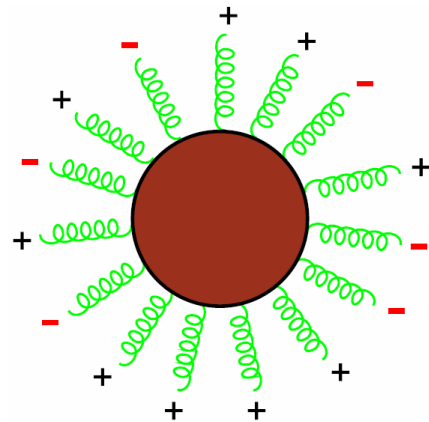


# QCD at Colliders

## Lecture 4



Lance Dixon, SLAC

Graduate College in Mass, Spectrum and Symmetry

Berlin 2 Oct. 2009

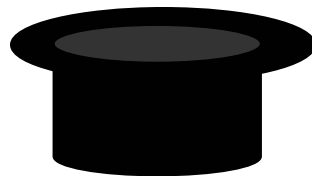
# Lecture 4 Outline

- NLO  $W + 3$ -jet production at hadron colliders
  - Lessons about scales
- QCD corrections and the Higgs [ $\rightarrow \gamma$ ] at the LHC

# $W + 3$ jets at NLO

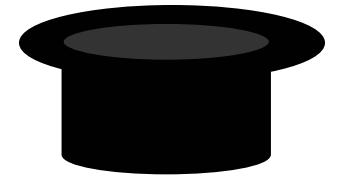
C.F.Berger, Z. Bern, LD, D. Forde, F. Febres Cordero, T. Gleisberg, H. Ita, D. Kosower, D. Maître, 0902.2760, 0907.1984

- Motivated as a background to SUSY searches in the “JETS + MET” channel, when the charged lepton in  $W \rightarrow l \nu_l$  is lost
- Also closely related to  $Z + 3$  jets (in progress), another SUSY background when  $Z \rightarrow \nu \nu$
- Many different kinematic configurations can appear in final state – have to be careful to choose scale  $\mu$  correctly to avoid pathologies!
- First phenomenologically interesting results from BlackHat [+Sherpa]

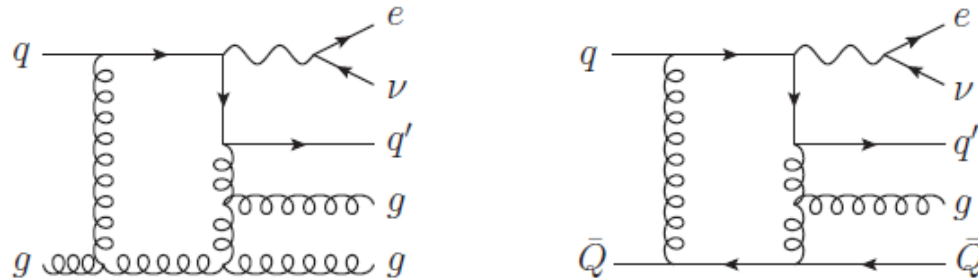


Related work by  
Ellis, Melnikov, Zanderighi  
0901.4101, 0906.1445

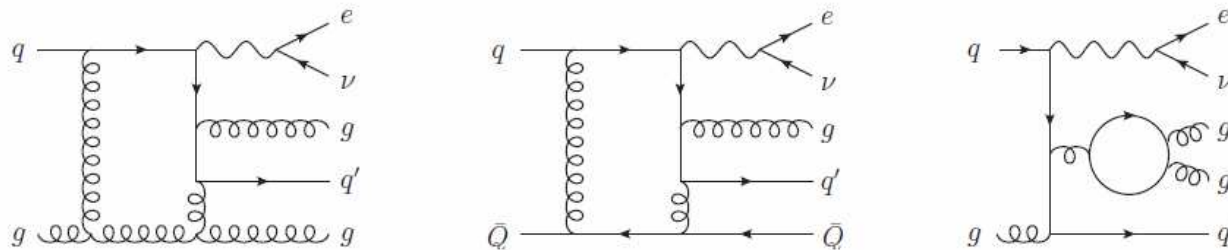
# Virtual Corrections



- Divide into leading-color terms, such as:



- and subleading-color terms, such as:



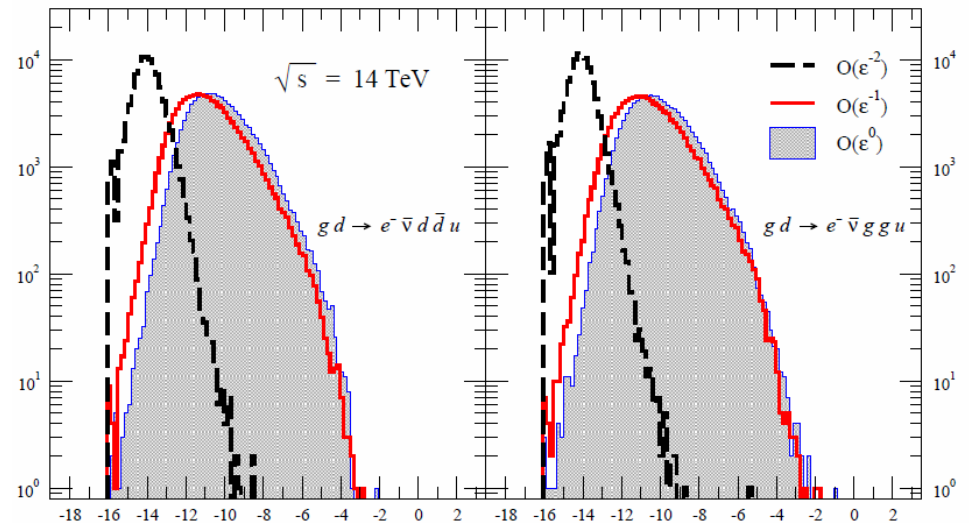
The latter include many more terms, and are much more time-consuming for computer to evaluate. But they are much smaller ( $\sim 1/30$  of total cross section) so evaluate them much less often.

# Numerical Stability of Virtual Terms

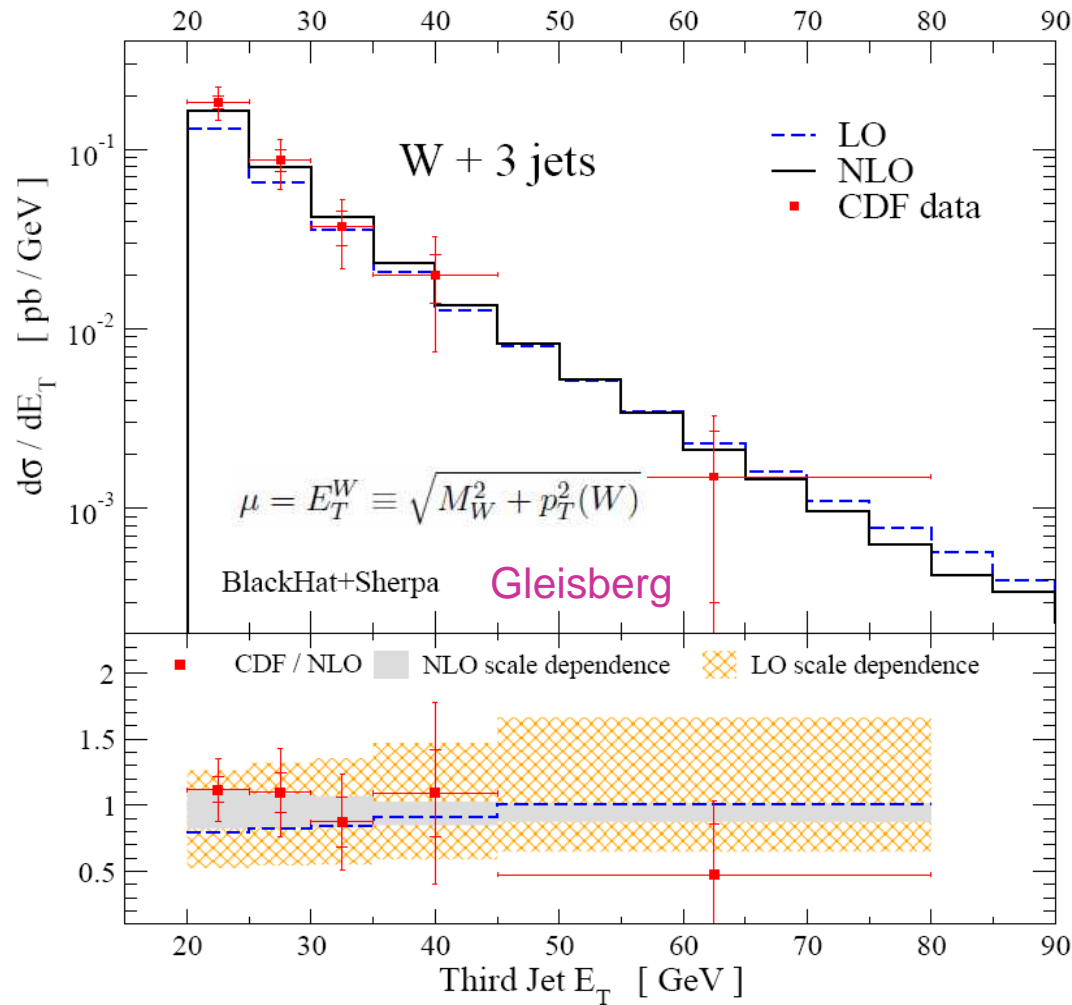
- Nontrivial because there are many kinematic regions where there are large cancellations between terms in this expansion, leading to roundoff error:

$$A^{1\text{-loop}} = \sum_i d_i \text{[box diagram]} + \sum_i c_i \text{[triangle diagram]} + \sum_i b_i \text{[bubble diagram]} + R + \mathcal{O}(\epsilon)$$

- BlackHat has a lot of tests for instability; if a piece of  $A^{1\text{-loop}}$  is unstable, it recomputes that piece with higher precision (~32 digits).
- Resulting distributions of  $\log(\text{relative error})$  →



# W + 3 jets at Tevatron at NLO



same cuts as CDF

$$E_T^{\text{jet}} > 20 \text{ GeV}, \quad |\eta^{\text{jet}}| < 2$$

$$E_T^e > 20 \text{ GeV}, \quad |\eta^e| < 1.1,$$

$$\cancel{E}_T > 30 \text{ GeV}, \quad M_T^W > 20 \text{ GeV}$$

$$M_T^W = \sqrt{2E_T^e E_T^\nu (1 - \cos(\Delta\phi_{e\nu}))}$$

Except: we use SIScone;  
 CDF used IR unsafe  
 JETCLU

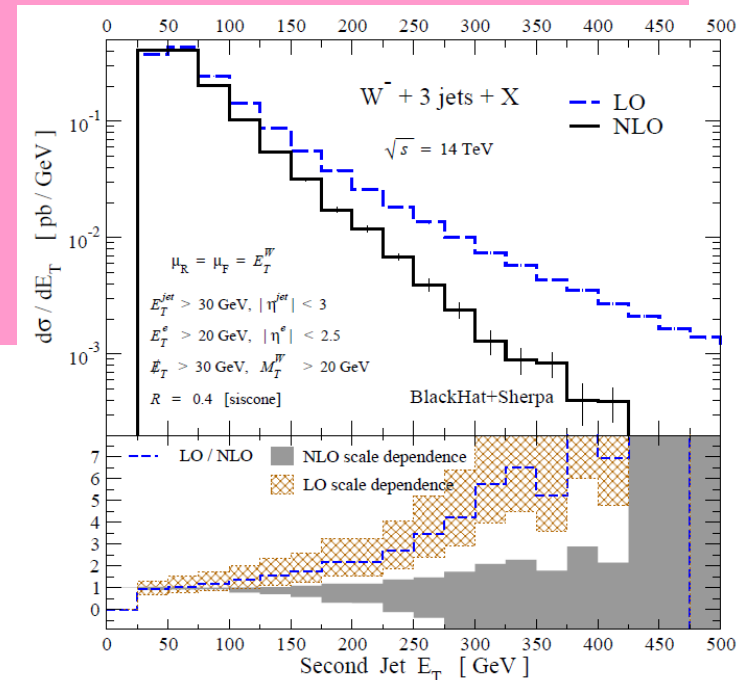
- Much smaller uncertainties than at LO.
- Agrees well with data; more data coming soon.

# W + 3 jets at LHC at NLO

- LHC has a much greater dynamic range; there will be events with jet  $E_T$ s much larger than  $M_W$ . Have to be more careful how we choose the renormalization + factorization scale.
- Indeed, the scale we used at the Tevatron,

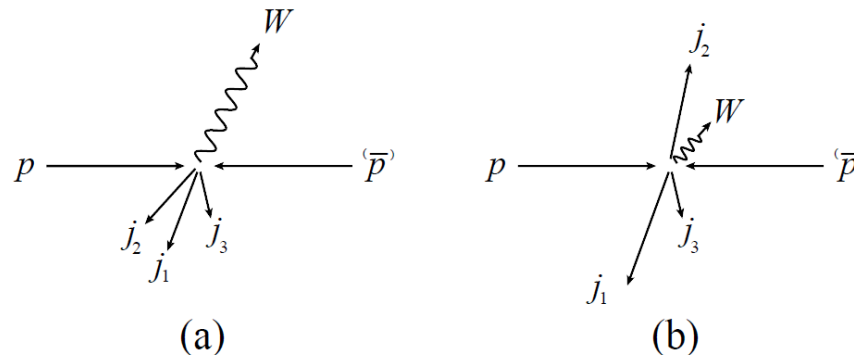
$$\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$$

which was also used in several other LO studies, turns out to be a really bad choice – NLO cross section can even dive negative! →



# Better Scale Choice(s)

- What's going on? Consider these 2 configurations:



- If (a) dominates, then  $\mu = E_T^W \equiv \sqrt{M_W^2 + p_T^2(W)}$  is OK

- But if (b) dominates, then the scale  $E_T^W$  is way too low.
- Looking at large  $E_T$  for the 2<sup>nd</sup> jet forces configuration (b).

- The total (partonic) transverse energy is a better variable; gets large properly for both (a) and (b)

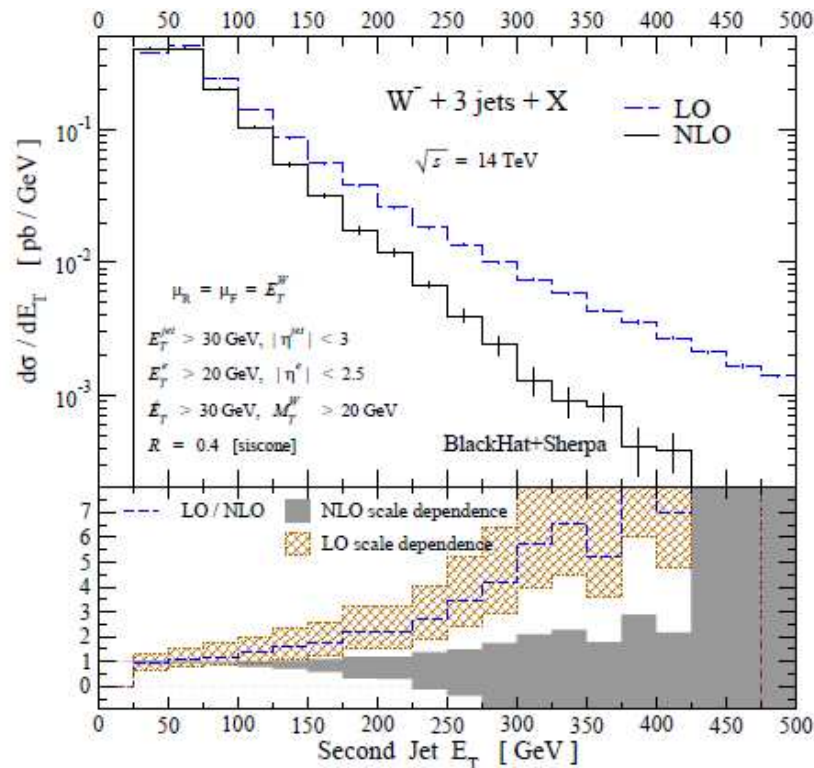
$$\hat{H}_T = \sum_p E_T^p + E_T^e + E_T^\nu$$

- Another reasonable scale is invariant mass of the n jets

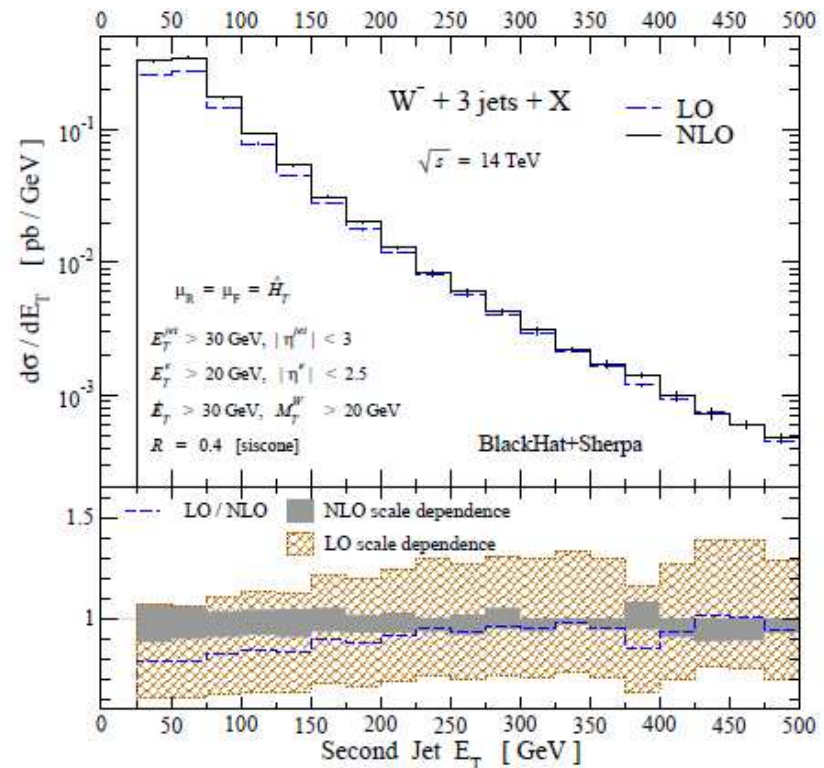
Bauer, Lange  
0905.4739



# Compare the Two Scale Choice(s)



$\mu = E_T^W$  very poor



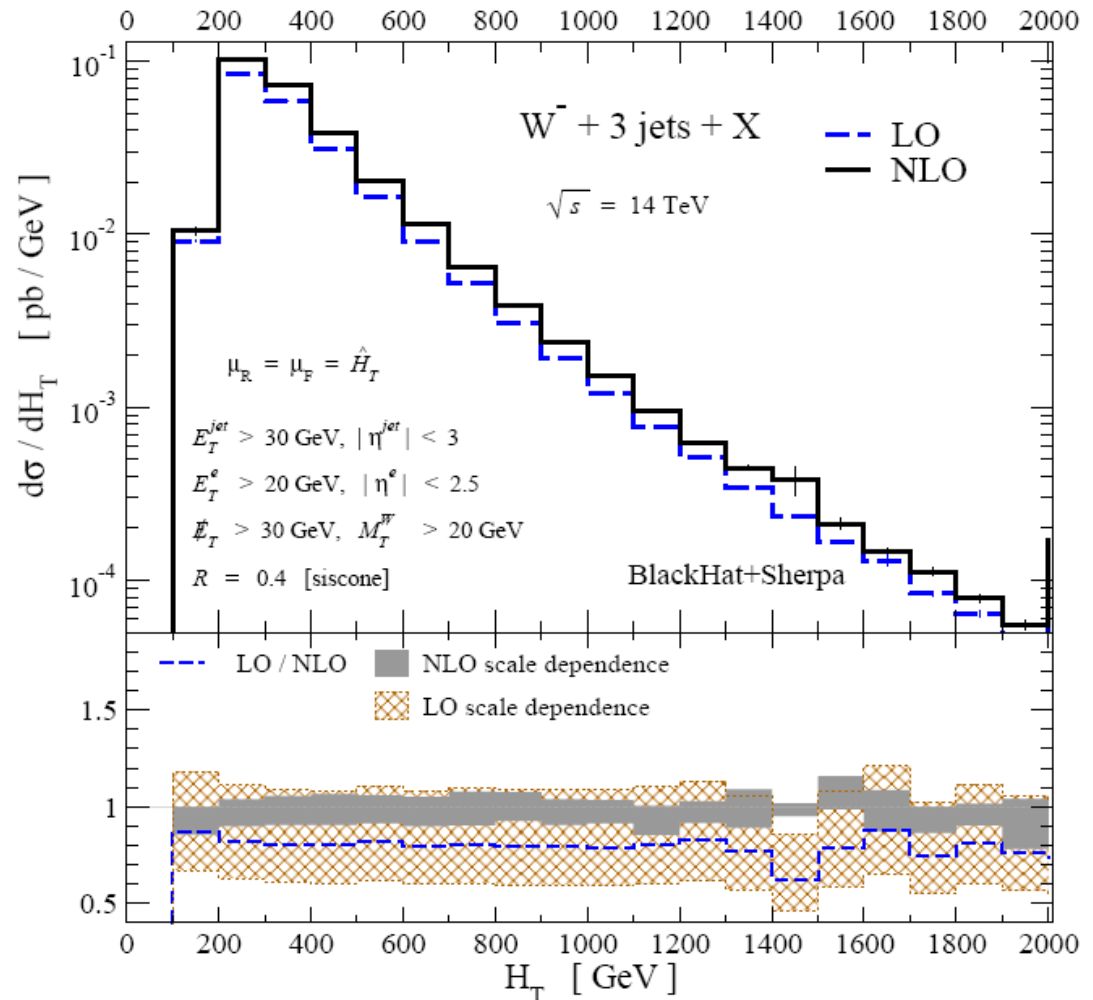
$\mu = \hat{H}_T$  excellent!

– LO/NLO quite flat, and also for many other observables

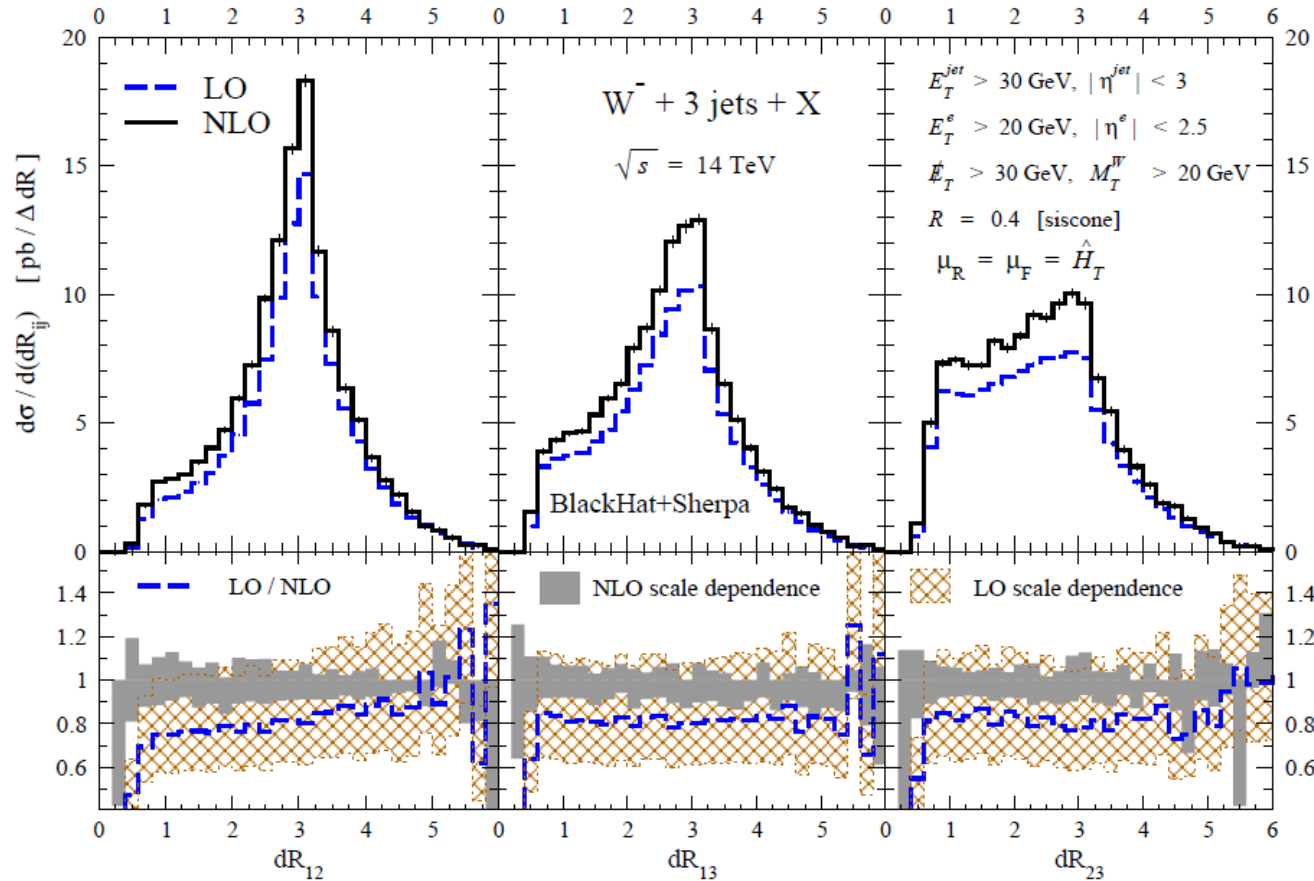
# Total Transverse Energy $H_T$ at LHC

$$H_T = \sum_j E_{T,j}^{\text{jet}} + E_T^e + E_T^\nu$$

often used in  
supersymmetry  
searches



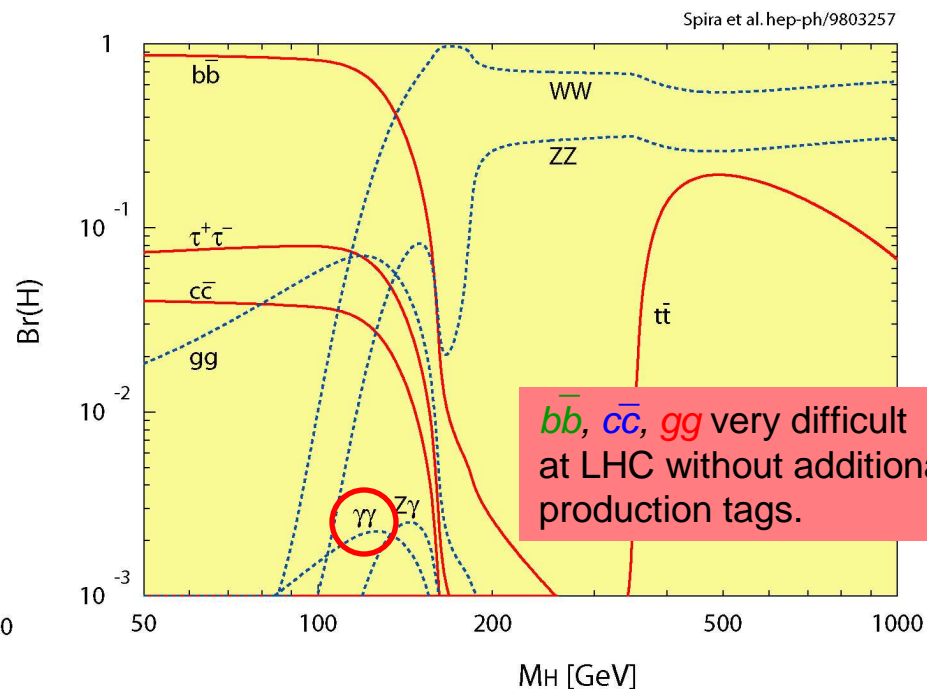
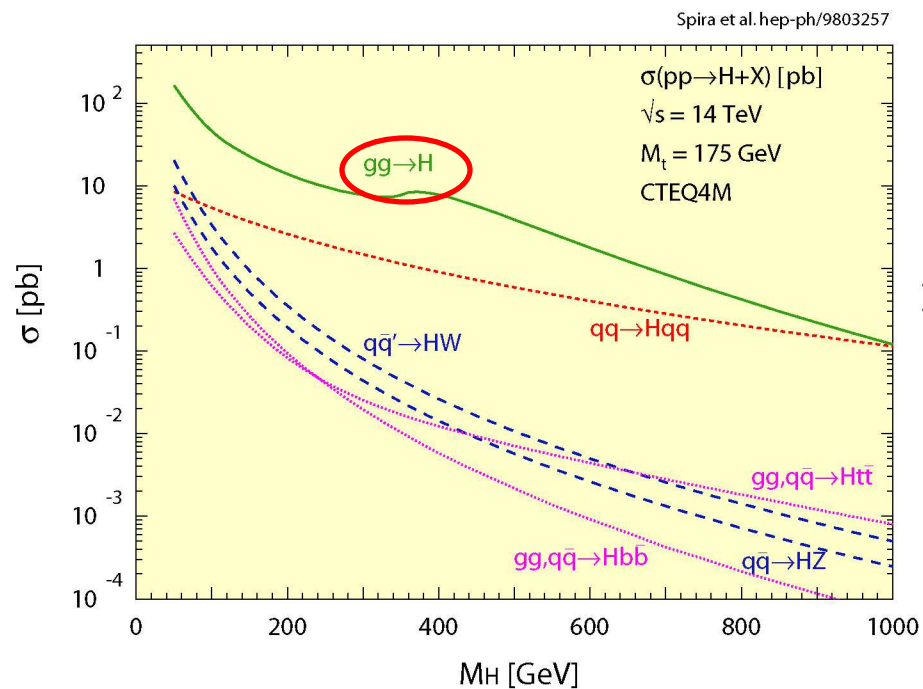
# Jet Separations $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$



Pretty well modeled by LO, except for first 2 jets

# QCD Corrections and the Higgs [ $\rightarrow \mu$ ] at the LHC

# Production & decay mechanisms



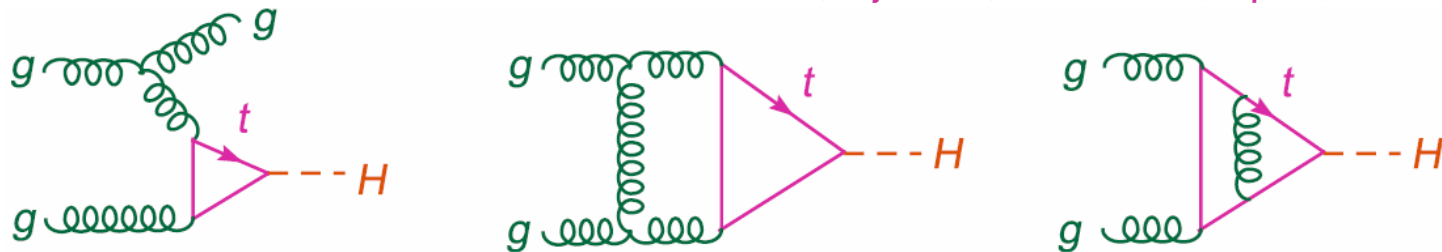
Here focus on:

$gg \rightarrow H$  largest, but largest QCD uncertainties

$H \rightarrow \gamma\gamma$  very small, but also clean bump

# $gg \rightarrow H \rightarrow \gamma\gamma$ Signal

- Height proportional to  $\sigma(pp \rightarrow H + X) \times \text{Br}(H \rightarrow \gamma\gamma)$
- Compute  $\sigma$  and  $\text{Br}$  as expansion in  $\alpha_s$
- Series for  $\text{Br}$  is quite **convergent**,  $\sim 6\%$  uncertainty from  $m_b$  in  $\Gamma_H$
- Series for  $\sigma$  is **poorly** behaved: first correction (NLO) is 80% !  
 Dawson; Djouadi, Graudenz, Spira, Zerwas (1991)

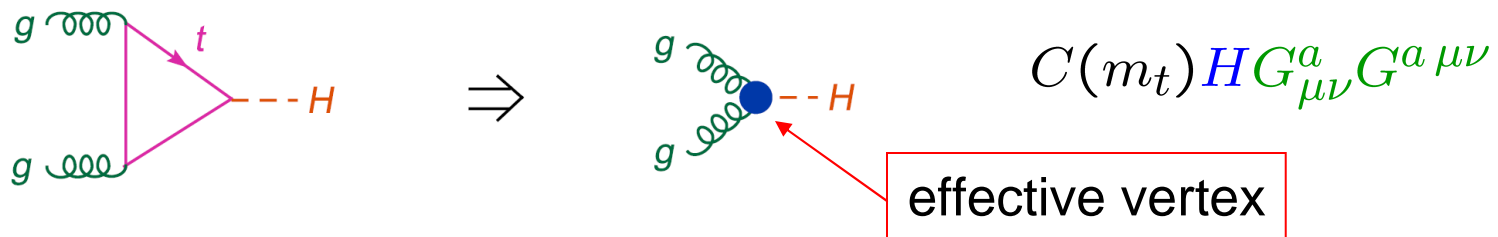


- Drove big theoretical effort to compute NNLO term

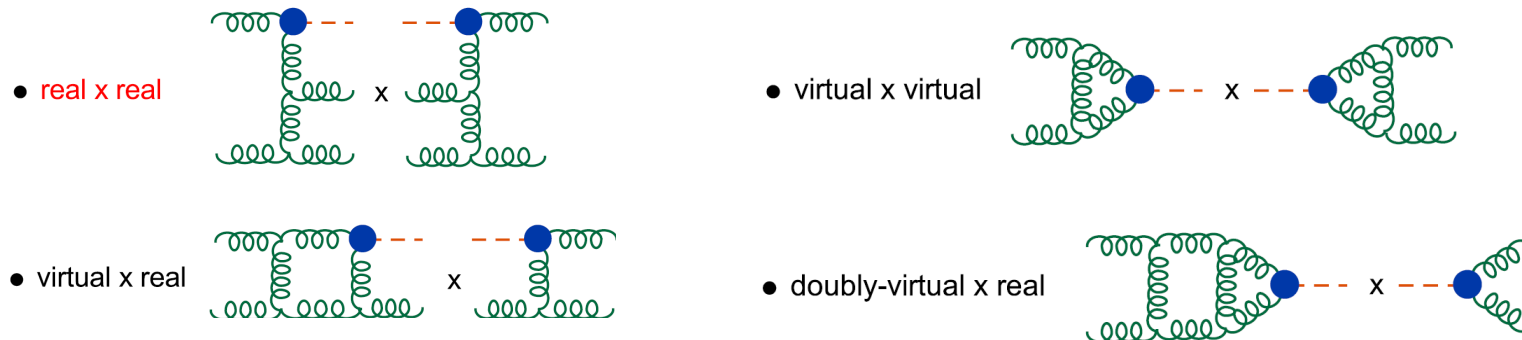
Catani, DeFlorian, Grazzini; Harlander, Kilgore; Anastasiou, Melnikov;  
 Ravindran, Smith, van Neerven (2001–03)

# $\sigma(pp \rightarrow H + X)$ at NNLO

To make tractable, use large  $m_t$  approximation:  
reduces number of loops by 1

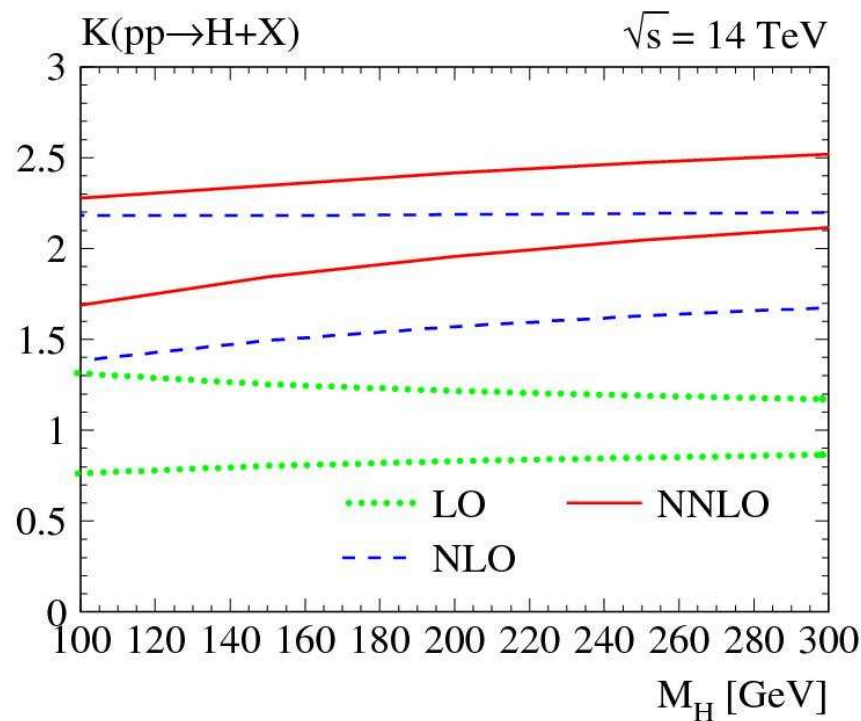


Still many amplitude interferences, with different numbers of final state gluons (or quarks). Each diverges; only sum is finite.



# $\sigma(pp \rightarrow H + X)$ at NNLO

Results expressed as *K factor*:



$$K_{\text{NNLO}} = \frac{\sigma_{\text{NNLO}}}{\sigma_{\text{LO}}}$$

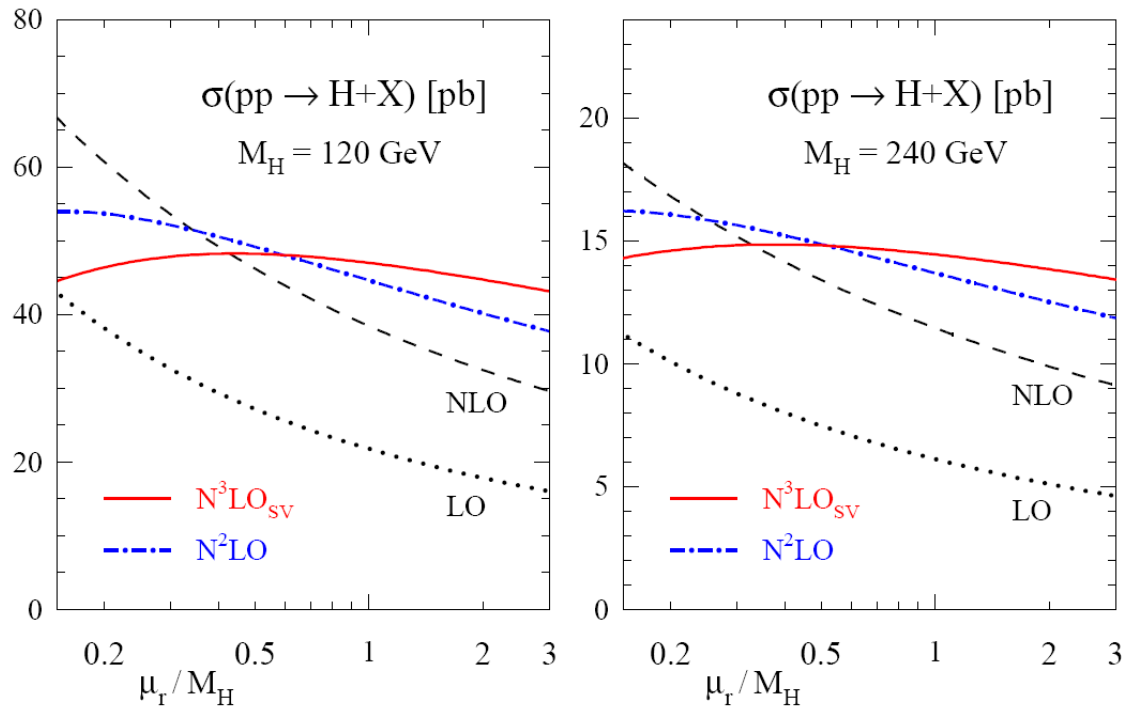
$$K_{\text{NLO}} = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$$

Series stabilized;  
residual uncertainty  
 $\sim 10\text{-}20\%$



# $\sigma(pp \rightarrow H + X)$ at $N^3LO_{SV}$

Using partial (logarithmically enhanced) results at  $N^3LO$ , even more stable results are obtained :



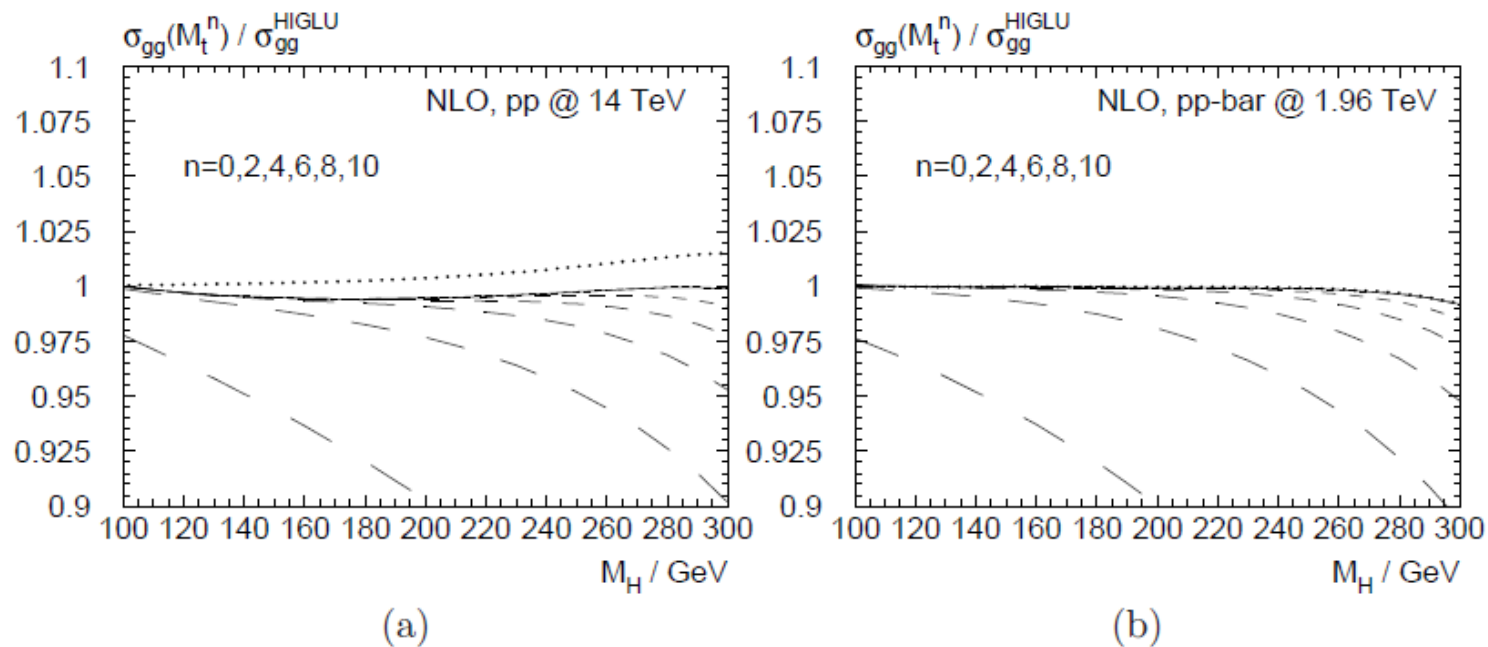
Moch, Vogt,  
 hep-ph/0508265  
 (also Ravindran,  
 hep-ph/0603041)

- Now residual uncertainty on  $\sigma^{\text{tot}} \sim \pm 5\%$
- Plus gluon pdf uncertainty (next slide)

# Corrections to large $m_t$ approximation

Harlander, Ozeren, 0909.3420

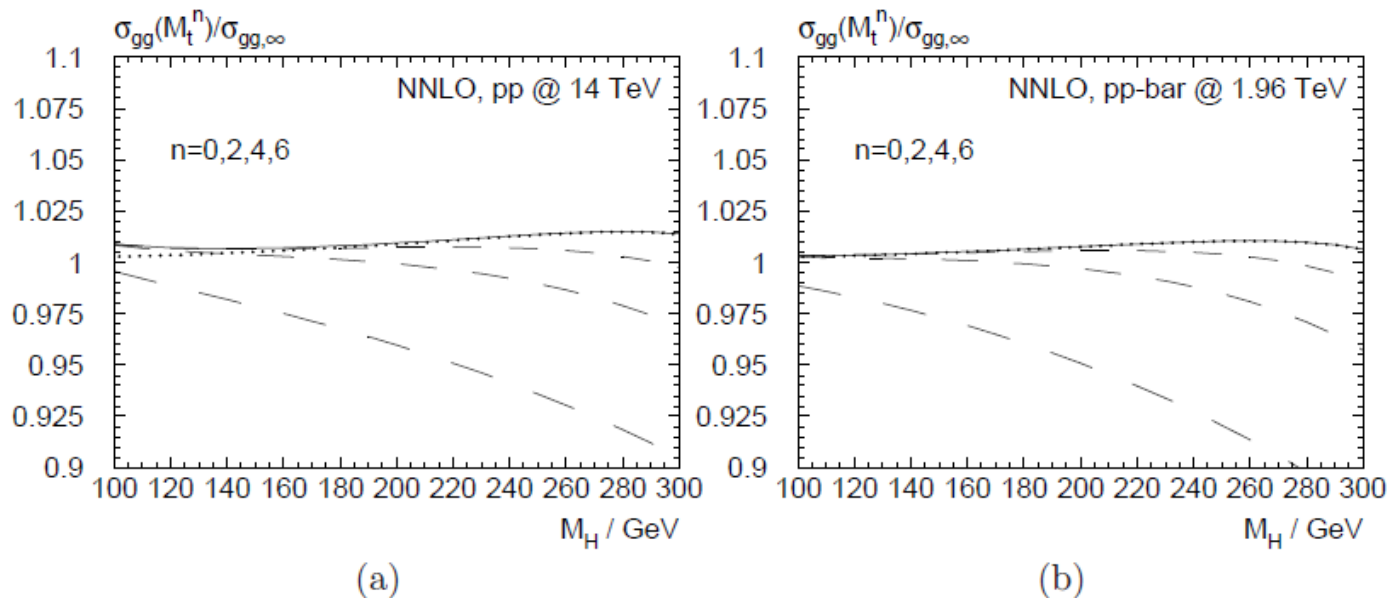
Recent evaluation of these corrections at NLO (to check convergence against the known exact result):



# Corrections to large $m_t$ approx. (cont.)

Harlander, Ozeren, 0909.3420

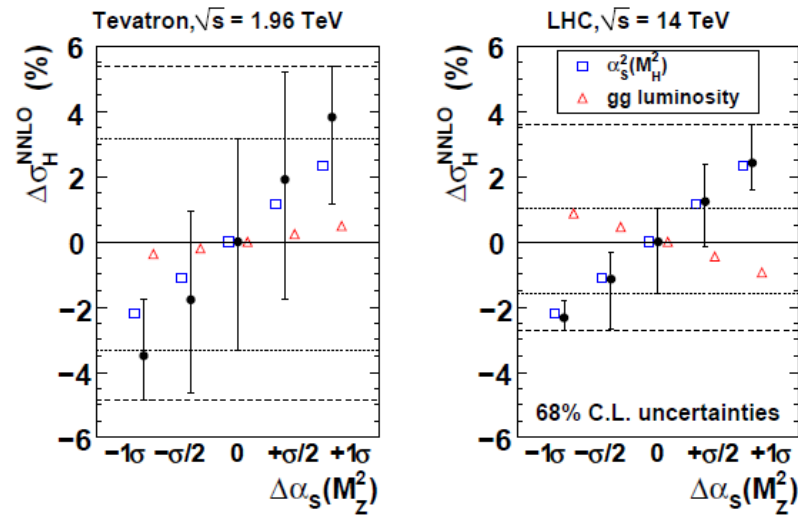
And then at NNLO:



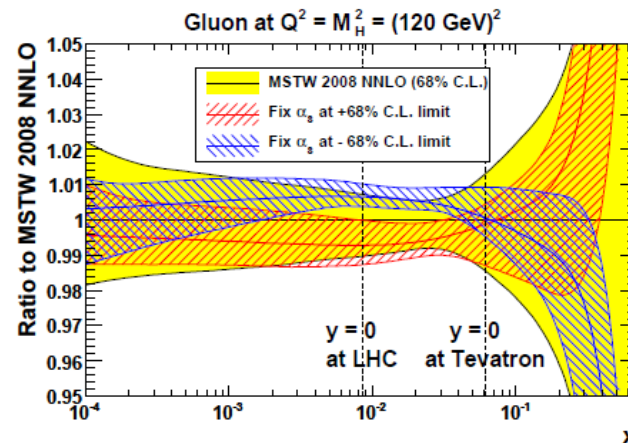
Showing that these corrections are quite small,  $\sim 1-2\%$ .

# $\sigma(pp \rightarrow H + X)$ pdf uncertainty

Higgs ( $M_H = 120$  GeV) with MSTW 2008 NNLO PDFs



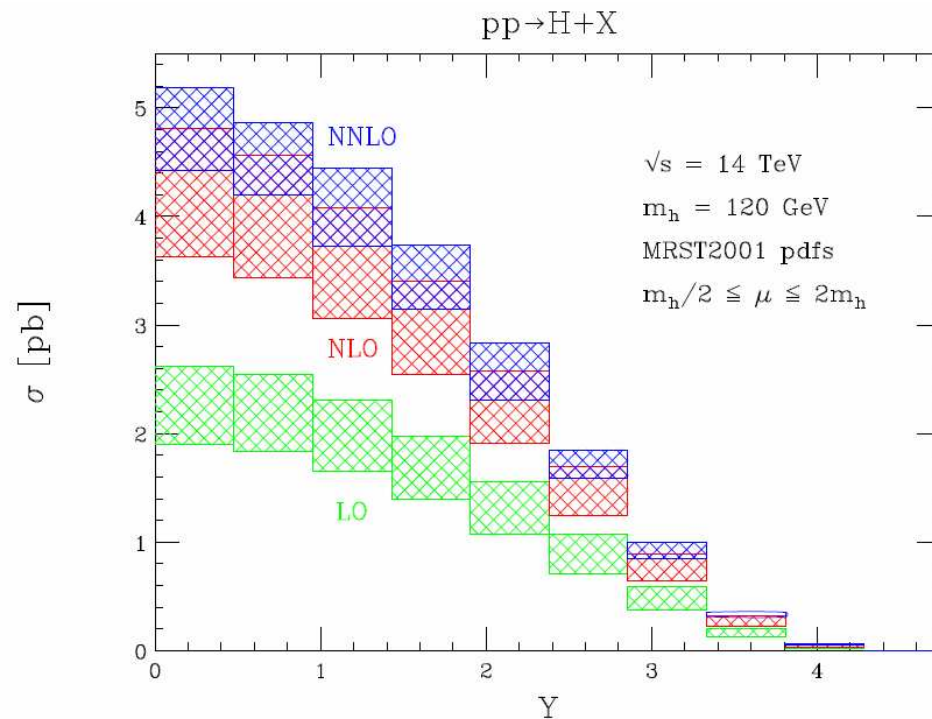
Martin, Stirling,  
Thorne, Watt,  
0905.3531



Uncertainty  
< 4% at LHC  
Highly correlated  
with value of  $\alpha_s$

# $\frac{d\sigma}{dY}(pp \rightarrow H + X)$ at NNLO

Anastasiou, Melnikov, Petriello, hep-ph/0501130



$Y$  = rapidity  
of Higgs boson

- Can include parton-level event cuts (photon kinematics, isolation, jet or jet veto, ...)
- Can use NNLO information to reweight event generators (PYTHIA, MC@NLO) in bins of  $p_T, Y$

Davatz, et al., hep-ph/0604077

# $gg \rightarrow H \rightarrow \gamma\gamma$ interference?

LD, M.S. Siu, hep-ph/0302233

- In principle as important as signal, since it **contaminates peak**

$$\text{Is } \text{Signal} = \sigma_{ii \rightarrow H} \times \text{Br}(H \rightarrow ff) = \frac{\Gamma_i \Gamma_f}{\Gamma} \quad ?$$

- Resonance-continuum interference would **negate** this:

$$\mathcal{A}(gg \rightarrow \gamma\gamma) = \frac{-\mathcal{A}(gg \rightarrow H) \mathcal{A}(H \rightarrow \gamma\gamma)}{\hat{s} - m_H^2 + im_H \Gamma_H} + \mathcal{A}_{\text{cont}}(gg \rightarrow \gamma\gamma)$$

- Normally interference effects are **small** for a **narrow** resonance, and  $\Gamma_H \sim \text{few MeV}$  for Higgs.
- However, this resonance is pretty **wimpy**,  $\sim (\text{one-loop})^2$

# $gg \rightarrow H \rightarrow \gamma\gamma$ interference

- Total  $gg \rightarrow \gamma\gamma$  amplitude

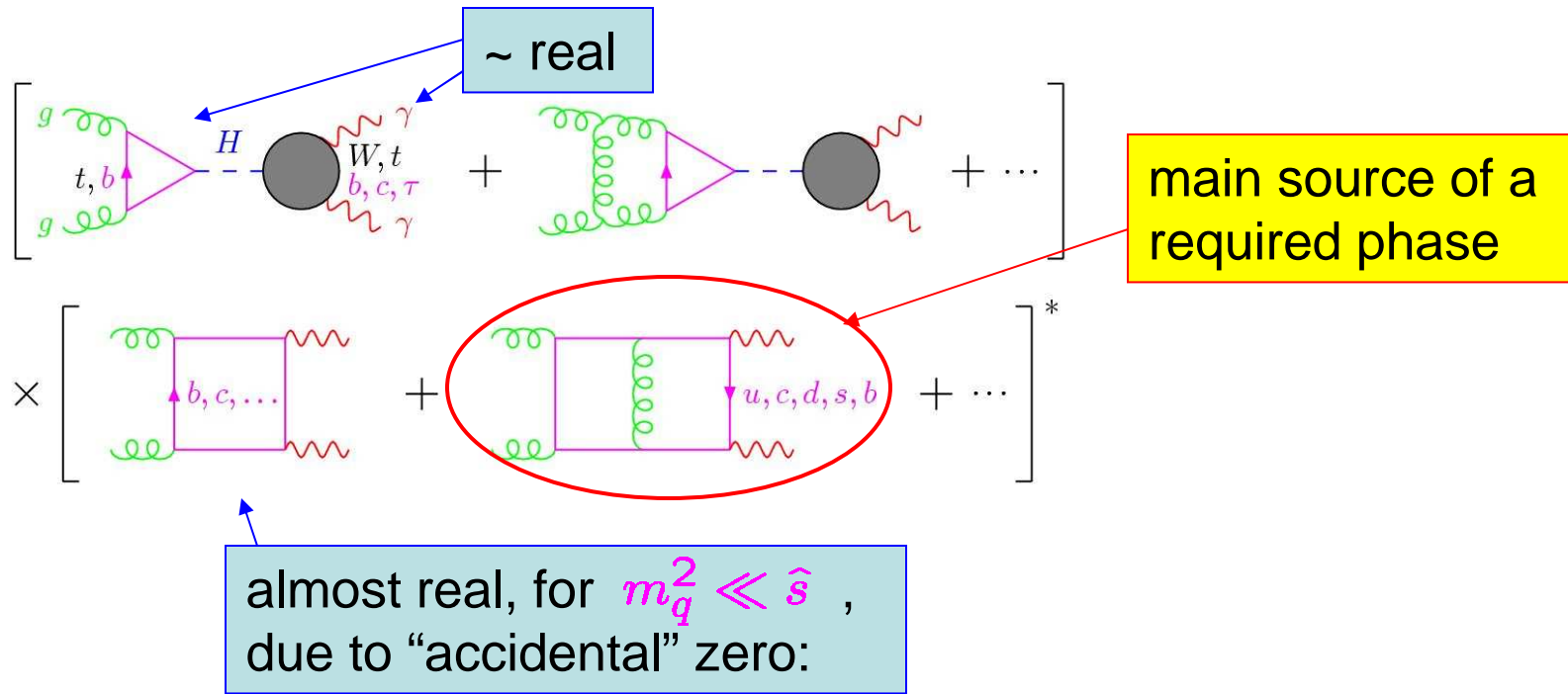
$$\mathcal{A}_{gg \rightarrow \gamma\gamma} = \frac{-\mathcal{A}_{gg \rightarrow H} \mathcal{A}_{H \rightarrow \gamma\gamma}}{\hat{s} - m_H^2 + im_H \Gamma_H} + \mathcal{A}_{\text{cont}}$$

- Interference term has 2 pieces

$$\begin{aligned} \delta\hat{\sigma}_{gg \rightarrow H \rightarrow \gamma\gamma} = & -2(\hat{s} - m_H^2) \frac{\text{Re}(\mathcal{A}_{gg \rightarrow H} \mathcal{A}_{H \rightarrow \gamma\gamma} \mathcal{A}_{\text{cont}}^*)}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} \\ & - 2m_H \Gamma_H \frac{\text{Im}(\mathcal{A}_{gg \rightarrow H} \mathcal{A}_{H \rightarrow \gamma\gamma} \mathcal{A}_{\text{cont}}^*)}{(\hat{s} - m_H^2)^2 + m_H^2 \Gamma_H^2} \end{aligned}$$

- “Re” term vanishes upon integration over  $\hat{s}$ , provided that  $\mathcal{A}_{gg \rightarrow H}$ ,  $\mathcal{A}_{H \rightarrow \gamma\gamma}$ ,  $\mathcal{A}_{\text{cont}}$  do not vary too quickly. Dicus, Willenbrock
- “Im” term needs **relative phase**, resonance vs. continuum.

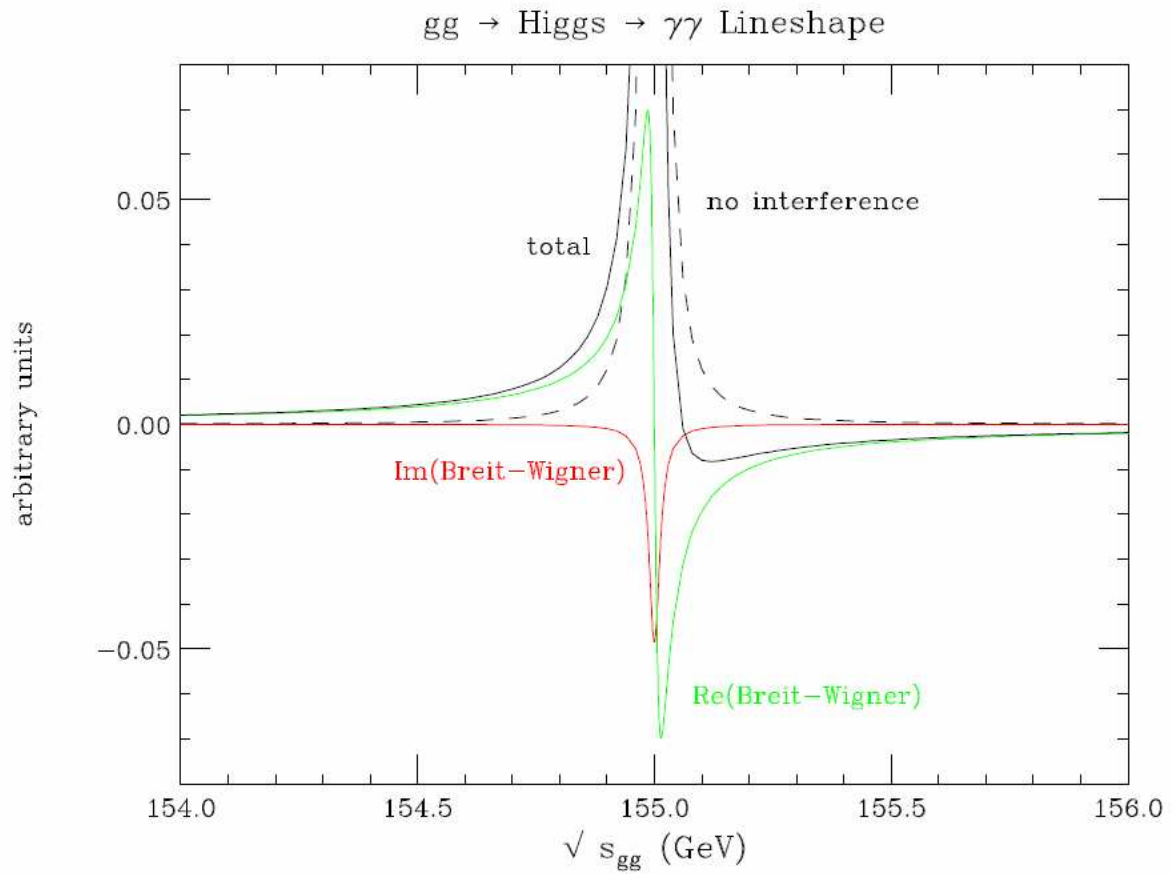
# $gg \rightarrow H \rightarrow \gamma\gamma$ interference

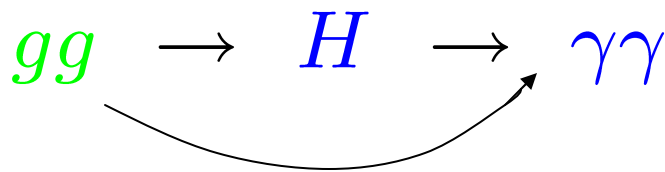


$$\mathcal{A}^{\text{tree}}(g^\pm g^\pm \rightarrow q\bar{q}) = \mathcal{A}^{\text{tree}}(q\bar{q} \rightarrow \gamma^\pm \gamma^\pm) = 0 \text{ for } m_q = 0$$

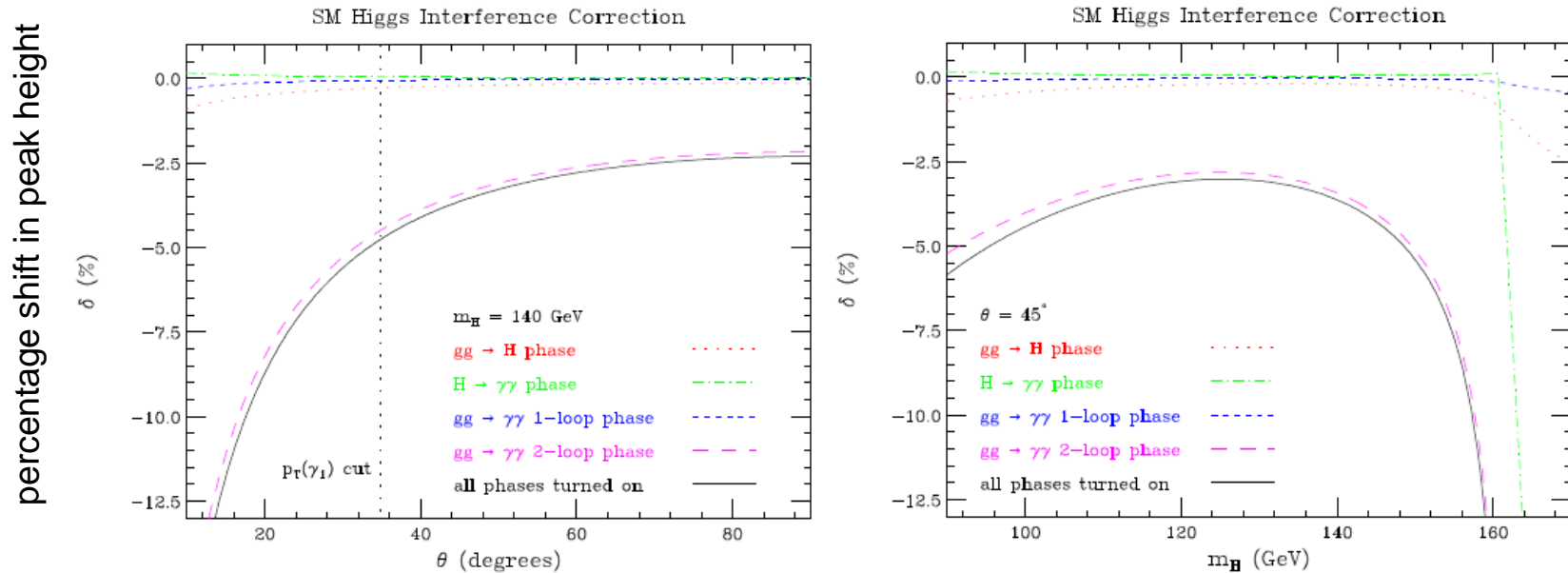


$gg \rightarrow H \rightarrow \gamma\gamma$  interference





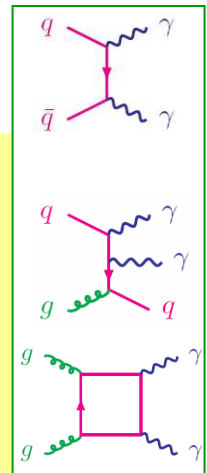
# interference



- Effect is  $-(2 - 6)\%$  over region where  $\gamma\gamma$  is visible.
- Gets very large near  $WW$  threshold.  
But visibility of  $\gamma\gamma$  is very poor there

# $\gamma\gamma$ backgrounds

- Multiple components, working from the “outside” in:
- Non-photons faking photons:  $e^\pm$  , hadrons
- Real photons from hadronic decay, principally  $\pi^0 \rightarrow \gamma\gamma$
- Photons from QED bremsstrahlung from  $e^\pm$
- Photons from **fragmentation** – “long” distance radiation from within a hadronic jet,  $p_T^{\% \text{ jet}} < 1 \text{ GeV}$
- Photons from QCD **hard process** – “short” distance radiation off quark lines,  $p_T^{\% q} > 1 \text{ GeV}$ . **Hard processes** include:
  - $q\bar{q} \rightarrow \gamma\gamma$  LO
  - $qg \rightarrow \gamma\gamma q$  NLO, enhanced by  $g(x)$ , FS singularity
  - $gg \rightarrow \gamma\gamma$  NNLO, enhanced by  $g(x)$



**Photon isolation** a key tool for suppressing many of the components

# Photon isolation criteria

Cut on nearby transverse hadronic energy

- Required to reject **jets** or  $\pi^0$ s faking  $\gamma$ s, reduce **fragmentation** terms

- Either

- **standard cone**: Circle, radius  $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$  in  $(\eta, \phi)$  plane. Require  $E_T < E_{T \max}$  inside circle.
- **smooth (Frixione) cone**: Require  $E_T(r)$  less than

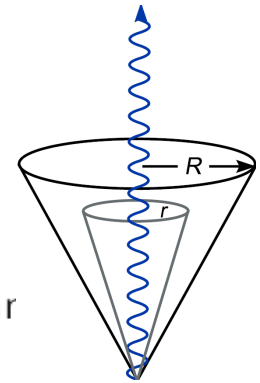
$$E_{T \max}(r) \equiv p_T(\gamma) \epsilon \frac{1 - \cos r}{1 - \cos R}$$

for all circles with  $r < R$ .

Frixione, hep-ph/9801442

**Smooth** cone has **no fragmentation** contribution, but may be difficult to exploit experimentally (transverse shower extent, finite detector granularity).

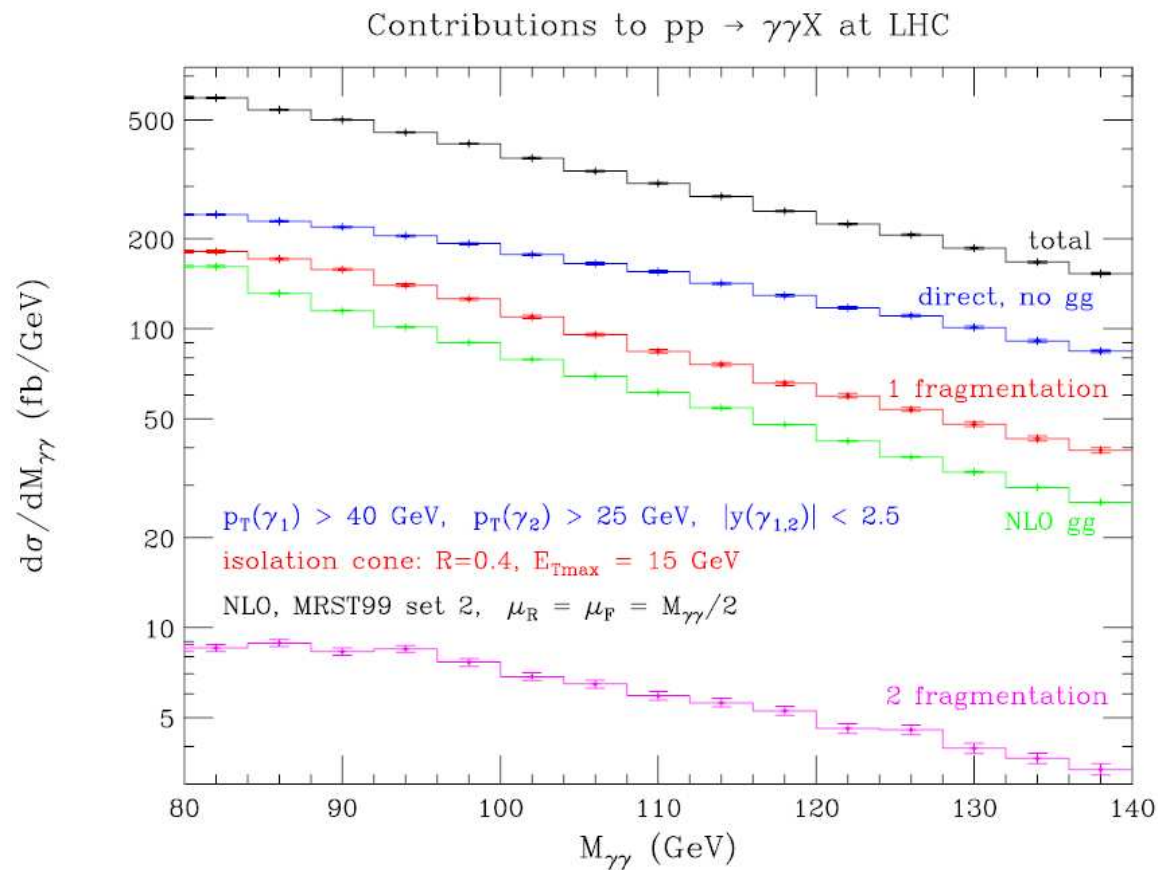
- **Too strict isolation**: a) is not infrared safe, and b) lowers efficiency at high  $L$



# Irreducible $\gamma\gamma$ background

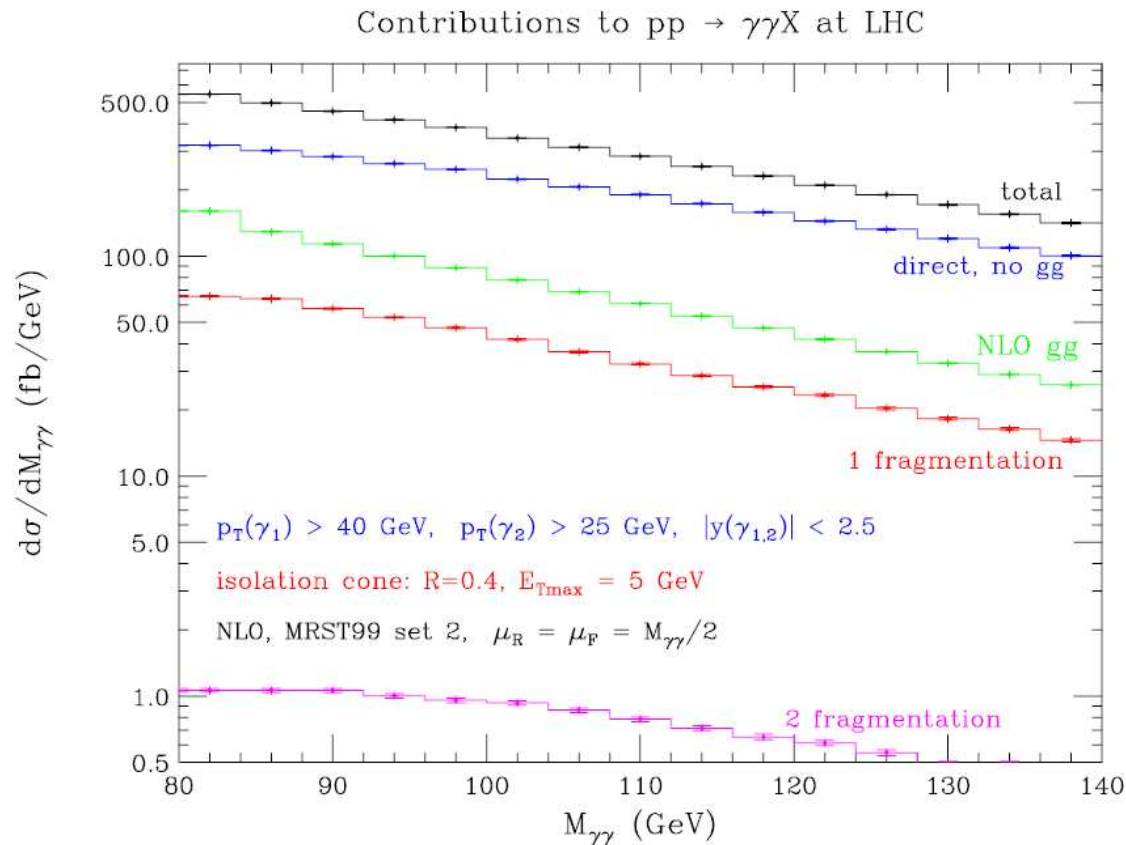
Bern, LD, Schmidt, hep-ph/0206194;

DIPHOX: Binnoth, Guillet, Pilon, Werlen, hep-ph/9911340



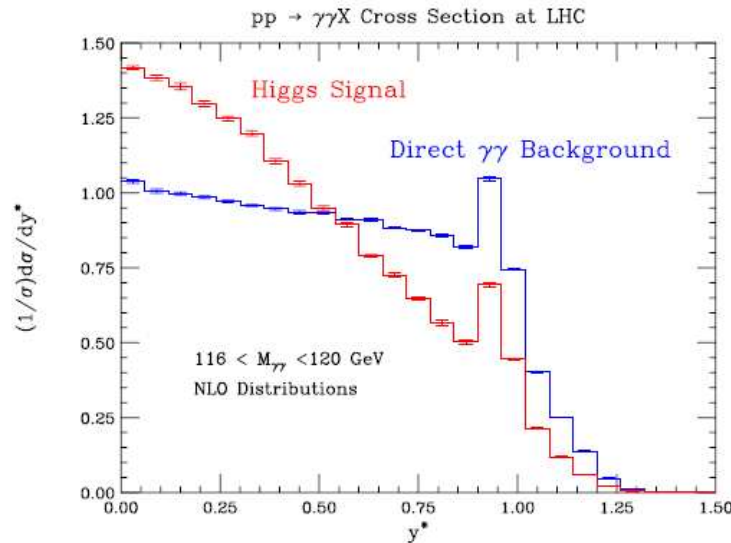
# Irreducible $\gamma\gamma$ background

With a tighter  $E_T$  cut:



# Angular distribution of $\gamma$ 's

- Scalar Higgs decay is flat in  $\cos \theta^*$ .  
At LO,  $\cos \theta^* = \tanh y^*$ , where  $y^* \equiv \frac{1}{2}(y(\gamma_1) - y(\gamma_2))$ .
- QCD background from  $q\bar{q}$ ,  $qg$ , and  $gg$  enhanced at small  $\theta^*$ , large  $y^*$ , due to  $t$ -channel fermion exchange.  
Information can be used to improve  $S/\sqrt{B}$ , but only marginally.  
better to use in likelihood function than as hard cut



# Require additional jet activity?

Abdullin et al., hep-ph/9805341

## Motivation:

- Signal dominantly  $gg \rightarrow H$ , really likes to radiate ( $8 + 8 \rightarrow 1$ ).
- Background has  $q\bar{q} \rightarrow \gamma\gamma$ , smaller color charge, less radiation

$M_H = 120$ GeV	no $\sqrt{\hat{s}}$ cut	$\sqrt{\hat{s}} > 210$ GeV	$\sqrt{\hat{s}} > 300$ GeV
$\sigma_S^{QCD}$ fb	10.6	7.0	4.0
$\sigma_S^{EW}$ fb	1.1	0.81	0.58
$\sigma_B$ fb/GeV	25.3	9.1	2.9

de Florian, Kunszt, hep-ph/9905283;

de Florian, Kulesza Vogelsang, hep-ph/0511205

- Can play a similar game using transverse momentum of photon pair,  $q_T(\gamma\gamma)$ .
- Maybe best done via likelihoods as well?



# Conclusions

- Higher order **QCD** for the LHC is an extremely rich field – we only scratched the surface of it here
- At **hadron colliders**, the physics **is QCD**  
– up to **small, electroweak** corrections!
- To uncover new physics of **electroweak** strength, we will need to understand **QCD** at colliders quite well
- Fortunately, there are lots of energetic young researchers
- But still, lots of work to be done, even as the data begin to roll in from the energy frontier!