Gravitational Renormalization in Large Extra Dimensions

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D. Ebert, J. Plefka, and AR *J. High Energy Phys.* **0902** (2009) 028.
T. Schuster and AR *Phys. Rev. D* **79** (2009) 125017.
arXiv:0908.2422 [hep-th].

Outline

Contents

- loop diagrams  are in general UV divergent
- Regularization by momentum cut-off Λ or dimensional regularization $d = 4 - \epsilon$
- Renormalization by adding counterterms introduces renormalization scale μ
- comparison with bare theory yield running of couplings $g(\mu)$ and other parameters, e. g. masses, which is encoded in the β function

$$\beta_g \equiv \mu \frac{\mathrm{d}g}{\mathrm{d}\mu}$$

Gravity as an effective field theory

- Perturbatively quantized Einstein gravity is famously nonrenormalizable
- Nevertheless it can be treated consistently as effective field theory [Donoghue'95]

$$S_{grav} = \int \mathrm{d}^d x \sqrt{-g} \left\{ \Lambda + \frac{1}{16\pi G_N} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right\}$$

- The higher derivative terms arise in loop expansions, the values of their couplings are largely undetermined, e.g. $c_1, c_2 \leq 10^{74}$ [Stelle'78]
- Question in this framework:
  Does inclusion of gravity alter the running of the Standard Model couplings

$$g_{\text{Maxwell}}(\mu),\, g_{\text{YM}}(\mu),\, \mathcal{Y}_{\text{Yukawa}}(\mu),\, \lambda_{\varphi^4}(\mu),\ ?$$

[Robinson,Wilczek'06]

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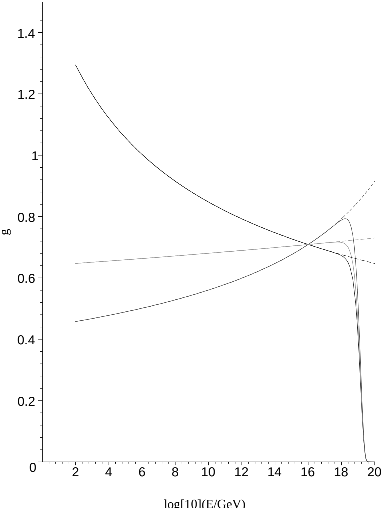
In four dimensions



[Robinson,Wilczek'06] in background field method and cut-off regularization:

$$\beta_g\Big|_{\kappa^2} = -\frac{1}{16\pi^2}\frac{3}{2}E^2\,\kappa^2\,g$$

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$$\beta_g\Big|_{\kappa^2} = 0$$

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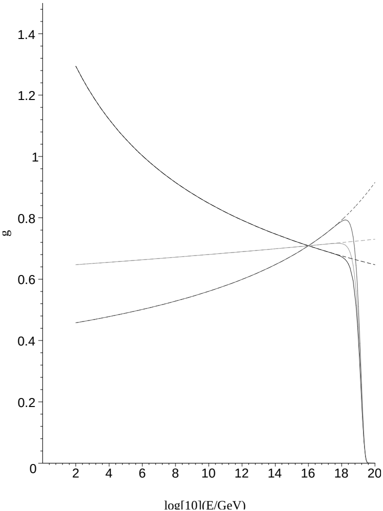

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The Einstein-Hilbert Action

$$S = \int \mathrm{d}^d x \sqrt{-\mathbf{g}} \frac{2}{\kappa^2} \mathbf{R} + \mathcal{L}_{\text{matter}}(g_{\mu\nu}, \Phi) + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}}$$

dimensionful coupling $\kappa = \sqrt{32\pi G_N}$, $[\kappa] = 1 - \frac{d}{2}$
Splitting the metric in flat background and graviton field:

$$\mathbf{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\sqrt{-\mathbf{g}} = 1 + \frac{\kappa}{2} h + \frac{\kappa^2}{8} \left(h^2 - 2h^{\alpha\beta} h_{\alpha\beta} \right) + \mathcal{O}(\kappa^3)$$
$$\mathbf{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\alpha} h_\alpha{}^\nu + \mathcal{O}(\kappa^3)$$
$$\mathbf{R} = \kappa \left(\Box h - \partial_\mu \partial_\nu h^{\mu\nu} \right) + \mathcal{O}(\kappa^2)$$

with $h \equiv h^\alpha_\alpha$.

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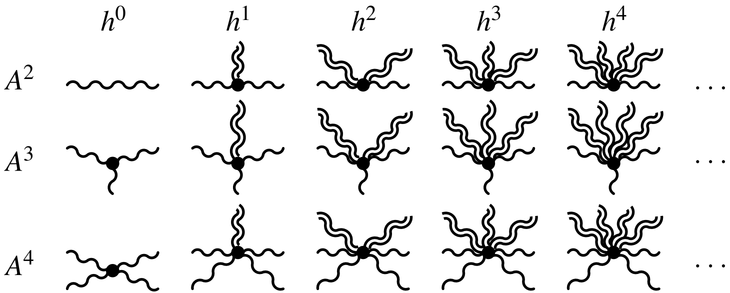
We use de Donder gauge: $\partial^\nu h_{\mu\nu} - \frac{1}{2} \partial_\mu h = 0$

Expansion in the field variables yields Feynman rules for vertices of arbitrary order in h:



z. B. $\mathcal{L}\big|_{\mathcal{O}(h,A^2)} = \kappa\partial_\mu A^a_\nu \partial_{[\rho} A^a_{\sigma]} \left(h^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\rho}h^{\nu\sigma} - \frac{1}{2}h\,\eta^{\mu\rho}\eta^{\nu\sigma}\right)$

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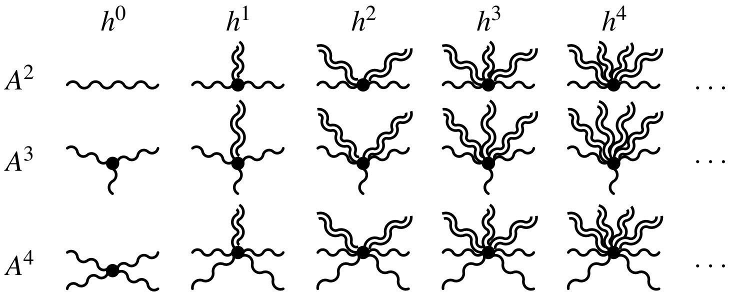

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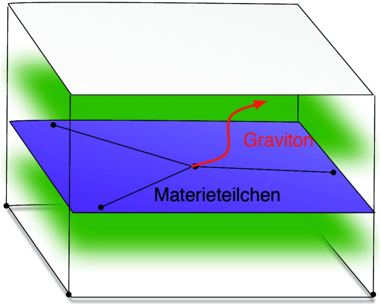
Large Extra Dimensions



- Space Time topology

$$\mathcal{M} = \mathbb{R}^{1,3} \times T^{\delta}$$

- The coordinates are denoted by $X^M = (x^\mu, y^i)$
- In large extra dimension senarios the actual gravity scale can be much lower as the experienced Planck scale on the brane [ADD'98]:

$$M^2_{(d=4)} = \underbrace{(2\pi R)^\delta}_{V_\delta} M^{D-2}_{(D)}$$

$$S = \int \mathrm{d}^D x \mathcal{L}_{\text{bulk}} + \int \mathrm{d}^d x \mathcal{L}_{\text{brane}}$$

- For a generic Bulk field Φ:

$$\int \mathrm{d}^D x \tfrac{1}{2}\partial_M \Phi \partial^M \Phi \Longrightarrow \int \mathrm{d}^d x \sum_{\vec{n}\in\mathbb{Z}^\delta} \tfrac{1}{2}\partial_\mu \Phi_{(\vec{n})}\partial^\mu \Phi_{(\vec{n})} + \tfrac{1}{2} m^2_{(\vec{n})}\Phi^2_{(\vec{n})}$$

with $\Phi(X) = V_\delta^{-1/2}\sum_{\vec{n}\in\mathbb{Z}^\delta}\Phi^{(\vec{n})}(x)e^{i\frac{\vec{n}\cdot\vec{y}}{R}}$ and $m^2_{(\vec{n})} = \frac{\vec{n}\cdot\vec{n}}{R^2}$

- The induced brane metric

$$G_{MN}(X) = \eta_{MN} + \kappa_{(D)} h_{MN}(X) \Longrightarrow g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa_{(d)} \sum_{\vec{n}\in\mathbb{Z}^\delta} h^{(\vec{n})}_{\mu\nu}(x)$$

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Contents

The Yang-Mills Lagrangian

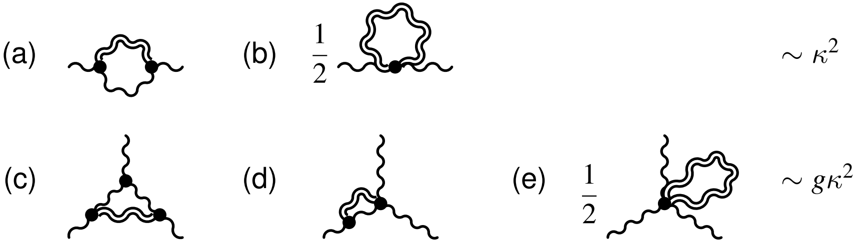
$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}\sqrt{-\mathbf{g}}g^{\mu\nu}g^{\rho\sigma}\,\mathrm{tr}[F_{\mu\rho}F_{\nu\sigma}]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

- Due to Slavonov-Taylor indentities only the renormalization of the two-point function is needed to calculate the gravity contribution to β_g
- The higher derivative (HD) terms expected to appear at one-loop are

$$\mathrm{tr}\left\{D^\mu F_{\mu\rho}D_\nu F^{\nu\rho}\right\} \quad \text{and} \quad \mathrm{tr}\left\{F_\alpha{}^\beta F_\beta{}^\gamma F_\gamma{}^\alpha\right\}.$$

Graviton Loop Corrections to the Gluon Two and Three Point Function



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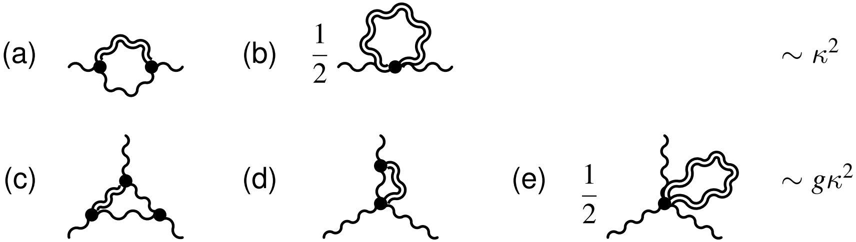

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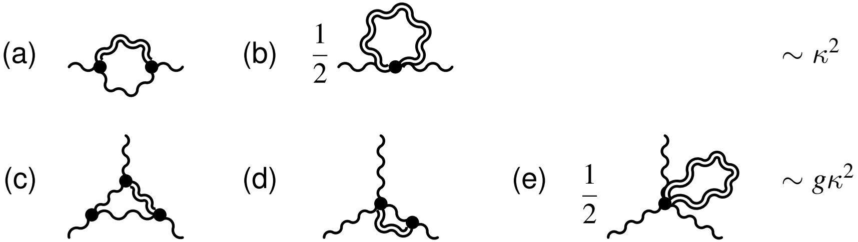

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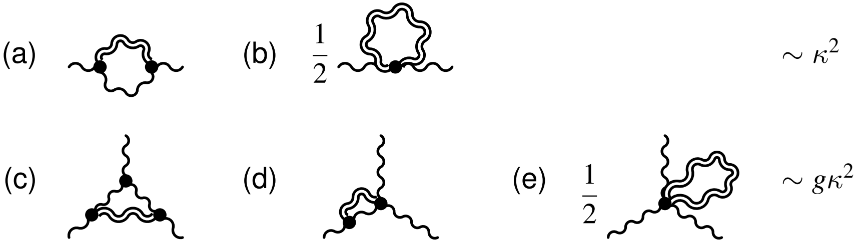

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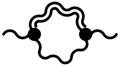
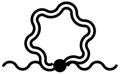
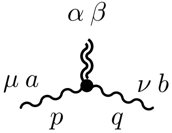
(a) 

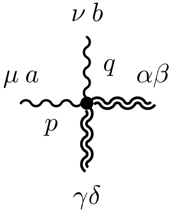
(b) $\frac{1}{2}$ 

$\sim \kappa^2$

The Gluon–Graviton Vertices with Two Gluon Lines



$$= -i\kappa\delta^{ab}\Big[P^{\mu\nu,\alpha\beta}\, p\cdot q + \eta^{\mu\nu}p^{(\alpha}q^{\beta)} + \tfrac{1}{2}\eta^{\alpha\beta}p^{\nu}q^{\mu} - p^{\nu}\eta^{\mu(\alpha}q^{\beta)} - q^{\mu}\eta^{\nu(\alpha}p^{\beta)}\Big]$$



$$\begin{aligned} = \frac{i}{2}\kappa^2\delta^{ab}\Big[&(p^{\nu}q^{\mu} - p\cdot q\,\eta^{\mu\nu})P^{\alpha\beta,\gamma\delta} \\ &+p\cdot q(2I^{\mu\nu,\alpha(\gamma}\eta^{\delta)\beta} + 2I^{\mu\nu,\beta(\gamma}\eta^{\delta)\alpha} \\ &\quad -I^{\mu\nu,\alpha\beta}\eta^{\gamma\delta} - I^{\mu\nu,\gamma\delta}\eta^{\alpha\beta}) \\ &+2p^{(\alpha}q^{\beta)}P^{\mu\nu,\gamma\delta} + 2p^{(\gamma}q^{\delta)}P^{\mu\nu,\alpha\beta} \\ &+\big\{2p^{\alpha}\eta^{\nu[\mu}\eta^{\beta](\gamma}q^{\delta)} + 2p^{\gamma}\eta^{\nu[\mu}\eta^{\delta](\alpha}q^{\beta)} \\ &\quad -p^{\nu}(q^{\alpha}P^{\mu\beta,\gamma\delta} + q^{\beta}P^{\alpha\mu,\gamma\delta} \\ &\qquad +q^{\gamma}P^{\alpha\beta,\mu\delta} + q^{\delta}P^{\alpha\beta,\gamma\mu})\big\} \\ &+\{(p,\mu)\leftrightarrow(q,\nu)\}\Big] \end{aligned}$$

where we have defined $P^{\mu\nu,\alpha\beta} \equiv \frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta})$.

Field-Strength Renormalization

$$= i\delta^{ab}(q^2\eta^{\mu\nu} - q^\mu q^\nu)\Delta + i\delta^{ab} q^2 (q^2\eta^{\mu\nu} - q^\mu q^\nu)\tilde{\Delta} + \cdots$$

$$\Delta = i\kappa^2 \frac{d-4}{8d}\left(d^2 - 4d + 8 + \frac{(d-2)(d-8)}{(D-2)}\right)\sum_{\vec{n}}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{k^2 - m_{\vec{n}}^2}$$

$$\tilde{\Delta} = i\kappa^2 \frac{(d-2)^2}{2(d+2)(D-2)}\Big(3 - d + \frac{\delta}{d}(4 - \tfrac{3}{2}d)\Big)\sum_{\vec{n}}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{1}{k^2(k^2 - m_{\vec{n}}^2)}$$

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Gravitational contribution to the YM β function in $4+\delta$ dimensions

$$\beta_g\Big|_{\kappa^2} = 0$$

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$$\Delta = \frac{2}{(4\pi)^{D/2-1}\Gamma(\frac{D}{2})} \frac{d-4}{(D-2)^2} \Big((d-3)(d-2) + \frac{\delta}{d}(d^2 - 4d + 8)\Big) \frac{\Lambda^{D-2} - \mu^{D-2}}{M_{(D)}^{D-2}}$$

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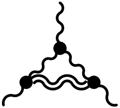
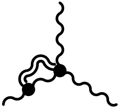
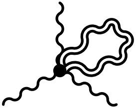
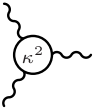
(c) 

(d) 

(e) $\frac{1}{2}$  $\sim g\kappa^2$

The Divergences of the Three Gluon Graphs



$$= g f^{abc} \Big[\eta^{\mu\nu} \big(p^\rho (2p\cdot q + p\cdot k + 3q\cdot k) - q^\rho (2q\cdot p + q\cdot k + 3p\cdot k) \big) + \ldots - \big(k^\mu k^\nu (p-q)^\rho + \ldots \big) - 3 \big(p^\rho q^\mu k^\nu - p^\nu q^\rho k^\mu \big) \Big] \tilde{\Delta}$$

In four (brane) dimensions the only counterterm needed in one-loop Einstein-Yang-Mills theory is

$$\mathcal{L}^{\text{c.t.}}_{\text{YM}} = \frac{1}{(4\pi)^{1+\delta/2}\Gamma\left(\frac{\delta}{2}+1\right)} \frac{8}{3\delta} \frac{\Lambda^\delta - \mu^\delta}{M_{(4+\delta)}^{\delta+2}} \operatorname{tr}\left\{ D^\mu F_{\mu\rho} D_\nu F^{\nu\rho} \right\}$$

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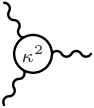

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$$

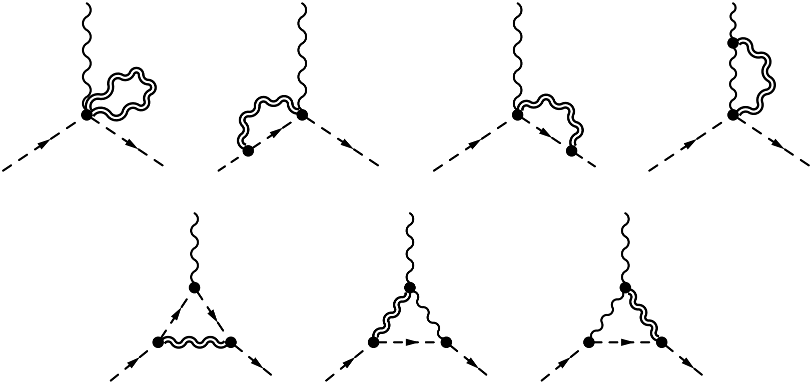
$$\mathcal{L}_s = \sqrt{-g} g^{\mu\nu} \left[(D_\mu \phi)^\dagger D_\nu \phi - m_\phi^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \right]$$

$$D_\mu = \partial_\mu - i g A_\mu$$

The higher derivative terms expected to appear at one-loop are

$$(D^\mu \phi)^\dagger D_\mu \phi \,, \quad (D^2 \phi)^\dagger D^2 \phi \,, \quad ig (D_\mu \phi)^\dagger F^{\mu\nu} D_\nu \phi \,, \quad g^2 \phi^\dagger F^{\mu\nu} F_{\mu\nu} \phi \,.$$

$$\color{red}{(D^\mu\phi)^\dagger D_\mu\phi\,,\quad (D^2\phi)^\dagger D^2\phi\,,}\quad ig(D_\mu\phi)^\dagger F^{\mu\nu}D_\nu\phi\,,\quad g^2\phi^\dagger F^{\mu\nu}F_{\mu\nu}\phi$$



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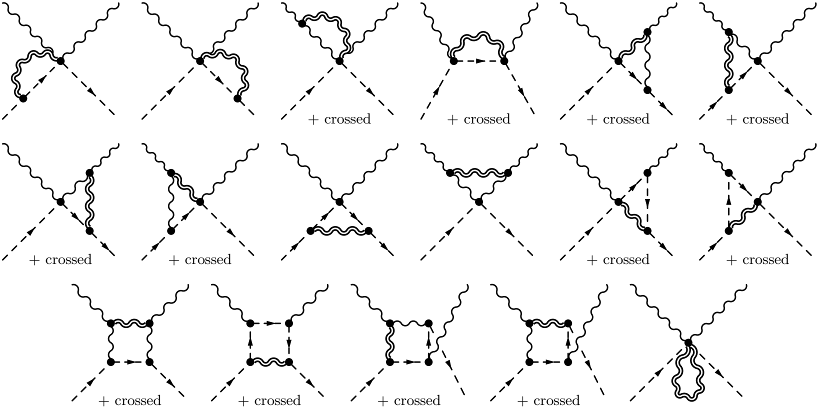
Scalar Field



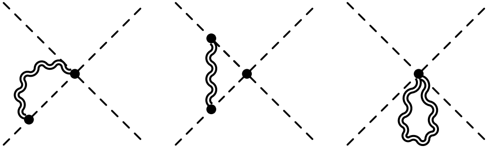
$$(D^{\mu}\phi)^{\dagger}D_{\mu}\phi\,,\quad (D^{2}\phi)^{\dagger}D^{2}\phi\,,\quad ig(D_{\mu}\phi)^{\dagger}F^{\mu\nu}D_{\nu}\phi\,,\quad g^{2}\phi^{\dagger}F^{\mu\nu}F_{\mu\nu}\phi$$

Scalar field-strength and HD renormalization $m_\phi = 0$

$$\mathcal{L}_s^{\text{c.t.}} = \frac{i}{(4\pi)^{1+\delta/2}\Gamma\left(\frac{\delta}{2}+2\right)}\left[\frac{2(8+5\delta)}{(\delta+2)^2}(D_\mu\phi)^\dagger D^\mu\phi\frac{\Lambda^{\delta+2}-\mu^{\delta+2}}{M_{(4+\delta)}^{\delta+2}}\right.$$
$$\left.-\left\{(D^2\phi)^\dagger D^2\phi - \tfrac{1}{3}ig(D_\mu\phi)^\dagger F^{\mu\nu}D_\nu\phi + \tfrac{1}{6}g^2\phi^\dagger F^{\mu\nu}F_{\mu\nu}\phi\right\}\frac{\Lambda^\delta-\mu^\delta}{M_{(4+\delta)}^{\delta+2}}\right]$$

Note: Without extra dimension ($\delta = 0$) the HD corrections vanish.

Renormalization of the φ^4 coupling – for a real scalar



$$\mathcal{L}_\mathrm{r} = -Z_\varphi \tfrac{1}{2}\varphi\partial^2\varphi - Z_{m_\varphi^2}\tfrac{1}{2}m_\varphi^2\varphi^2 - Z_{\varphi^4}\tfrac{\lambda}{4!}\varphi^4 + \dots$$

$$Z_\varphi - 1 = \frac{\kappa^2}{16\pi^2} m_\varphi^2 \frac{2}{d-4}$$

$$Z_{m_\varphi^2} - 1 = \frac{\kappa^2}{16\pi^2} m_\varphi^2 \frac{2}{d-4}$$

$$Z_{\varphi^4} - 1 = \frac{\kappa^2}{16\pi^2} 4m_\varphi^2 \frac{2}{d-4}$$

β Function

$$\beta_\lambda\big|_{\kappa^2} = -\frac{\kappa^2}{4\pi^2} m_\varphi^2 \lambda$$

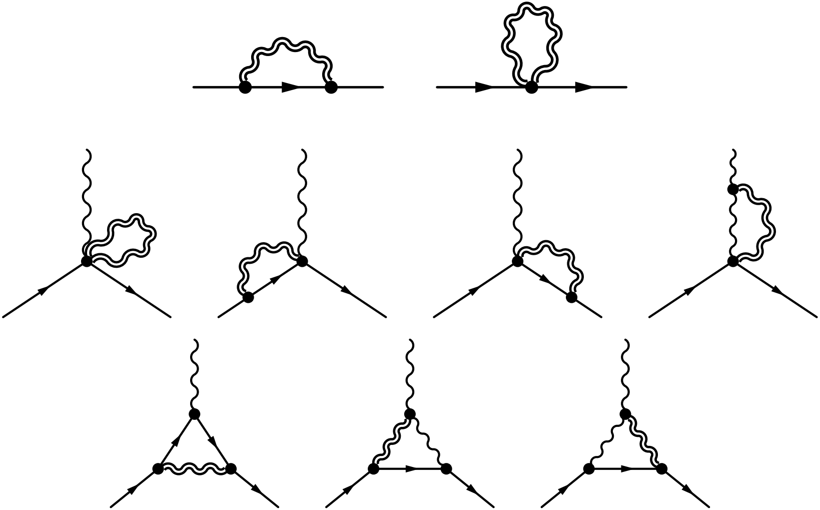
$$L_f = \sqrt{-g}\,\overline{\psi}(i\not{\mathcal{D}} - m_\psi - \mathcal{Y}\varphi)\psi$$

$$\not{\mathcal{D}} = \gamma^a e_a^\mu \mathcal{D}_\mu$$
$$\mathcal{D}_\mu = \partial_\mu - i\Omega_\mu - igA_\mu$$

The higher derivative terms expected to appear at one-loop are

$$i\overline{\psi}\not{D}\psi\,, \quad i\overline{\psi}\not{D}\not{D}\not{D}\psi\,, \quad i\overline{\psi}\not{D}D^2\psi\,, \quad i\overline{\psi}D^2\not{D}\psi\,, \quad i\overline{\psi}D_\mu\not{D}D^\mu\psi\,.$$

Fermions



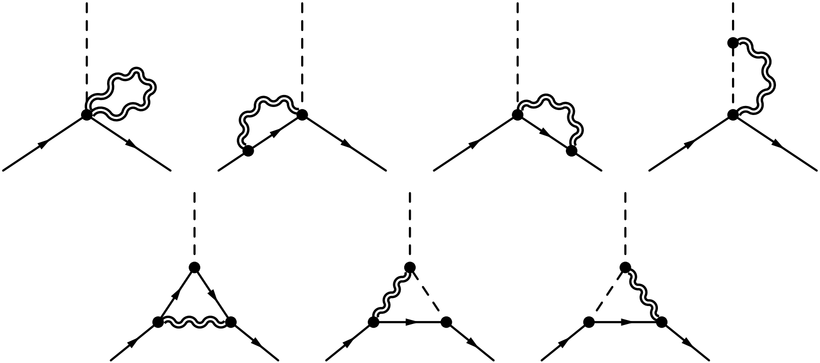
$$i\bar{\psi}\not{D}\psi\,, \quad i\bar{\psi}\not{D}\not{D}\not{D}\psi\,, \quad i\bar{\psi}\not{D}D^2\psi\,, \quad i\bar{\psi}D^2\not{D}\psi\,, \quad i\bar{\psi}D_\mu\not{D}D^\mu\psi$$

Fermionic field-strength and HD renormalization $m_\psi = 0$

$$\mathcal{L}_f^{\text{c.t.}} = \frac{i}{(4\pi)^{1+\delta/2}\Gamma\left(\frac{\delta}{2}+2\right)}\,\overline{\psi}\Bigg[\frac{3(11+9\delta)}{(\delta+2)^2}\,\frac{\Lambda^{\delta+2}-\mu^{\delta+2}}{M_{(4+\delta)}^{\delta+2}}\not{D}+ \\ +\Big\{-(10+\tfrac{49}{8}\delta)\not{D}\not{D}\not{D}-(\tfrac{58}{3}+\tfrac{143}{12}\delta)\not{D}D^2 \\ +(\tfrac{41}{3}+\tfrac{109}{12}\delta)D^2\not{D}+(\tfrac{41}{3}+\tfrac{109}{12}\delta)D_\mu\not{D}D^\mu\Big\}\frac{\Lambda^\delta-\mu^\delta}{\delta\,M_{(4+\delta)}^{\delta+2}}\Bigg]\psi$$

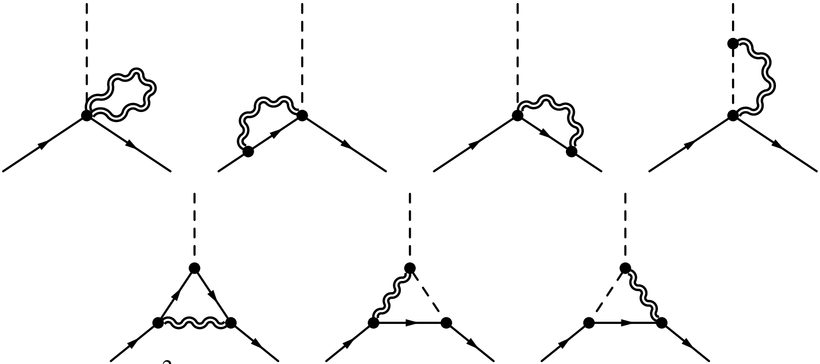
Renormalization of the Yukawa coupling

for uncharged fermion and scalar



$$\mathcal{L}_{\mathrm{r}} = Z_\psi \overline{\psi} i \partial\!\!\!/ \psi - Z_{m_\psi} m_\psi \overline{\psi}\psi - Z_{\varphi\overline{\psi}\psi} \mathcal{Y}\, \varphi \overline{\psi}\psi + \ldots$$

Fermions



$$Z_\psi - 1 = \frac{\kappa^2}{16\pi^2}\frac{1}{4}m_\psi^2\frac{2}{d-4}$$

$$Z_{m_\psi} - 1 = \frac{\kappa^2}{16\pi^2}\frac{1}{4}m_\psi^2\frac{2}{d-4}$$

$$Z_{\varphi\bar\psi\psi} - 1 = \frac{\kappa^2}{16\pi^2}\left(\tfrac{3}{4}m_\psi^2 + \tfrac{1}{4}m_\varphi^2\right)\frac{2}{d-4}$$

β Function

$$\beta_\mathcal{Y}\Big|_{\kappa^2} = -\frac{\kappa^2}{16\pi^2}\left(m_\psi^2 - \tfrac{1}{2}m_\varphi^2\right)\mathcal{Y}$$

Summary and Outlook

- The Yang-Mills β-function receives no contributions form gravitational self-coupling.
- Yukawa and φ^4 interactions receive gravitational corrections
- The spectrum of HD corrections is not restricted to Lee-Wick terms.

- Next step: Non-Perturbative Methods

Functional Renormalization Group Equation [Wetterich'93,Reuter'98]

$$\mu\frac{\partial}{\partial\mu}\Gamma_\mu[\Phi] = \frac{1}{2}\,\mathrm{Tr}\left[\mu\frac{\partial\mathcal{R}_\mu}{\partial\mu}\left(\Gamma^{(2)}_\mu[\Phi]\mathcal{R}_\mu\right)\right]$$

Thank you!