Finite Temperature QCD with Twisted Mass Fermions

Florian Burger

Humboldt-Universität zu Berlin

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GK "Masse, Spektrum, Symmetrie", Block Course 2009

Outline

1. Motivation
2. Why and how using twisted mass fermions
3. Some thermodynamics on the lattice
4. Goals

Motivation

- Heavy Ion Experiments: **RHIC** (future **LHC** and **FAIR**)
- Study matter under extreme conditions: near Big Bang,Cores of neutron stars
- For interpretation of experimental data: theoretical understanding of QCD at high temperatures and densities
- Lattice: Phase transition/crossover free quarks and gluons (Deconfinement)

  Quark-Gluon-Plasma

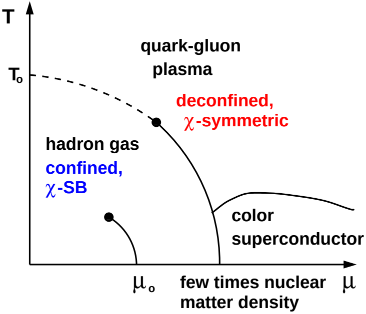
- Determine on lattice:

  Critical temperature (T_c)

  Equation of State (EoS) ($\epsilon(T)$, $p(T)$)

Qualitative Picture of QCD Phases

QCD phase diagram:



Plot taken from: F. Karsch[1]

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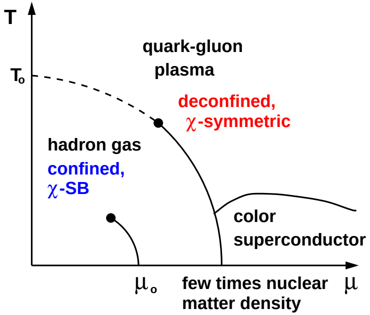

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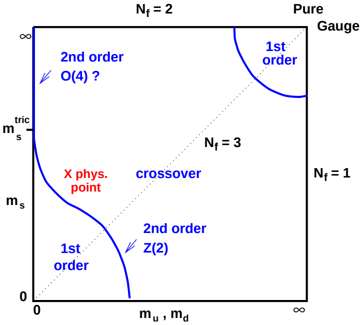
Order of transition:



Plot taken from: O. Philipsen[2]

[1] F. Karsch, *J. Phys. Conf. Ser.*, 46:122–131, 2006.

[2] O. Philipsen, *Eur. Phys. J. ST*, 152:29–60, 2007.

Finite temperatures on the lattice

Lattice: Discretize QCD on Euclidean space-time lattice with

lattice spacing a
volume $V = a^3 N_\sigma^3$

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$$\langle \mathcal{O} \rangle = \frac{\mathrm{Tr}\ \mathcal{O} e^{-\beta H}}{\mathrm{Tr}\ e^{-\beta H}} = \frac{1}{Z} \int_{\mathrm{p.}} DA \int_{\mathrm{a.p.}} D\psi\ D\bar{\psi}\ \ \mathcal{O} \exp\left(- \int_0^\beta d\tau \int d^3x\ \mathcal{L}^{\mathrm{E}}_{\mathrm{QCD}} \right)$$

$$\rightarrow \frac{1}{Z} \int_{\mathrm{p.}} DU \int_{\mathrm{a.p.}} D\psi\ D\bar{\psi}\ \ \mathcal{O} \exp\left(- S^{\mathrm{lattice}}_{\mathrm{QCD}} \right)$$

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Temperature T:

$$T \equiv \frac{1}{\beta} = \frac{1}{N_\tau a},$$

$$N_\tau < N_\sigma$$

Outline

Current status of research, staggered fermions

- **staggered fermions**:
results for :

T_c, EoS, finite density

[1] Y. Aoki et al., *Phys. Lett.* B643:46-54, 2006, [hep-lat/0609068]

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But:

T_c (physical result, $n_f = 2 + 1$, $a = 0$, phys. point):

Y. Aoki et. al.[1] : 151(3)(3) MeV   M. Cheng et. al.[2] : 192(7)(4) MeV

Also:

Chiral transition and deconfinement at same T_c?

Moreover:

Staggered fermions controversial (Rooting)

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→ **Wilson fermions:**

- Clover-improved Wilson fermions
- Twisted mass fermions ($\mathcal{O}(a)$ improvement)

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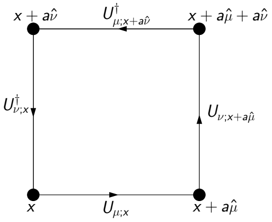
Wilson's way of discretization

- Fermion part:
  naive discretization → unphysical "doublers" in spectrum

$$\int d^4x \bar{\psi}(m + i\gamma_\mu D^\mu)\psi \rightarrow a^4 \sum_{x,y} \left(m\bar{\psi}_x \delta_{x,y}\psi_y + \bar{\psi}_x D^{W}_{x,y}\psi_y\right)$$
$$= a^4 \Big(\sum_x (m + 4r/a)\bar{\psi}_x \psi_x -$$
$$- \frac{1}{2a} \sum_{x,\mu} \left(\bar{\psi}_x (r - \gamma_\mu) U_{\mu;x}\psi_{x+a\hat{\mu}} + \bar{\psi}_{x+a\hat{\mu}}(r + \gamma_\mu) U^\dagger_{\mu;x}\psi_x\right)\Big)$$

- Gauge part:

$$\int d^4x (-\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}) \rightarrow S_{\mathrm{P}} = \beta \sum_P [1 - \frac{1}{N} \mathrm{Re}\ \mathrm{Tr}\,(U_P)]$$



→ bare parameters: β, $\kappa \equiv \frac{1}{2(4r+m)}$

Fermion sector:

Wilson twisted mass (tm) action:

$$S_f^{\mathrm{tm}} = \sum_{x,y} \bar{\chi}_x (D^{\mathrm{tm}})_{xy} \chi_y \qquad D^{\mathrm{tm}} = D^{\mathrm{W}} + m + i\mu_q \gamma_5 \tau^3$$

D^{W}: Wilson Dirac Operator

$\bar{\chi}$,χ: Quark fields in twisted basis
$\psi = \exp\left(i\omega\gamma_5\tau^3/2\right)\chi$

[1] R. Frezzotti und G. C. Rossi, JHEP, 08:007, 2004

Lattice regularization, tm fermions

Fermion sector:

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D^{W}: Wilson Dirac Operator

$\bar{\chi}, \chi$: Quark fields in twisted basis $\psi = \exp\left(i\omega\gamma_5\tau^3/2\right)\chi$

maximal twist $(\omega = \frac{\pi}{2})$: $\quad m_{\text{R}} = Z_m(m - m_{\text{cr}}) = 0$

$\rightarrow$ **automatic** $\mathcal{O}(a)$ improvement[1]

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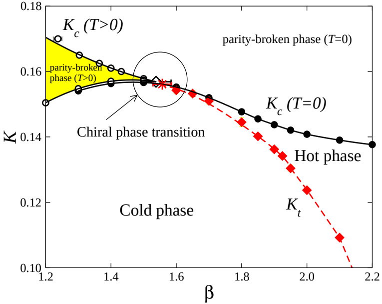
Phase diagram for twisted mass fermions



← Phase diagram for Wilson fermions from S. Ejiri et al.[1]

[1] S. Ejiri et al., arXiv:0909.2121 [hep-lat], 2009

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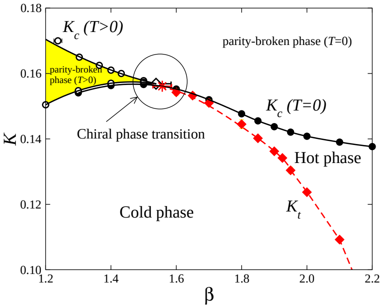

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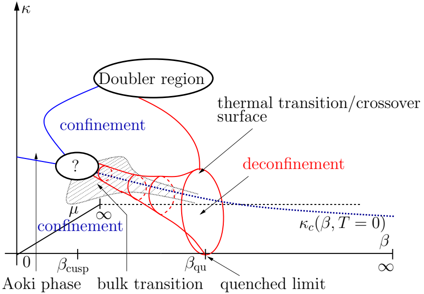
Same for tm fermions from E.-M. Ilgenfritz et al.[2] →



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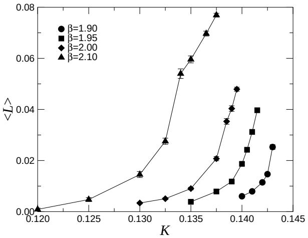
Outline

How detect transitions?

Deconfinement transition:

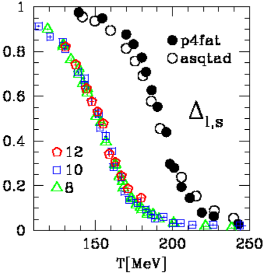
Order Parameter ($m_q = \infty$):
Polyakov loop $L(\mathbf{x}) = \langle \mathrm{Tr} \ \prod_{n_4}^{\text{per. BC}} U_4(\mathbf{x}, n_4) \rangle$



Taken from S. Ejiri et al.[1]

Chiral transition:

Order parameter ($m_q \to 0$):
Chiral condensate $\langle \bar{\psi}\psi \rangle$



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[2] Y. Aoki et al., *JHEP*, 06:088, 2009.

Finite chemical potential μ

- Implementation on lattice:

$$\nabla_4 \psi_x \to \frac{1}{2a}\left(U_{4;x} e^{a\mu} \psi_{x+\hat{4}a} - U^{\dagger}_{4;x-\hat{4}a} e^{-a\mu} \psi_{x-\hat{4}a} \right)$$

- "Sign problem" of fermionic determinant:

$$\det D \ngeq 0$$

- E.g. circumvent by reweighting and Taylor expansion:

$$\langle O \rangle_\mu = \langle O R(\mu) \rangle_0 / \langle R(\mu) \rangle_0, \qquad R(\mu) = \det M(\mu) / \det M(0)$$

Outline

Goals

- Thermodynamics with Wilson twisted mass fermions

[1] tmfT Collab.: M. Petschlies, M. P. Lombardo u.a.; E.-M. Ilgenfritz et al., arXiv:[0905.3112], 2009

[2] ETM Collab.: C. Urbach, K. Jansen, M. Wagner u.a.; Ph. Boucaud et al., arXiv:[0803.0224], 2009

Goals

- Thermodynamics with Wilson twisted mass fermions

- $n_f = 2$:
  - T_c and EoS at maximal twist
  - test of universality: staggered fermions

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- Important prerequisites:
  - known 3d phase diagram for twisted mass fermions[1]
  - highly efficient Code of ETM Collab.[2]

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- Extend to:
  - finite density
  - $n_f = 2 + 1 + 1$

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End

Thank you

▸ Start

End

Appendix

Pressure (large homogeneous systems):

$$p(T) = \frac{T}{V} \ln Z$$

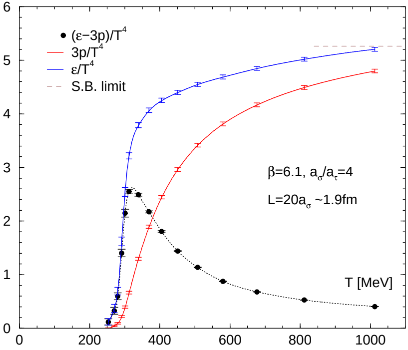
Energy density:

$$\epsilon(T) = \frac{T^2}{V} \frac{\partial \ln Z}{\partial T}\Big|_V$$

but: $V \propto a^3$, $T \propto \frac{1}{a}$ and $a \equiv a(\beta, \ldots)$

$\rightarrow$ Consider interaction measure:

$$I = \epsilon - 3p = -\frac{T}{V}\frac{d \ln Z}{d \ln a} \underbrace{=}_{\text{pure gauge}} \langle S_P \rangle \frac{d \ln g^2}{d \ln a}$$



Plot from T. Umeda et al.[1]

[1] T. Umeda et al., *PoS* **LAT2008** (2008) 174.