A New Regulariation of $\mathcal{N} = 4$ Super Yang-Mills Theory

Theodor Schuster

Humboldt Universität zu Berlin
Institut für Physik

10.07.2009

F. Alday, J. Henn, J. Plefka and T. Schuster, arXiv:0908.0684

Outline

Reasons to investigate $\mathcal{N} = 4$ Super Yang-Mills

- SUSY is a candidate for solving:
  - Coupling Constant Unification
  - Hierarchy Problem
  - Dark Matter Problem
- $\mathcal{N} = 4$ SYM is the maximally supersymmetric gauge theory in 4 dimensions
- AdS/CFT conjecture:
  - exact duality between $\mathcal{N} = 4$ SYM and a string theory on $AdS_5 \times S^5$
- posesses quantum superconformal symmetry: $\beta_g = 0$
  $\Rightarrow$controlled by the two parameters N and g
- Learn about non-supersymmetric gauge theories
  - tree-level QCD is effectively supersymmetric

Example: Gluon Tree Amplitudes from $\mathcal{N} = 4$ SYM

- Formula for all tree-level super-amplitudes in $\mathcal{N} = 4$ SYM [J.M. Drummond and J.M. Henn JHEP 04(2009)018]

$$\Downarrow$$

- Formula for all gluon tree amplitudes! [to be published]

Motivation For a New Regularization

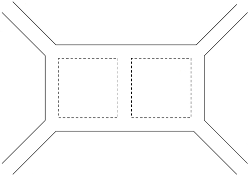
- Form of infrared divergences and the BDS ansatz suggest to calculate the logarithm of amplitudes
- Inconvenience of dimensional regularization: interference between pole terms and $\mathcal{O}(\epsilon)$ terms
  $\Rightarrow$higher order terms needed

An Alternative Regularization

Field Theory:

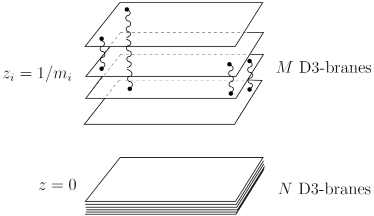
- Consider $U(N+M)$ symmetry with $N \gg M$
- Higgsing $U(N+M) \to U(N) \times U(1)^M$

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields



Dual string picture:

- Seperating M of the $D3$-branes serves as regulator
- Extended dual conformal symmetry of amplitudes



Higgsing $\mathcal{N} = 4$ Super Yang-Mills

- Field content: All fields in adjoint representation of $U(N+M)$
  - Gluons: $A_\mu, \quad \mu = 0,1,2,3$
  - 6 real scalars: $\Phi_I, \quad I = 1,\ldots,6$
  - 4 gluinos: $\Psi_A, \quad A = 1,2,3,4$

The Action: uniquely fixed by supersymmetry

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \, \mathrm{Tr}\Big(-\tfrac{1}{4}\hat{F}^2_{\mu\nu} - \tfrac{1}{2}(D_\mu \hat{\Phi}_I)^2 + \tfrac{g^2}{4}[\hat{\Phi}_I, \hat{\Phi}_J]^2 + \text{fermions}\Big),$$

Decompose all the fields into $N+M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a,b = 1,\ldots,N, \;\; i,j = N+1,\ldots,N+M,$$

thereby switching on a vacuum expectation value: $\hat{\Phi}_I = \delta_{I9}\,\langle\Phi_9\rangle + \Phi_I$

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

- Add R_ξ gauge fixing and appropriate ghost term
- Qudratic terms $(A_M := (A_\mu, \Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i - m_j)^2 \,(A_M)_{ij}\,(A^M)_{ji} - m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{fermions} \Big\}$$

⇒ light fields $\mathcal{O}_{ij}$ with mass $(m_i - m_j)$
⇒ heavy fields $\mathcal{O}_{ib}$ with mass m_i

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

Higgsing leads to new bosonic interactions

$$\hat{S}_{\mathcal{N}=4}\Big|_{\mathcal{O}(m_i)} = \int d^4x \Big\{ m_i \left([\Phi_9, A^\mu] A_\mu \right)_{ii} - m_i (A_\mu [\Phi_9, A^\mu])_{ii} + m_i \left([\Phi_9, \Phi_{I'}] \Phi_{I'} \right)_{ii} - m_i \left(\Phi_{I'} [\Phi_9, \Phi_{I'}] \right)_{ii} \Big\}$$

These vertices are neccesary in order to have extended dual conformal symmetry

One loop test of extended dual conformal symmetry

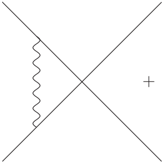
Consider the (special) colour ordered amplitude

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle$$

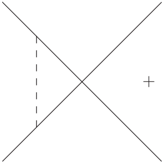
related to the leading color contribution of four scalar amplitude by

$$\mathcal{A}_4 = \sum_{\sigma \in S_4/Z_4} \delta^{j_{\sigma(1)}}_{i_{\sigma 1}}\delta^{j_{\sigma(2)}}_{i_{\sigma(2)}}\delta^{j_{\sigma(3)}}_{i_{\sigma(3)}}\delta^{j_{\sigma(4)}}_{i_{\sigma(4)}} A_4(\sigma(1),\sigma(2),\sigma(3),\sigma(4))$$

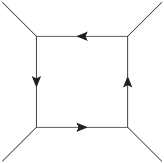
Relevant one-loop diagrams:



+ cyclic permutations

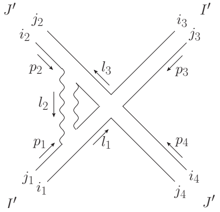


+ cyclic permutations

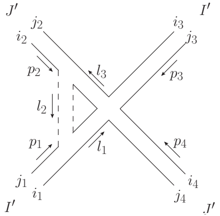


One loop test of extended dual conformal symmetry

The Triangle diagrams want to talk to each other:



$$= 2Ng^4 \int \frac{d^4 l}{(2\pi)^4} \frac{(l_1 + p_1) \cdot (l_3 - p_2)}{(l_1^2 + m_1^2)(l_2^2 + m_2^2)(l_3^2 + m_3^2)}$$

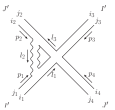
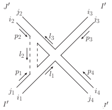


$$= 2Ng^4 \int \frac{d^4 l}{(2\pi)^4} \frac{(2m_1 - m_2)(2m_3 - m_2)}{(l_1^2 + m_1^2)(l_2^2 + m_2^2)(l_3^2 + m_3^2)}$$

One loop test of extended dual conformal symmetry

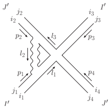
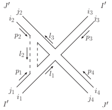
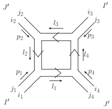
Both diagrams can be combined using the five dimensional momenta

$$\hat{p}_k = (p_k, m_k - m_{k+1}) \qquad \hat{l}_k = (l_k, m_k)\,,$$

 + 

$$= 2Ng^4 \int \frac{d^4 l}{(2\pi)^4} \frac{(\hat{l}_2 + 2\hat{p}_1)\cdot(\hat{l}_2 - 2\hat{p}_2)}{\hat{l}_1^2\,\hat{l}_2^2\,\hat{l}_3^2}$$

Numerator algebra is the same as in the massless case: $\hat{p}_k^2 = 0$

 +  + 

$$= -16Ng^4 \int \frac{d^4 l}{(2\pi)^4} \frac{(\hat{p}_1\cdot\hat{p}_2)(\hat{p}_2\cdot\hat{p}_3)}{\hat{l}_1^2\,\hat{l}_2^2\,\hat{l}_3^2\,\hat{l}_4^2}$$

All triangle integrals have canceled!

One loop test of extended dual conformal symmetry

- The four point amplitude is given by

$$A_4 = i g_{\mathrm{YM}}^2 \left(1 - \frac{a}{2} I^{(1)}(s,t,m_i) + O(a^2) \right)$$

here $I^{(1)}(s,t,m_i)$ is the Massive box integral

$$I^{(1)}(s,t,m_i) = c_0 \int \frac{d^4 l}{(2\pi)^4} \frac{(\hat{p}_1 \cdot \hat{p}_2)(\hat{p}_2 \cdot \hat{p}_3)}{\hat{l}_1^2 \, \hat{l}_2^2 \, \hat{l}_3^2 \, \hat{l}_4^2}$$

- Reexpressed in 5d dual variables $\hat{x}_{ij} = \hat{x}_i - \hat{x}_j$: $\boxed{\hat{p}_i = \hat{x}_{i\,i+1}}$

$$\boxed{I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5 \hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\Big[2x_{i\mu}\left(x_i^\nu \frac{\partial}{\partial x_i^\nu} + m_i \frac{\partial}{\partial m_i}\right) - (x_i^2 + m_i^2)\frac{\partial}{\partial x_i^\mu}\Big] I^{(1)}(s,t,m_i) = 0$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry!

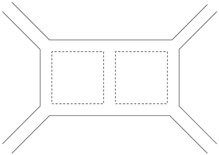
Only 4 particle invariants: $\frac{m_1 m_3}{\hat{x}_{13}^2}$ and $\frac{m_2 m_4}{\hat{x}_{24}^2}$

$$\Rightarrow \quad I^{(1)}(x_{13}^2, x_{24}^2, m_i) = f\left(\frac{m_1 m_3}{\hat{x}_{13}^2}, \frac{m_2 m_4}{\hat{x}_{24}^2}\right)$$

Indeed one finds in the $m_i \to 0$ limit: $f(u,v) = 2\ln(u)\ln(v) - \pi^2 + O(m^2)$

Higher Loops and More External Legs

- At 2 loops: Only one integral is allowed by extended dual conformal symmetry:



Bubbles and triangles forbidden. Similarly restricts possible integrals at higher loops and more external legs.

- Checked exponentiation in Higgs regularization: results agree with dimensional regularization

Summary and Outlook

- New IR regularization of planar amplitudes in $\mathcal{N} = 4$ SYM.
- Breaking of dual conformal invariance at loop level can be repaired by regularization via Higgsing with an exact extended dual conformal symmetry.
- Regularization via Higgsing restricts allowed integrals at higher loops. Is of pratical help for higher loop calculations.
- Calculation of regulated integrals is relativly easy.

- Proof of exact dual conformal symmetry?
- Proof of anomalous dual conformal Ward identity?
- Study exponentiation in the new regularization?
- Can breaking of standard conformal invariance at loop level be controlled?

Summary and Outlook

- New IR regularization of planar amplitudes in $\mathcal{N} = 4$ SYM.
- Breaking of dual conformal invariance at loop level can be repaired by regularization via Higgsing with an exact extended dual conformal symmetry.
- Regularization via Higgsing restricts allowed integrals at higher loops. Is of pratical help for higher loop calculations.
- Calculation of regulated integrals is relativly easy.

- Proof of exact dual conformal symmetry?
- Proof of anomalous dual conformal Ward identity?
- Study exponentiation in the new regularization?
- Can breaking of standard conformal invariance at loop level be controlled?

Thank you for your attention!