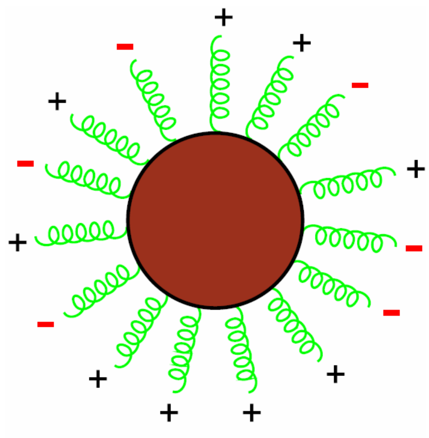
QCD at Colliders

Lecture 3



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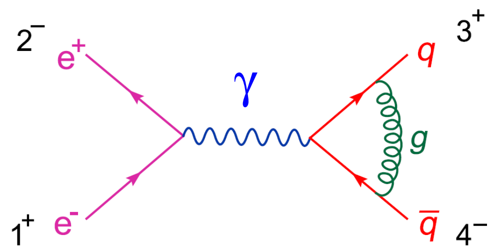
Berlin   1 Oct. 2009

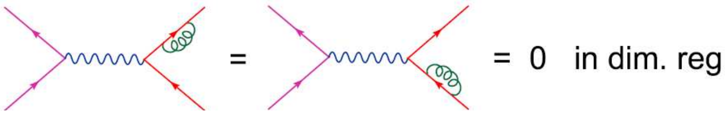
Lecture 3 Outline

- Virtual corrections
- NLO Drell-Yan example
- Catani-Seymour dipole formalism
- Why are K factors so big?
- Resummations
- NNLO status

Virtual Corrections

The simplest process:





cancellation of UV & IR divergences!

overlap of soft & collinear IR divergences

$$\mathcal{A}_4^{1-\text{loop}} = \mathcal{A}_4^{\text{tree}} \frac{\alpha_s}{4\pi} \exp[\epsilon(\ln(4\pi) - \gamma_E)] \times 2C_F \left(\frac{\mu^2}{-s_{12}}\right)^{\epsilon} \left[-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - \frac{7}{2} - \frac{\delta_R}{2} + \frac{\pi^2}{12}\right]$$

for $2 - 2\epsilon\delta_R$ virtual-gluon helicity states

$\delta_R = 1$ for CDR & HV schemes; $\delta_R = 0$ for FDH $\approx$ $\overline{\text{DR}}$ scheme

More complicated 1-loop amplitudes

$ggggg$

$+++++$: $= \frac{i}{96\pi^2} \frac{s_{12}s_{23} + s_{23}s_{34} + s_{34}s_{45} + s_{45}s_{51} + s_{51}s_{12} + \varepsilon(1,2,3,4)}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,1\rangle},$

$-++++$: $= \frac{i}{48\pi^2} \frac{1}{[1\,2]\,\langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,5\rangle\,[5\,1]} \Big[(s_{23}+s_{34}+s_{45})[2\,5]^2 - [2\,4]\,\langle 4\,3\rangle\,[3\,5]\,[2\,5] - \frac{[1\,2]\,[1\,5]}{\langle 1\,2\rangle\,\langle 1\,5\rangle} \Big(\langle 1\,2\rangle^2 \langle 1\,3\rangle^2 \frac{[2\,3]}{\langle 2\,3\rangle} + \langle 1\,3\rangle^2 \langle 1\,4\rangle^2 \frac{[3\,4]}{\langle 3\,4\rangle} + \langle 1\,4\rangle^2 \langle 1\,5\rangle^2 \frac{[4\,5]}{\langle 4\,5\rangle} \Big) \Big]$

$$V^g = -\frac{1}{\epsilon^2}\sum_{j=1}^{5}\left(\frac{\mu^2}{-s_{j,j+1}}\right)^{\epsilon} + \sum_{j=1}^{5}\ln\left(\frac{-s_{j,j+1}}{-s_{j+1,j+2}}\right)\ln\left(\frac{-s_{j+2,j-2}}{-s_{j-2,j-1}}\right) + \frac{5}{6}\pi^2 - \frac{\delta_R}{3}$$

the following functions for the $(1^-, 2^-, 3^+, 4^+, 5^+)$ helicity configuration,

~ 1 page

$--+++$:

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{23}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right)\right] - 2, \qquad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

$$F^f = -\frac{1}{2}\frac{\langle 1\,2\rangle^2 (\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle + \langle 2\,4\rangle\,[4\,5]\,\langle 5\,1\rangle)}{\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle} \frac{\mathrm{L}_0\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}}$$

$$F^s = -\frac{1}{3}\frac{[3\,4]\,\langle 4\,1\rangle\,\langle 2\,4\rangle\,[4\,5]\,(\langle 2\,3\rangle\,[3\,4]\,\langle 4\,1\rangle + \langle 2\,4\rangle\,[4\,5]\,\langle 5\,1\rangle)}{\langle 3\,4\rangle\,\langle 4\,5\rangle} \frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{1}{3}F^f - \frac{1}{3}\frac{\langle 3\,5\rangle\,[3\,5]^3}{[1\,2]\,[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]} + \frac{1}{3}\frac{\langle 1\,2\rangle\,[3\,5]^2}{[2\,3]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[5\,1]} + \frac{1}{6}\frac{\langle 1\,2\rangle\,[3\,4]\,\langle 4\,1\rangle\,\langle 2\,4\rangle\,[4\,5]}{s_{23}\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,s_{51}}$$

and the corresponding ones for the $(1^-, 2^+, 3^-, 4^+, 5^+)$ helicity configuration,

$-+-++$:

$$V^f = -\frac{5}{2\epsilon} - \frac{1}{2}\left[\ln\left(\frac{\mu^2}{-s_{34}}\right) + \ln\left(\frac{\mu^2}{-s_{51}}\right)\right] - 2, \qquad V^s = -\frac{1}{3}V^f + \frac{2}{9}$$

$$F^f = -\frac{\langle 1\,3\rangle^2\langle 4\,1\rangle[2\,4]^2}{\langle 4\,5\rangle\langle 5\,1\rangle}\frac{\mathrm{Ls}_1\left(\frac{-s_{23}}{-s_{51}}, \frac{-s_{34}}{-s_{51}}\right)}{s_{51}^2} + \frac{\langle 1\,3\rangle^2\langle 5\,3\rangle[2\,5]^2}{\langle 3\,4\rangle\langle 4\,5\rangle}\frac{\mathrm{Ls}_1\left(\frac{-s_{12}}{-s_{34}}, \frac{-s_{51}}{-s_{34}}\right)}{s_{34}^2} - \frac{1}{2}\frac{\langle 1\,3\rangle^3(\langle 1\,5\rangle\,[5\,2]\,\langle 2\,3\rangle - \langle 3\,4\rangle\,[4\,2]\,\langle 2\,1\rangle)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}}$$

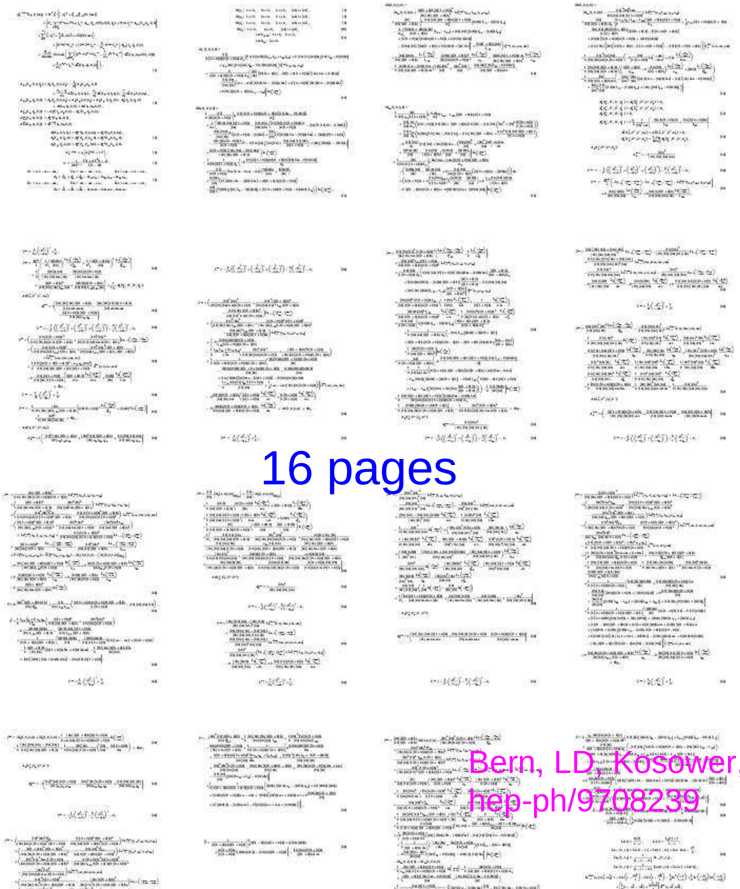
$$\begin{aligned} F^s = &-\frac{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,1\rangle^2[2\,4]^2}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle^2}\frac{2\,\mathrm{Ls}_1\left(\frac{-s_{23}}{-s_{51}}, \frac{-s_{34}}{-s_{51}}\right) + \mathrm{L}_1\left(\frac{-s_{23}}{-s_{51}}\right) + \mathrm{L}_1\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^2} \\ &+ \frac{\langle 3\,2\rangle\langle 2\,1\rangle\langle 1\,5\rangle\langle 5\,3\rangle^2[2\,5]^2}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle^2}\frac{2\,\mathrm{Ls}_1\left(\frac{-s_{12}}{-s_{34}}, \frac{-s_{51}}{-s_{34}}\right) + \mathrm{L}_1\left(\frac{-s_{12}}{-s_{34}}\right) + \mathrm{L}_1\left(\frac{-s_{51}}{-s_{34}}\right)}{s_{34}^2} \\ &+ \frac{2}{3}\frac{\langle 2\,3\rangle^2\langle 4\,1\rangle^3[2\,4]^3}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{23}}{-s_{51}}\right)}{s_{51}^3} - \frac{2}{3}\frac{\langle 2\,1\rangle^2\langle 5\,3\rangle^3[2\,5]^3}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle}\frac{\mathrm{L}_2\left(\frac{-s_{12}}{-s_{34}}\right)}{s_{34}^3} \\ &+ \frac{\mathrm{L}_2\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}^3}\Big(\frac{1}{3}\frac{\langle 1\,3\rangle\,[2\,4]\,[2\,5]\,(\langle 1\,5\rangle\,[5\,2]\,\langle 2\,3\rangle - \langle 3\,4\rangle\,[4\,2]\,\langle 2\,1\rangle)}{\langle 4\,5\rangle} \\ &+ \frac{2}{3}\frac{\langle 1\,2\rangle^2\langle 3\,4\rangle^2\,\langle 4\,1\rangle\,[2\,4]^3}{\langle 4\,5\rangle\,\langle 5\,1\rangle\,\langle 2\,4\rangle} - \frac{2}{3}\frac{\langle 3\,2\rangle^2\langle 1\,5\rangle^2\,\langle 5\,3\rangle\,[2\,5]^3}{\langle 5\,4\rangle\,\langle 4\,3\rangle\,\langle 2\,5\rangle}\Big) \\ &+ \frac{1}{6}\frac{\langle 1\,3\rangle^3(\langle 1\,5\rangle\,[5\,2]\,\langle 2\,3\rangle - \langle 3\,4\rangle\,[4\,2]\,\langle 2\,1\rangle)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\frac{\mathrm{L}_0\left(\frac{-s_{34}}{-s_{51}}\right)}{s_{51}} + \frac{1}{3}\frac{[2\,4]^2[2\,5]^2}{[1\,2][2\,3][3\,4]\langle 4\,5\rangle[5\,1]} \\ &- \frac{1}{3}\frac{\langle 1\,2\rangle\langle 4\,1\rangle^2[2\,4]^3}{\langle 4\,5\rangle\langle 5\,1\rangle\langle 2\,4\rangle[2\,3][3\,4]s_{51}} + \frac{1}{3}\frac{\langle 3\,2\rangle\langle 5\,3\rangle^2[2\,5]^3}{\langle 5\,4\rangle\langle 4\,3\rangle\langle 2\,5\rangle[2\,1][1\,5]s_{34}} + \frac{1}{6}\frac{\langle 1\,3\rangle^2\,[2\,4]\,[2\,5]}{s_{34}\,\langle 4\,5\rangle\,s_{51}} \end{aligned}$$

Bern, LD, Kosower,
hep-ph/9302280

More legs, or massive legs, rapidly increases complexity!

Some helicity config's more complex than others

$Vq\bar{q}gg$ $\qquad V = W, Z, \gamma^*$

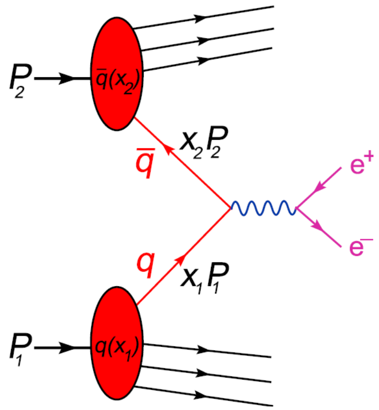


16 pages

Bern, LD, Kosower,
hep-ph/9708239

Drove many theorists "numerical" eventually

The Drell-Yan process



LO partonic cross section: $\hat{s} = x_1 x_2 s = M^2_{e^+e^-}$

$$\hat{\sigma}(q\bar{q} \to e^+e^-) = \frac{1}{2\hat{s}} \frac{1}{4N_c^2} \sum_{h,c} |\mathcal{A}_4|^2$$

$$= \frac{4\pi\alpha^2}{3} \frac{1}{N_c} Q_q^2$$

$$\frac{d\hat{\sigma}}{dM^2} = \frac{\sigma_0}{N_c} Q_q^2 \delta(\hat{s} - M^2), \qquad \sigma_0 \equiv \frac{4\pi\alpha^2}{3M^2}$$

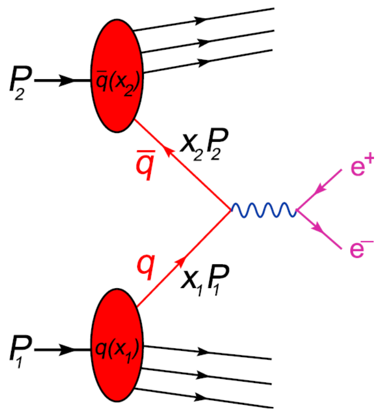
LO hadronic cross section:

$$\frac{d\sigma}{dM^2} = \int_0^1 dx_1 dx_2 \sum_q [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)] \frac{d\hat{\sigma}}{dM^2}$$

$$= \frac{\sigma_0}{N_c} \int_0^1 dx_1 dx_2 \delta(x_1 x_2 s - M^2) \sum_q Q_q^2 [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)]$$

$$= \frac{\sigma_0\, s}{N_c} \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) \sum_q Q_q^2 [q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)], \qquad \tau \equiv \frac{M^2}{s}$$

Drell-Yan rapidity distribution



rapidity $$Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

$$\exp(2Y) = \frac{E+p_z}{E-p_z} = \frac{P_2\cdot P_Z}{P_1\cdot P_Z} = \frac{\frac{1}{x_2}p_{\bar{q}}\cdot P_Z}{\frac{1}{x_1}p_q\cdot P_Z} = \frac{x_1}{x_2}$$

combined with mass measurement,

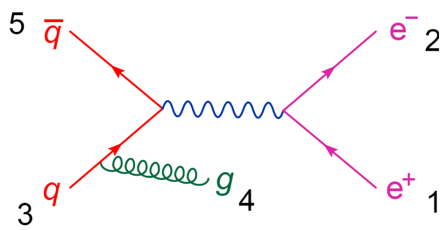
$$x_1x_2 = \tau = \frac{M^2}{s}$$

double distribution $$\frac{d^2\sigma}{dM^2dY} = \frac{\sigma_0}{N_c\,s}\sum_q Q_q^2[q(x_1)\bar{q}(x_2)+\bar{q}(x_1)q(x_2)]$$

measures product of quark and antiquark distributions at

$$x_1 = \sqrt{\tau}e^{Y} \qquad x_2 = \sqrt{\tau}e^{-Y}$$

NLO QCD corrections to Drell-Yan production



$$|A_5|^2 = \frac{s_{13}^2 + s_{15}^2 + s_{23}^2 + s_{25}^2}{s_{12}s_{34}s_{45}}$$

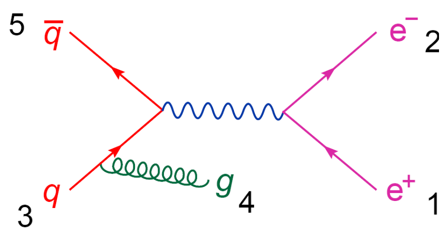
As at LO, average over decay direction of e^+ and e^-:

$$\langle k_1^\mu k_1^\nu \rangle_\Omega \equiv \int \frac{d\Omega_{e^+e^-}}{4\pi} k_1^\mu k_1^\nu = -\frac{s_{12}}{12}\eta^{\mu\nu} + \frac{1}{3}(k_1+k_2)^\mu (k_1+k_2)^\nu = \langle k_2^\mu k_2^\nu \rangle_\Omega$$

$$\langle s_{13}^2 \rangle_\Omega = \langle s_{23}^2 \rangle_\Omega = \frac{1}{3}(s_{13}+s_{23})^2 = \frac{1}{3}(s_{34}+s_{35})^2$$

$$\Rightarrow \quad \langle |A_5|^2 \rangle_\Omega = \frac{2}{3}\frac{(s_{34}+s_{35})^2 + (s_{35}+s_{45})^2}{s_{12}s_{34}s_{45}}$$

Phase space for DY @ NLO



Could use gluon energy, angle in CM frame, E_4, θ

Trade for z,y ε [0,1] defined by:

$$z = \frac{s_{12}}{s_{35}}$$
$$y = \frac{1-\cos\theta}{2}$$

$$E_4 = -\frac{s_{34}+s_{45}}{2\sqrt{s_{35}}} = -\frac{s_{12}-s_{35}}{2\sqrt{s_{35}}} = \frac{1-z}{2}\sqrt{s_{35}}$$
$$s_{34} = -\sqrt{s_{35}}E_4(1-\cos\theta) = -y(1-z)s_{35}$$
$$\Rightarrow \quad s_{45} = -\sqrt{s_{35}}E_4(1-\cos\theta) = -(1-y)(1-z)s_{35}$$
$$s_{12} = M^2 = z s_{35}$$

cross section:

$$\langle |A_5|^2 \rangle_\Omega = \frac{2}{3\,M^2} \frac{(1-y(1-z))^2 + (1-(1-y)(1-z))^2}{y(1-y)\,(1-z)^2}$$

P.S. measure in D=4-2ε:

$$\propto \left(\frac{\mu^2}{s_{35}}\right)^\epsilon \frac{d^{3-2\epsilon}p_4}{2E_4} \propto \left(\frac{\mu^2\, z}{M^2}\right)^\epsilon dE_4\, E_4^{1-2\epsilon}\, d\cos\theta(\sin^2\theta)^{-\epsilon} d\Omega^{1-2\epsilon}$$
$$\propto \left(\frac{\mu^2}{M^2}\right)^\epsilon dy\, dz\, [y(1-y)]^{-\epsilon}\, z^\epsilon\, (1-z)^{1-2\epsilon}$$

QCD corrections to DY (cont.)

Integral to do:

$$I = \left(\frac{\mu^2}{M^2}\right)^{\epsilon} z^{\epsilon}(1-z)^{-1-2\epsilon} \times \int_0^1 dy\, [y(1-y)]^{-\epsilon} \frac{(1-y(1-z))^2 + (1-(1-y)(1-z))^2}{y(1-y)}$$

Hard collinear divergences are at *y = 0,1*

related by symmetry

Separate using

$$\frac{1}{y(1-y)} = \frac{1}{y} + \frac{1}{1-y}$$

Expand *1/y* term in cross section about *y=0*

$$I = 2\left(\frac{\mu^2}{M^2}\right)^{\epsilon} z^{\epsilon}(1-z)^{-1-2\epsilon} \int_0^1 dy\, y^{-1-\epsilon}\left[1+z^2-2y(1-y)(1-z)^2\right] \times(1-\epsilon\ln(1-y))$$

$$= 2\left(\frac{\mu^2}{M^2}\right)^{\epsilon} z^{\epsilon}(1-z)^{-1-2\epsilon}\left[-\frac{1+z^2}{\epsilon} - (1-z)^2 + \mathcal{O}(\epsilon)\right]$$

QCD corrections to DY (cont.)

Including a few other omitted prefactors:

divergence absorbed into *q(x)* in $\overline{\text{MS}}$ factorization scheme

$$\frac{d\hat{\sigma}^{\text{NLO, real}}}{dM^2} = \frac{\sigma_0}{N_c\, s} Q_q^2 \frac{\alpha_s}{2\pi} C_F \Bigg[2\left(-\frac{1}{\epsilon} - \ln(4\pi) + \gamma\right)\frac{1+z^2}{1-z} - 2\frac{1+z^2}{1-z}\left(-2\ln(1-z) + \ln z - \ln\frac{M^2}{\mu^2}\right) - 2(1-z)^2 \Bigg]$$

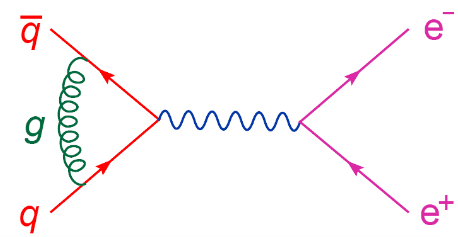
correction to cross section

artifact of my using unconventional FDH scheme with 2 gluon helicities, vs. standard 2-2ε of CDR – drop!

$$\begin{aligned} q(x,\mu) &= q_0(x) + \frac{\alpha_s(\mu)}{2\pi}\left(-\frac{1}{\epsilon} - \ln(4\pi) + \gamma_E\right)\int_x^1 \frac{d\xi}{\xi}\left[P_{qq}^{(0)}(x/\xi) q_0(\xi) + P_{qg}^{(0)}(x/\xi) g_0(\xi)\right] \\ &= q_0(x) + \frac{\alpha_s(\mu)}{2\pi}\left(-\frac{1}{\epsilon} - \ln(4\pi) + \gamma_E\right)\int_x^1 \frac{dz}{z}\left[C_F \frac{1+z^2}{1-z} q_0(x/z) + P_{qg}^{(0)}(z) g_0(x/z)\right] \end{aligned}$$

QCD corrections to DY (cont.)

Finally, virtual graph has support only at *z=1*.
-- kinematics same as at LO. Regulates *1/(1-z)* into plus distribution. Final result:



$$\frac{d\sigma^{\rm NLO}}{dM^2} = \frac{\sigma_0}{N_c\, s}\int_0^1 dx_1 dx_2 dz\, \delta(x_1 x_2 z - \tau) \sum_q Q_q^2 \Big[q(x_1,\mu_F)\bar{q}(x_2,\mu_F)\Big(\delta(1-z) + \frac{\alpha_s(\mu_R)}{2\pi} C_F D_q(z,\mu_F)\Big) + g(x_1,\mu_F)(q(x_2,\mu_F)+\bar{q}(x_2,\mu_F))\frac{\alpha_s(\mu_R)}{2\pi} T_R D_g(z,\mu_F) + (x_1 \leftrightarrow x_2)\Big]$$

where

$$D_q(z,\mu_F) = 4(1+z^2)\left(\frac{\ln(1-z)+\ln(M/\mu_F)}{1-z}\right)_+ -2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2}{3}\pi^2 - 8\right)$$

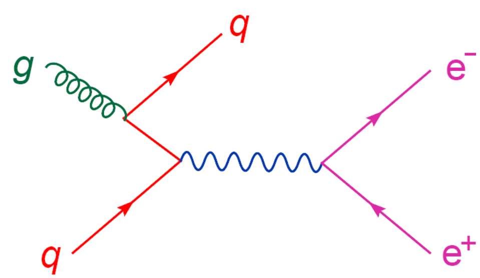
singular distribution as $z \longrightarrow 1$

QCD corrections to DY (cont.)

and

$$D_g(z,\mu_F) = (z^2+(1-z)^2)\left[\ln\frac{(1-z)^2}{z} + 2\ln\frac{M}{\mu_F}\right] + \frac{1}{2} + 3z - \frac{7}{2}z^2$$

comes from the *qg -> qγ** subprocess:

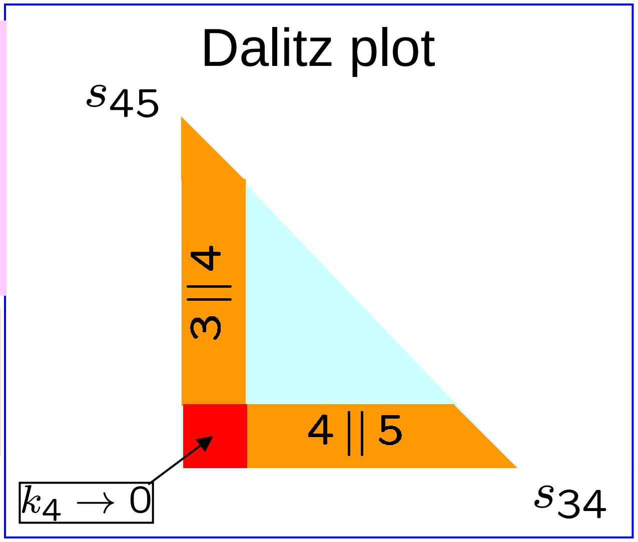


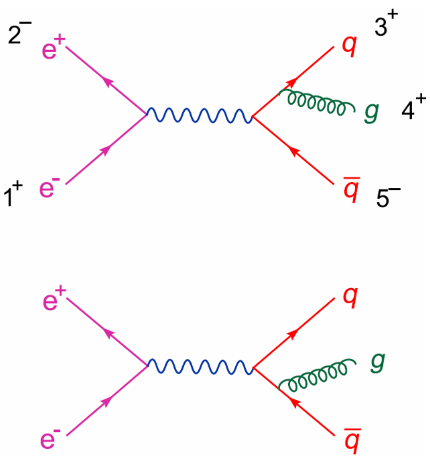
- Cross section related by crossing to *$\bar{q}$q -> gγ**
- Remove *g -> q$\bar{q}$* collinear singularity in same way
- Note that there is no *1/(1-z)* (soft gluon) singularity in this term.

Real radiation in general case

Cannot perform the phase-space integral analytically in $D=4-2\varepsilon$, especially not for generic experimental cuts

Also can't do it numerically, because of $1/\varepsilon^2$ poles





2 solutions:

1. Slice out singular regions of phase-space, with (thin) width s_{min} Perform integral there approximately. Rest of integral done numerically. Check cancellation of s_{min} dependence.
2. Subtract a function that mimics the soft/collinear behavior of the radiative cross section, and which you can integrate (analytically). Integral of the difference can be done numerically.

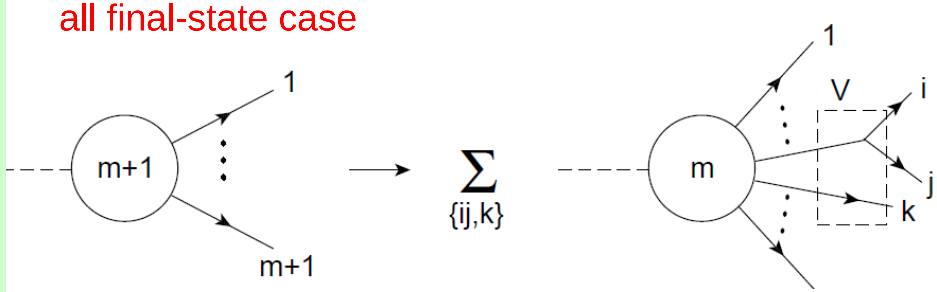
Dipole formalism

Catani, Seymour, hep-ph/9602277, hep-ph/9605323. Recently automated by several groups

Popular version of the subtraction method

Using Altarelli-Parisi kernels, build dipole subtraction function $D_{ij,k}$ for each pair of partons i,j that can get singular, and for each "spectator" parton k

all final-state case



$$\mathcal{D}_{ij,k}(p_1,...,p_{m+1}) = -\frac{1}{2p_i \cdot p_j}$$

$$\cdot {}_m\!< 1,..,\widetilde{ij},..,\widetilde{k},..,m+1| \frac{\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2} \boldsymbol{V}_{ij,k} |1,..,\widetilde{ij},..,\widetilde{k},..,m+1>_m$$

The $D_{ij,k}$ multiply the LO cross section, at a boosted phase-space point:

$$\widetilde{p}_k^\mu = \frac{1}{1-y_{ij,k}} p_k^\mu \ , \quad \widetilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}} p_k^\mu$$

$$\tilde{z}_i = \frac{p_i p_k}{p_j p_k + p_i p_k} = \frac{p_i \widetilde{p}_k}{\widetilde{p}_{ij} \widetilde{p}_k} \qquad y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i}$$

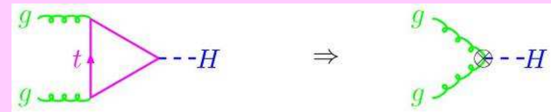
$$< s|\boldsymbol{V}_{q_i g_j,k}(\tilde{z}_i; y_{ij,k})|s' > \ = \ 8\pi\mu^{2\epsilon}\alpha_S\, C_F \left[\frac{2}{1-\tilde{z}_i(1-y_{ij,k})} - (1+\tilde{z}_i) - \epsilon(1-\tilde{z}_i)\right] \delta_{ss'}$$

All dipole integrals can be done analytically

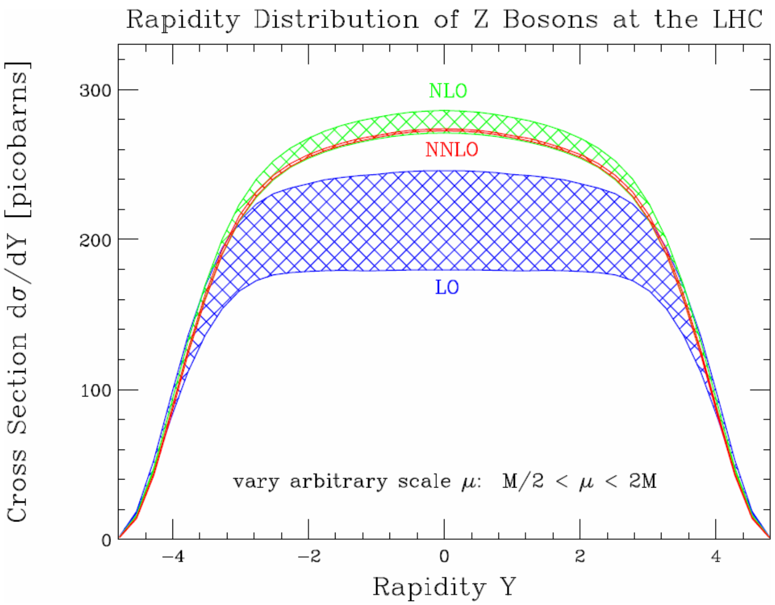
Why are (N)NLO corrections large?

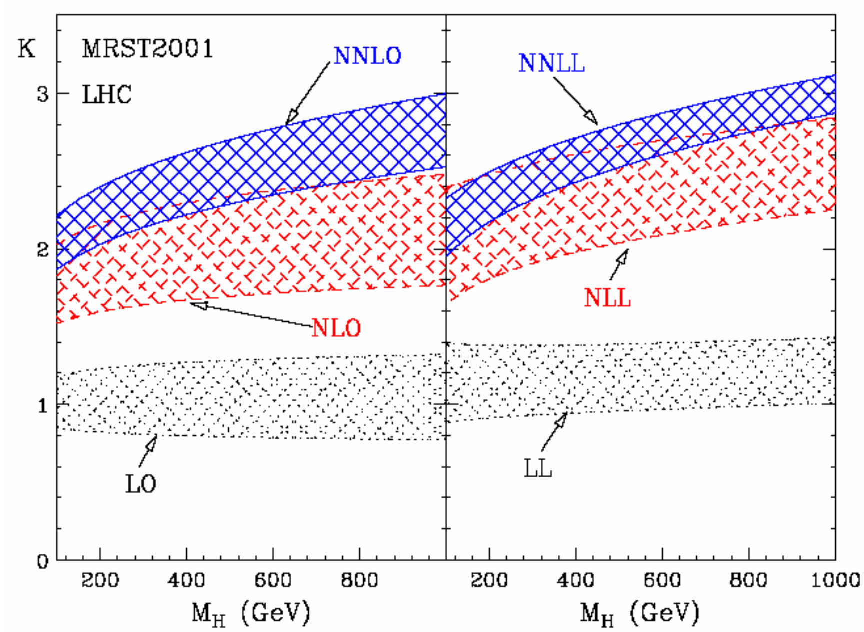
+ 30% typical for
quark-initiated (*W/Z*, …)

+ 80-100%
for some



gluon-initiated ($gg \to \text{Higgs} + X$)



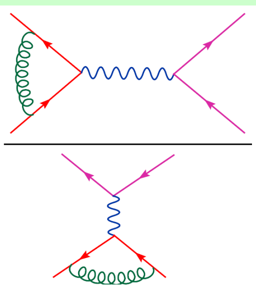


This is much bigger than $R_{e^+e^-} = 1 + \frac{\alpha_s}{\pi} \approx 1 + \frac{0.1}{\pi} \approx 1 + 0.03$!!

Some answers (not all for all processes)

1. LO parton distribution fits not very reliable due to large theory uncertainties
2. New processes can open up at NLO. In *W/Z* production at Tevatron or LHC, *qg -> γ^*q* opens up, and *g(x)* is very large – but correction is negative!
3. Large π^2 from analytic continuation from space-like region where pdfs are measured (DIS) to time-like region (Drell-Yan/W/Z):

2 Re



$$= 1+\frac{\alpha_s}{\pi}C_F\left(-\frac{1}{\epsilon^2}\right)\mathrm{Re}\left[\left(\frac{\mu^2}{-Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{+Q^2}\right)^{\epsilon}\right]$$

$$= 1+\frac{\alpha_s}{\pi}C_F\left(-\frac{1}{\epsilon^2}\right)\mathrm{Re}\left[\exp(i\pi\epsilon)-1\right] = 1+\frac{\alpha_s}{\pi}C_F\frac{\pi^2}{2}$$

4. Soft-gluon/Sudakov resummation

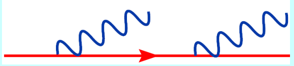
- A prevalent theme in QCD whenever one is at an edge of phase space.
- Infrared-safe but sensitive to a second, smaller scale
- Same physics as in (high-energy) QED: $e^+e^- \; = \; e^+e^-(\gamma)$
- What is prob. of no γ with $E > \Delta E$, $\theta > \Delta\,\theta$?

$$P = 1 - \frac{\alpha}{\pi}\int_{\Delta E}\frac{dE}{E}\int_{\Delta\theta}\frac{d\theta}{\theta} + \cdots \;=\; 1 - \frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta + \cdots$$
$$= \exp\left(-\frac{\alpha}{\pi}\ln\Delta E\ln\Delta\theta\right) + \cdots$$

soft

collinear

leading double logarithms -- in contrast to single logs of renormalization group, DGLAP equations.

exponentiation because soft emissions are independent 

Hadron collider examples

$p_T(Z)$, important application to $p_T(W)$, m_W measurement at Tevatron

Production of heavy states, like
- top quark at the Tevatron (W and Z production less so),
- even a light Higgs boson at the LHC, via $gg \to H$

Called threshold resummation or $z \longrightarrow 1$ limit,
where $z = M^2/s$.
Can be important for $z \ll 1$ though.
For m_H = 120 GeV at 14 TeV LHC, x = 10^{-4} !
Radiation is being suppressed because you are running out of phase space – parton distributions are falling fast.

Threshold Resummation

Can see the first threshold log in the NLO corrections to Drell-Yan/*W/Z* production:

$$C_F D_q(z,\mu_F) \;=\; 4C_F(1+z^2)\left(\frac{\ln(1-z)+\ln(M/\mu_F)}{1-z}\right)_+ \\ -2\frac{1+z^2}{1-z}\ln z \;+\; \delta(1-z)\left(\frac{2}{3}\pi^2-8\right)$$

It is a double-log expansion:

$$D_q^{(n)}(z,\mu_F) \;\propto\; (C_F\alpha_s)^n\left[\left(\frac{\ln^{2n+1}(1-z)}{1-z}\right)_+ + \cdots\right]$$

For *gg* -> *H*, same leading behavior at large *z*.
Except color factor is much bigger: $C_A = 3$, not $C_F = 4/3$

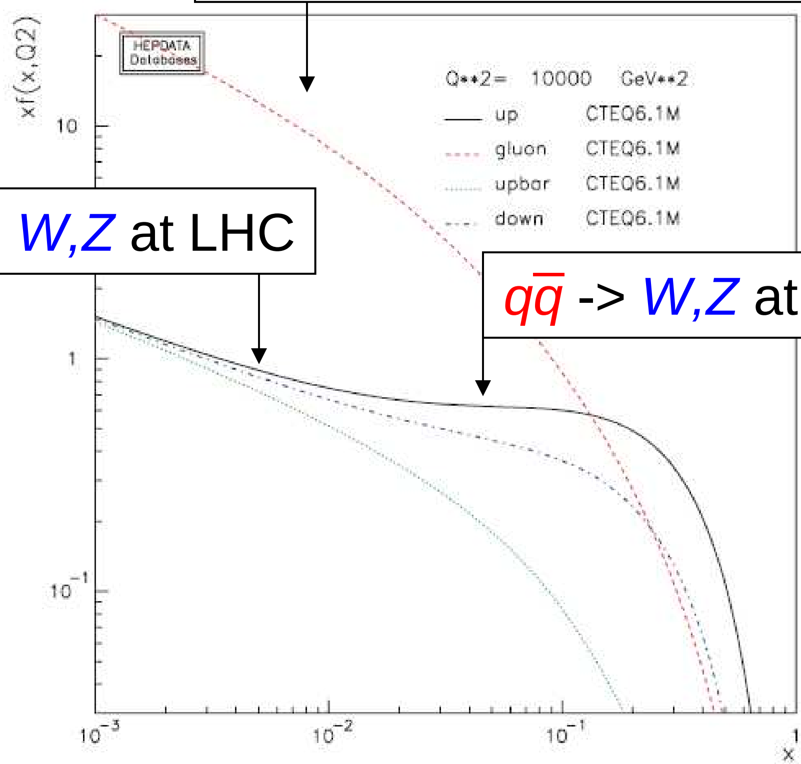
$$D_{gg\to H}^{(n)}(z,\mu_F) \;\propto\; (C_A\alpha_s)^n\left[\left(\frac{\ln^{2n+1}(1-z)}{1-z}\right)_+ + \cdots\right]$$

Fast falling pdfs -- worse for gluons

gg -> *light Higgs* at LHC

$q\bar{q}$ -> *W,Z* at LHC

$q\bar{q}$ -> *W,Z* at Tevatron



Q: If it is called Sudakov suppression, why does it increase the cross section?

A. Because the same suppression happens in the DIS process used to measure the pdfs. Both parton distributions "bigger than you thought": 2 – 1 > 0.

pdfs → 2

partonic cross section → 1

The NNLO Frontier

NNLO QCD required for high precision at LHC:

- parton distributions
  - evolution (NNLO DGLAP kernels)
  - fits to DIS, Drell-Yan, and jet data
- LHC production of single *W*s and *Z*s
  - “partonic” luminosity monitor
  - precision m_W
- Higgs production via gluon fusion and extraction of Higgs couplings

- NNLO progress historically in terms of number of scales: 0,1,2, ∞
- More scales tougher, but more flexible applications

No-scale, inclusive problems

$R(e^+e^- \to \text{hadrons})$ & $R(\tau \to \nu_\tau + \text{hadrons})$

Gorishnii, Kataev, Larin;
Surguladze, Samuel (1990)

DIS sum rules: $\int_0^1 dx\, F_n(x)$

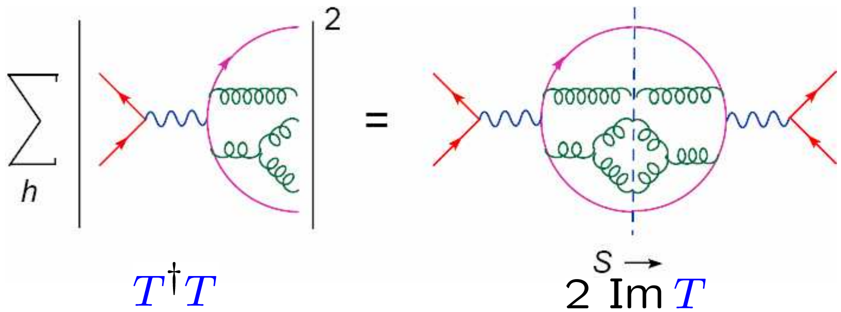
Bjorken $(\bar{\nu}p - \nu p)$ Larin, Tkachov, Vermaseren (1990)

Bjorken $(\vec{e}p)$ & Gross-Llewellyn-Smith $(\nu p + \bar{\nu}p)$ Larin, Vermaseren (1991)

Use unitarity of *S*-matrix to relate real production of hadrons to imaginary part of virtual-photon forward scattering:

$$S = 1 + iT$$
$$1 = S^\dagger S = (1 - iT^\dagger)(1 + iT)$$
$$2\,\mathrm{Im}\,T = T^\dagger T$$

$\Rightarrow$

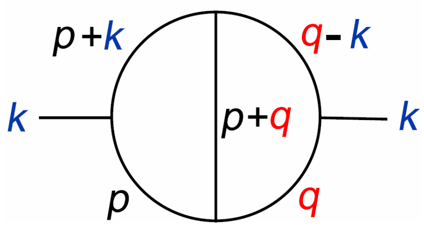


cut imposes $\delta(p_i^2 - m_i^2)$

Transforms inclusive phase-space integrals into loop integrals for virtual photon propagator. *s* (or Q^2) factors out by dim. analysis

No-scale problems (cont.)

- Multi-loop integral technology: Integration by parts (IBP)



Chetyrkin, Tkachov (1981)

$$0 = \int d^D p d^D q \ldots \frac{\partial}{\partial q^\mu} \frac{k^\mu}{p^2 q^2 (p+q)^2 \ldots}$$

- Reduces problem to system of linear equations, solved recursively by MINCER, in terms of few "master integrals"

Gorishnii, Larin, Surguladze, Tkachov (1989)

No-scale $\Rightarrow$ analytic simplicity – pure numbers

$$\begin{aligned} \frac{R_{e^+e^-}}{R^{(0)}} = \; & 1 + \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-11\zeta(3) + \frac{365}{24} + n_f\left(\frac{2}{3}\zeta(3) - \frac{11}{12}\right)\right] \\ & + \left(\frac{\alpha_s}{\pi}\right)^3 \left[\frac{275}{6}\zeta(5) - \frac{1103}{4}\zeta(3) - \frac{121}{8}\zeta(2) + \frac{87029}{288} \right. \\ & + n_f\left(-\frac{25}{9}\zeta(5) + \frac{262}{9}\zeta(3) + \frac{11}{6}\zeta(2) - \frac{7847}{216}\right) \\ & \left. + n_f^2\left(-\frac{19}{27}\zeta(3) - \frac{1}{18}\zeta(2) + \frac{151}{162}\right)\right] \end{aligned}$$

1-scale, inclusive problems

Drell-Yan, W, Z total cross section

$\sigma^{\text{tot}}(pp \to V + X)$ Hamberg, van Neerven, Matsuura (1990)

Higgs total cross section $(m_t \to \infty)$

$\sigma^{\text{tot}}(pp \to H + X)$ Harlander, Kilgore; Anastasiou, Melnikov (2002); Ravindran, Smith, van Neerven (2003)

$\hat{\sigma}$ depends on $z = M^2_{V,H}/\hat{s}$

DIS coefficient functions $C_i(z)$ Van Neerven, Zijlstra (1991)

F_L -- Moch, Vermaseren, Vogt (2004)

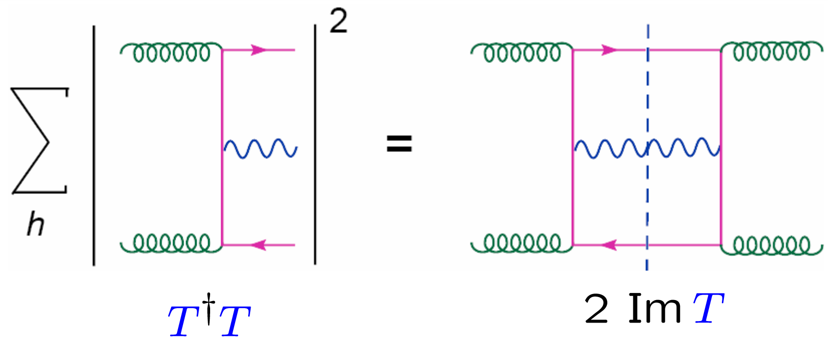
Leading-twist anomalous dimensions

DGLAP kernels $P_{ij}(x)$ Moch, Vermaseren, Vogt (2004)

1-scale problems (cont.)

- Can apply unitarity and multi-loop integral technology to DY/Higgs production too: Anastasiou, Melnikov (2002)
  forward 2 → 2 scattering instead of propagator

$$\hat{\sigma}(gg \to V q\bar{q}) = 2\,\text{Im}\,\mathcal{A}(gg \to gg)|_{Vq\bar{q}}$$

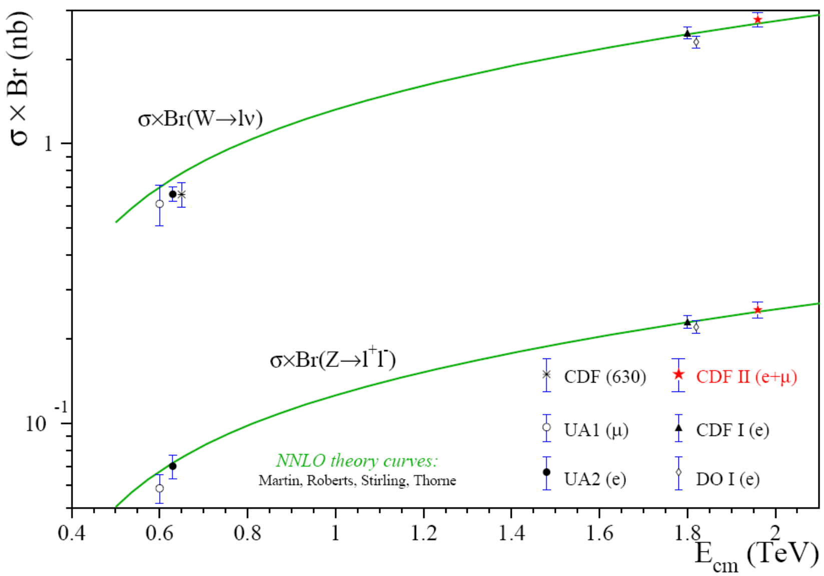


cut imposes $\delta(p_i^2 - m_i^2)$, which includes $\delta(m_V^2 - z\hat{s})$

analytic structure of 1-scale integrals: (harmonic) polylogarithms $\text{Li}_n(z)$

1-scale applications

- Precise prediction of total cross sections σ_W and σ_Z at Tevatron (and LHC) – use ratio to measure Br($W \to l\nu$)



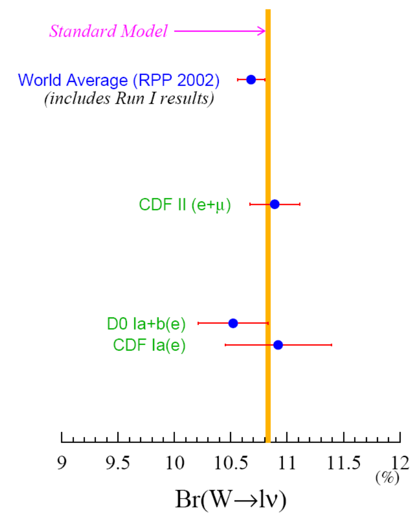
Hamberg, van Neerven, Matsuura (1991); Harlander, Kilgore (2002)

$$R \equiv \frac{\sigma_W}{\sigma_Z} \frac{\text{Br}(W \to \ell\nu)}{\text{Br}(Z \to \ell\ell)} = 10.92 \pm 0.15 \pm 0.14$$

NNLO theory

LEP

→



CDF, hep-ex/0508029

2-scale, semi-inclusive problem

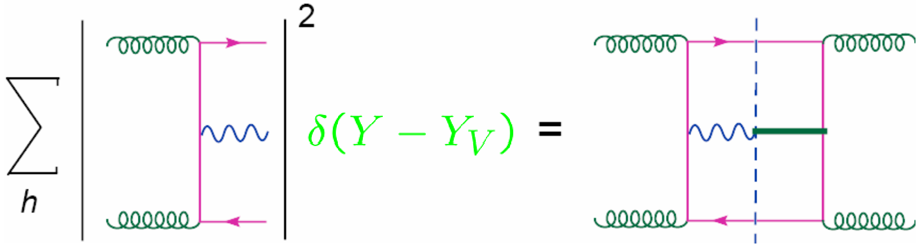
Drell-Yan, W, Z rapidity distribution $\frac{d\sigma(pp\to V+X)}{dY_V}$

$$Y_V = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

Anastasiou, LD, Melnikov, Petriello (2003)

$\hat{\sigma}$ depends on $z = M^2_{V,H}/\hat{s}$ and Y_V

- Unitarity + multi-loop IBP method still works – one more "propagator" to implement rapidity δ function



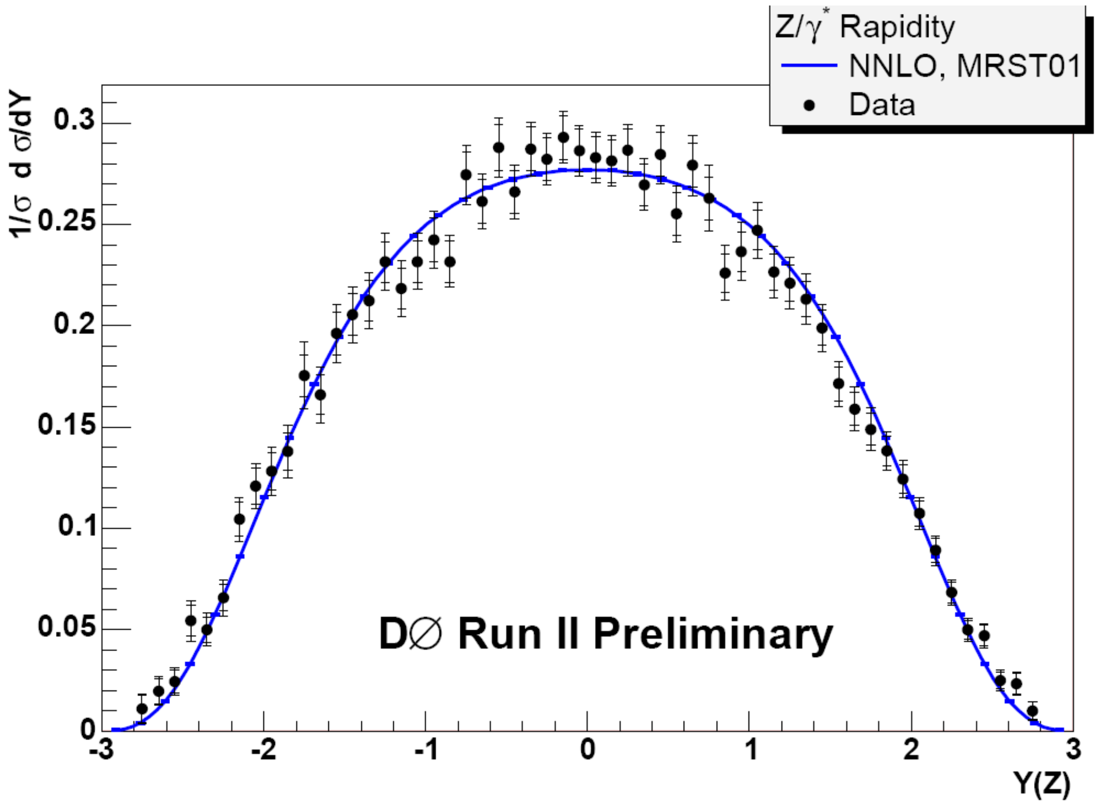
- Complicated analytic structure:

$$\text{Li}_2\left(\frac{u-1-i\sqrt{(4u^2-z(1+u)^2)/z}}{2u}\right)$$

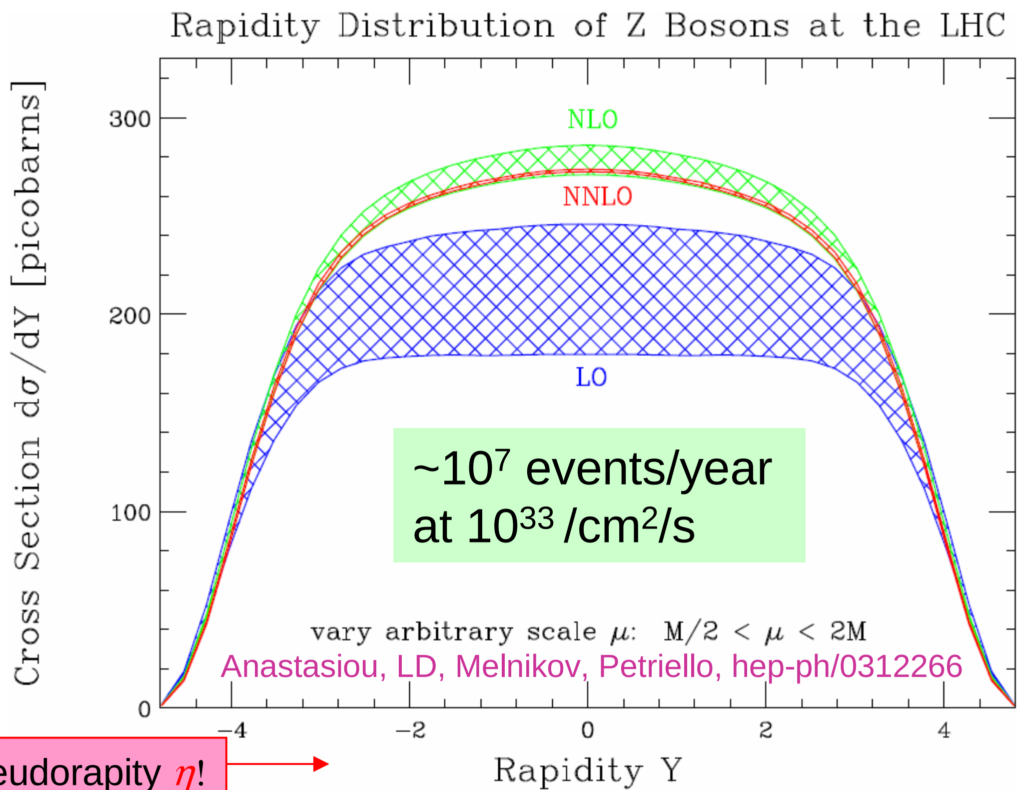
$$u \equiv \frac{x_1}{x_2}\exp(-2Y_V)$$

2-scale applications

$\frac{d\sigma(pp \to V + X)}{dY_V}$ at Tevatron



2-scale applications (cont.)



not pseudorapity η!

- Uncertainty due to omitted higher-order terms very small

- Permits use of $d\sigma_{W,Z}/dY$ as "partonic luminosity monitor": use it to normalize other cross sections – pp luminosity, some detector efficiencies, drop out of ratio

$$\frac{d\sigma}{dY} \sim \sum_q [q_1(x_1)\bar{q}_2(x_2) + \bar{q}_1(x_1)q_2(x_2)]$$

$$x_1 = \frac{M_V}{\sqrt{s}}\, e^{Y} \qquad x_2 = \frac{M_V}{\sqrt{s}}\, e^{-Y}$$

Dittmar, Pauss, Zurcher, hep-ex/9705004

∞-scale problem

- "Holy grail": Flexible method for arbitrary (infrared-safe) observable at NNLO. Include isolation, p_T, rapidity cuts, jet algorithm dependence, …

Partial wish list:

Now done!
Gehrmann, Gehrmann-de Ridder, Glover, Heinrich; Weinzierl

- e^+e^- event-shape observables
- pp or ep $\rightarrow$ inclusive jets, dijets, multijets
- $pp \rightarrow (W, Z, H) + X$ with parton-level cuts
- $pp \rightarrow (W, Z)$ + jets

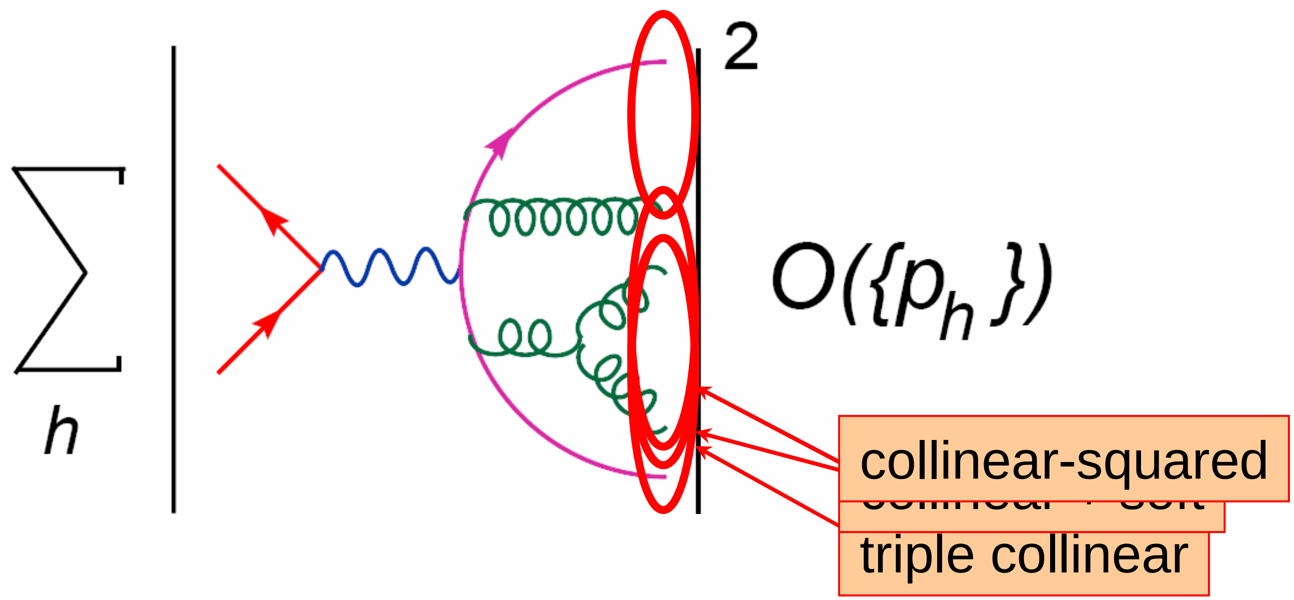
- Amplitudes known for many of these processes
- Phase-space integration is the stumbling block

- Analytic structure too complicated; go numerical

Numerical phase-space integration

- Integration has to be done in *D=4-2ε* due to severe infrared divergences ($1/\varepsilon^4$)

Example: e^+e^- event-shapes



Phase-space integration (cont.)

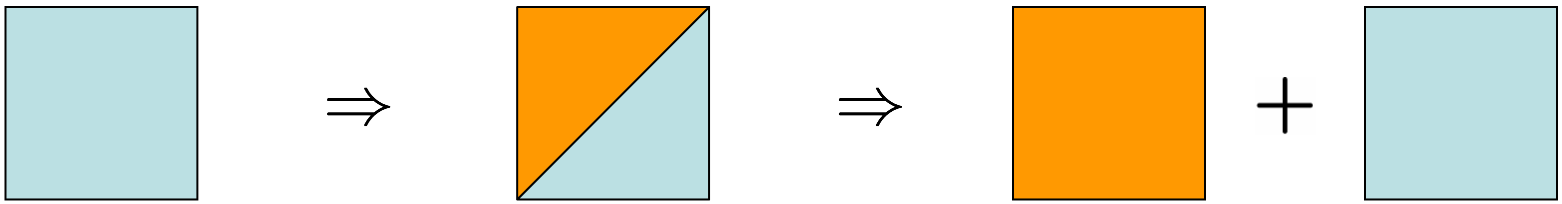
Two basic approaches at present:

Method 1. Iterated sector decomposition

Partition integration region and remap to make divergences "1-dimensional", let computer find subtraction terms

Binoth, Heinrich; Anastasiou, Melnikov, Petriello (2003,2004)

Simple example: $I = \int_0^1 \int_0^1 dx\,dy \frac{x^\epsilon y^\epsilon}{(x+y)^2}$



Phase-space integration (cont.)

Method 2. Use known factorization properties of amplitudes to build subtraction terms for general processes, and integrate them, a la NLO

Many authors (~1997-2003)

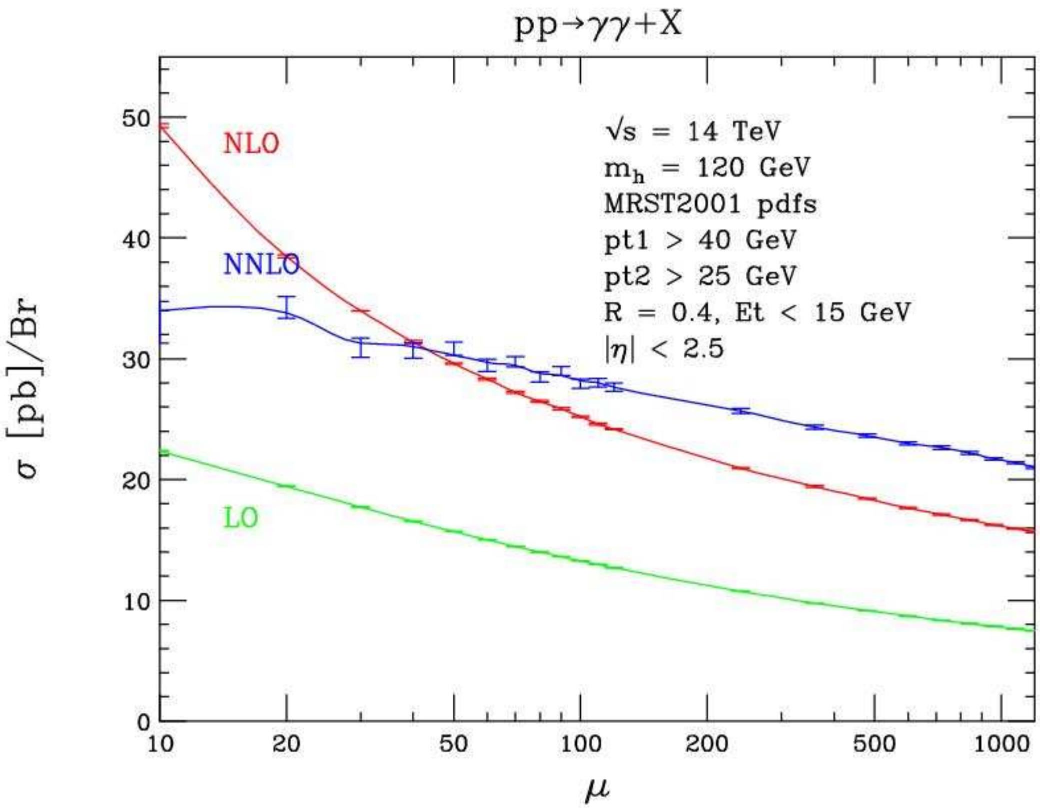
Kosower; Weinzierl; Gehrmann, Gehrmann-de Ridder, Heinrich (2003)
Frixione, Grazzini (2004); Gehrmann, Gehrmann-de Ridder, Glover (2004,2005);
Del Duca, Somogyi, Trocsanyi (2005)

used for NNLO $e^+e^- \rightarrow$ 3 jets

LHC applications

- So far, just from method 1

- $pp \to H + X \to \gamma\gamma + X$ with parton-level cuts



Anastasiou, Melnikov, Petriello, hep-ph/0409088; hep-ph/0501130

LHC applications (cont.)

- $pp \to W + X \to \ell\nu + X$ with parton-level cuts

Important to constrain observed lepton rapidity (not W rapidity) and to impose realistic cuts on lepton p_T and missing E_T

Melnikov, Petriello, hep-ph/0603182

$p_\perp^e > p_\perp^{e,\min}, \quad |\eta^e| < 2.5, \quad E_\perp^{\rm miss} > 20$ GeV

$p_\perp^{e,\min}$	LO	NLO	NNLO
Inc	11.70,13.74,15.65	16.31,16.82,17.30	16.31, 16.40, 16.50
20	5.85,6.96,8.01	7.94,8.21,8.46	8.10,8.07,8.10
30	4.305, 5.12,5.89	6.18,6.36,6.54	6.18,6.17,6.22
40	0.628,0.746,0.859	2.07,2.10,2.11	2.62,2.54,2.50
50	0,0,0	0.509,0.497,0.480	0.697,0.651,0.639

TABLE I: The lepton invariant mass distribution $d\sigma/dM^2$, $M = m_W$, for on-shell W production in the reaction $pp \to W^-X \to e^-\bar{\nu}W$, in pb/GeV^2, for various choices of $p_\perp^{e,\min}$, GeV and $\mu = m_W/2, m_W, 2m_W$.

As in more inclusive computations, scale uncertainties highly reduced at NNLO

End of Lecture 3