integrals

soft singularities: $k_s \to 0$

$$\sigma^{\text{real}} \sim \int \frac{dk_s^2}{k_s^{2(1+\epsilon)}}\, \sigma^{\text{LO}}(k_s = 0)$$

collinear singularities: $k_{ab}^2 \to 0 \quad (k_a \;||\; k_b)$

$$\sim \int \frac{dk_{ab}^2}{k_{ab}^{2(1+\epsilon)}}\, \sigma^{\text{LO}}(k_P)$$

virtual soft/collinear singularities:

$$\sigma^{\text{virt}} \sim \Big[-\frac{1}{\epsilon^2}\sum_i C_i - \frac{1}{\epsilon}\sum_{i,j} D_{ij} \ln\Big(\frac{\mu^2}{-s_{ij}}\Big)\Big]\, \sigma^{\text{LO}}$$

- Virtual corrections cancel real singularities, but only for quantities insensitive to soft/collinear radiation → **infrared-safe observables *O***

Infrared safety

infrared-safe observables *O*:

- Behave smoothly in soft limit as any parton momentum → 0
- Behave smoothly in collinear limit as any pair of partons → parallel (||)

$$O_n(\ldots, k_s, \ldots) \quad \to \quad O_{n-1}(\ldots, \cancel{k_s}, \ldots) \qquad k_s \to 0$$

$$O_n(\ldots, k_a, k_b, \ldots) \quad \to \quad O_{n-1}(\ldots, k_P, \ldots) \qquad k_a \,||\, k_b$$

- Cannot predict perturbatively any infrared-unsafe quantity, such as:
  - the number of partons (hadrons) in an event
  - observables requiring no radiation in some region (rapidity gaps or overly strong isolation cuts)
  - $p_T(W)$ precisely at $p_T = 0$

Exercise: Can you think of other IR unsafe types of observables?

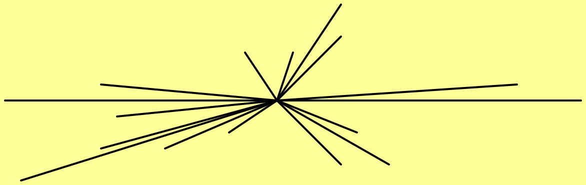
Infrared safety (cont.)

Examples of IR safe quantities:

- jets, defined by cluster or (suitable) cone algorithm
- most kinematic distributions of “electroweak” objects, *W, Z, Higgs* (photons tricky because they can come from fragmentation)

k_T jet cluster algorithm:

- Construct a list of objects, starting with particles *i* (or maybe calorimeter towers), plus “the beam” *b*
- Define a “distance” between objects, which vanishes in soft/collinear limits: $d_{ij} = 2\,\mathrm{min}\{k_T^{(i)}, k_T^{(j)}\}^2\,[\cosh(\eta^{(i)} - \eta^{(j)}) - \cos(\phi^{(i)} - \phi^{(j)})]$
- Cluster together the 2 objects with smallest distance; combine their 4-momenta into one. $d_{ib} = k_{\mathrm{T}}^2$
- Repeat until all $d_{ij} > d_{ij}^{\mathrm{cut}}$
- The remaining objects are jets



Cacciari
Salam

Anti-**k_T** is ~same except **k_T^2** → 1/ **k_T^2** New ATLAS default algorithm; Clusters hardest partons first. Catchment areas very round.

Infrared safety (cont.)

Cones are/were tricky to get right.

- JETCLU (CDF) + D0 cone algorithms were IR unsafe for NLO W + 2 jets
- Midpoint OK for W + 2 jets, but fails for W + 3 jets

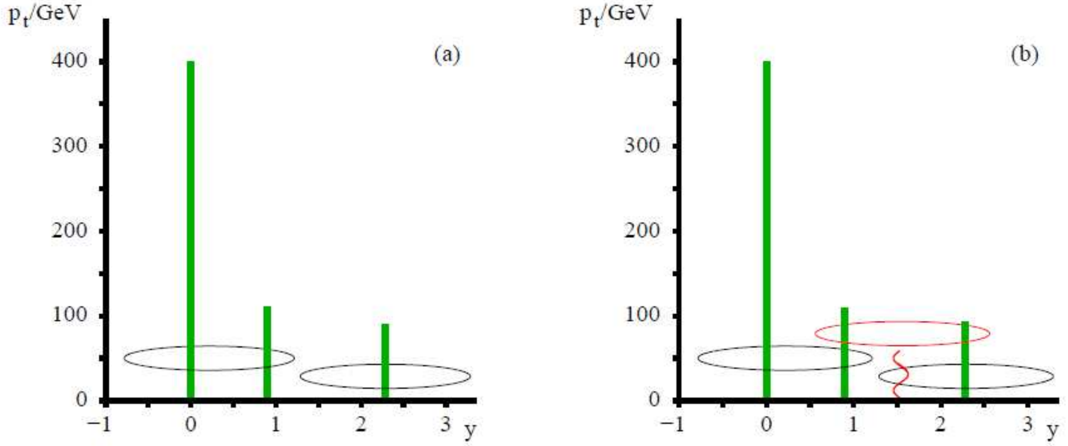


Figure 1: Configuration illustrating one of the IR unsafety problems of the midpoint jet algorithm ($R = 1$); (a) the stable cones (ellipses) found in the midpoint algorithm; (b) with the addition of an arbitrarily soft seed particle (red wavy line) an extra stable cone is found.

SIScone is a practical (fast enough) seedless cone algorithm that avoids these problems

Salam, Soyez

MC@NLO

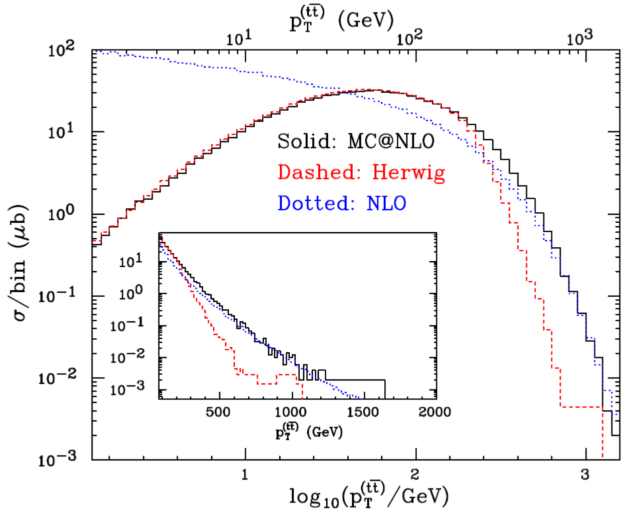
- As with LO matching of MEs to MCs, goal is to combine best features of two approaches: more accurate normalization of event rates (NLO) and hadron-level event descriptions (MC).
- More intricate than LO matching – must perform an exact NLO subtraction, then correct it to remove the parton-shower double-count

Working example: **MC@NLO** Frixione, Webber, hep-ph/0204244

Based on HERWIG MC

LHC processes available to date:

- single vector and Higgs bosons
- vector boson pairs
- heavy quark pairs
- single top
- lepton pairs
- Higgs bosons in association with a W or Z.



NNLO

- Based on sum of 2-loop *n*-parton process, plus 1-loop (*n*+1)-parton process, tree-level (*n*+2)-parton processes
- Required for high precision at LHC, because NLO results often have 10% or more residual uncertainties
- Where is high precision warranted?
- parton distributions
  - evolution (NNLO DGLAP kernels)
  - fits to DIS, Drell-Yan, and jet data
- LHC production of single *W*s and *Z*s
  - "partonic" luminosity monitor
  - precision m_W
- Higgs production via gluon fusion and extraction of Higgs couplings
- Inclusive jets? *Z* + 1 jet? Vector boson pairs? Not yet available
- Can use NNLO studies to reweight MC[@NLO]

Davatz et al.
hep-ph/0604077

End of Lecture 1