Oct. 01. 2009 · Humboldt-University of Berlin

Lattice study of pion pion scattering

Xu Feng
NIC, DESY / Münster University

in collaboration with **Karl Jansen** and **Dru B. Renner**
on behalf of **ETMC**

1 Scattering: from experiment to theory

- scattering is a main method in studying hadron-hadron interactions
- cross section σ are measured in experiment
- in the limit of $r \to \infty$, wave function $\psi(\vec{r})$ for two-particle state is expanded

$$\psi(\vec{r}) \to e^{i\vec{k}\cdot\vec{r}} + A(k,\theta) r^{-1} e^{ikr}$$

- σ can be given by scattering amplitude $A(k,\theta)$

$$\frac{d\sigma}{d\Omega} = A^2(k,\theta)$$

- according to partial wave analysis

$$A(k,\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)A_l(k)P_l(\cos\theta), \quad A_l(k) = \frac{e^{2i\delta_l(k)}-1}{2i}$$

- in the limit of $k \to 0$, scattering length a_l is extracted from $\delta_l(k)$

$$\delta_l(k) = a_l k^{2l+1} + O(k^{2l+3})$$

2 Pion pion scattering

- chiral symmetry spontaneously breaking: three Goldstone meson $\pi^{\pm}$, π^{0}
- property of π as Goldstone bonson:
  chiral symmetry play a central role in $\pi\pi$ scattering
- in χ-PT theory, chiral expansion of scattering amplitude $A(s)$ is given by:

$$A(s) = A(s)_2 + A(s)_4 + A(s)_6 + \cdots$$

- we need low energy constants (LECs) to determine $A(s)$
- $A(s)$ is specified by isospin $A^I(s)$
- single pion: isospin triplet

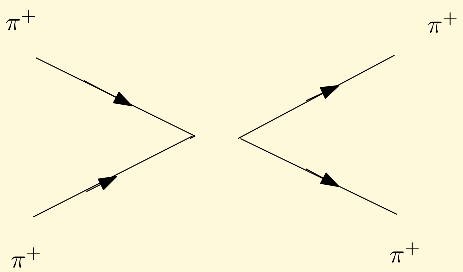
$$\begin{aligned} |\pi^+\rangle &= |I=1, I_z=+1\rangle, \\ |\pi^0\rangle &= |I=1, I_z=0\rangle, \\ |\pi^-\rangle &= |I=1, I_z=-1\rangle. \end{aligned}$$

- two pion system:

$$\begin{aligned} |\pi\rangle \otimes |\pi\rangle &\equiv |\pi\pi\rangle \\ |I=1\rangle \otimes |I=1\rangle &= |I=2\rangle \oplus |I=1\rangle \oplus |I=0\rangle \end{aligned}$$

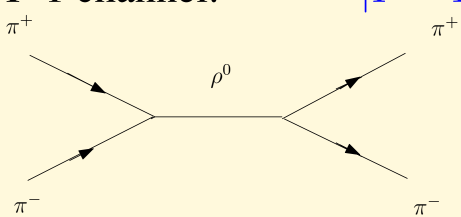
- I=2 channel, here we consider

$$|I=2, I_z=+2\rangle = |\pi^+\pi^+\rangle$$

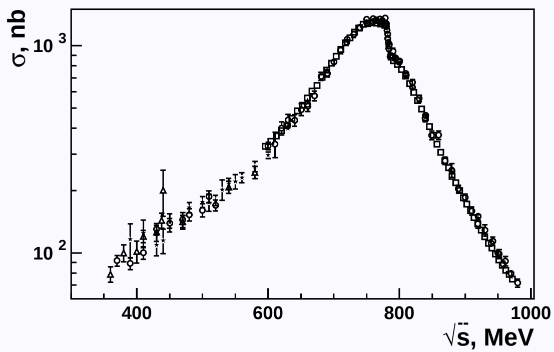


  - ⋆ no resonance appears in this channel
  - ⋆ LECs is required to determine χ-PT scattering amplitude

- I=1 channel: $|I=1, I_z=0\rangle = |\pi^+\pi^- - \pi^-\pi^+\rangle$

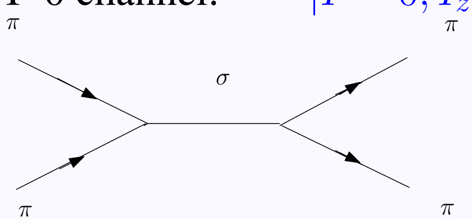


  - ⋆ $\rho(770)$ resonance appears in this channel



resonance mass: m_ρ=775 MeV, large width: Γ_ρ=149 MeV:

- I=0 channel: $|I=0, I_z=0\rangle = |\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0\rangle$



- ⋆ $\sigma(600)$ resonance appears in this channel
- ⋆ m_σ=513(32) MeV, Γ_σ=670(134) MeV [CLEO Collaboration, 2002] ??
- ⋆ width is as large as mass $\rightarrow$
  strong overlap between resonance and background
- ⋆ experimental existence for $\sigma(600)$ is not fully settled

$$m_\sigma = 400 - 1200 \ \ \mathbf{MeV}, \quad \Gamma_\sigma = 500 - 1000 \ \ \mathbf{MeV}. \ \ [\mathrm{PDG}]$$

访问主页
标题页
◀◀ ▶▶
◀ ▶
第 5 页 18
返回
全屏显示
关闭
退出

3 Motivation of Lattice QCD

- pros
  - ⋆ $\pi\pi$ scattering is non-perturbative in nature at low energies
  - ⋆ LQCD offers a non-perturbative method to study $\pi\pi$ scattering from first principle theory, QCD
    - ∗ extract parameters of $\delta_l^I(\sqrt{s})$ and a_l^I, provide LECs for χ-PT
    - ∗ obtain mass and decay width of resonance
- cons
  - ⋆ lattice artifacts arise from non-zero lattice spacing a
  - ⋆ finite volume constraints: no concepts of δ_l^I in a finite volume
- solutions:
  - ⋆ $N_f = 2$ maximally twisted fermions, automatically $O(a)$ improved
  - ⋆ Lüscher's finite size method: connecting finite volume to infinite volume

4 Lüscher's method

- assume that $V^I(\vec{r})$ describe the interaction of $\pi\pi$ in the isospin channel, I.
- infinite volume: continuous energy spectrum $\sqrt{s}$:

$$V^I(\vec{r}) \Rightarrow \psi^I(\vec{r}) \Rightarrow \delta_l^I(\sqrt{s})$$

- lattice simulation ask for finite volume: discrete energy spectrum $\sqrt{s_L}$

$$V_L^I(\vec{r}) = \sum_{\vec{m}\in\mathbb{Z}^3} V(\vec{r}+\vec{m}L) \Rightarrow \sqrt{s_L}$$

$$V_L^I(\vec{r}) \Rightarrow \psi_L^I(\vec{r}) \Rightarrow \left(\delta_l^I\right)_L(\sqrt{s_L})??$$

- Lüscher established a connection between $\sqrt{s_L}$ and δ

$$\frac{1}{\tan\delta(\sqrt{s_L})} = Z(1;k_L L/(2\pi))\ , \quad k_L = \sqrt{s_L/4 - m_\pi^2}$$

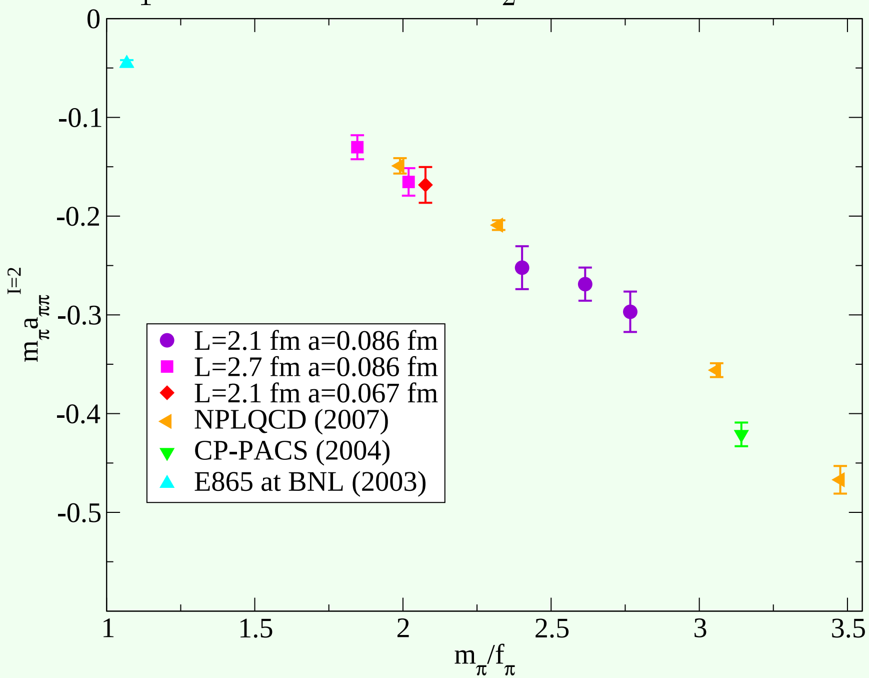
- $Z(1;k_L L/(2\pi))$ is a known function and universal for different interactions

5 I=2 channel

- twisted mass fermions:
  m_π: 270 MeV–480 MeV a: 0.086 fm, 0.067 fm
- calculate discrete energy spectrum $E_{\pi\pi}^{I=2}$ for each m_π
- Lüscher's method relates $E_{\pi\pi}^{I=2}$ to δ, and hence scattering length $a_{\pi\pi}^{I=2}$

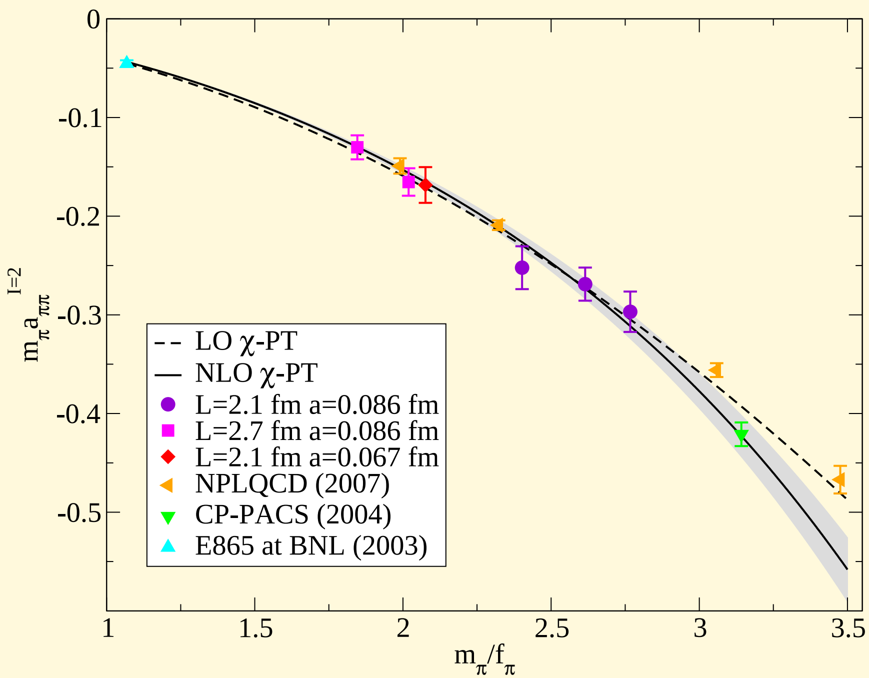
$$E_{\pi\pi}^{I=2} - 2m_\pi = -\frac{4\pi a_{\pi\pi}^{I=2}}{m_\pi L^3}\left[1 + c_1 \frac{a_{\pi\pi}^{I=2}}{L} + c_2 \left(\frac{a_{\pi\pi}^{I=2}}{L}\right)^2\right] + O(L^{-6})$$

where $c_1 = -2.837297$ and $c_2 = 6.375183$



- NLO χ-PT

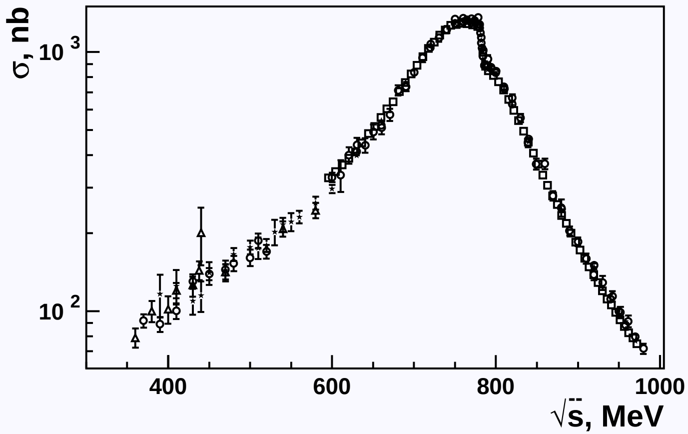
$$m_\pi a_{\pi\pi}^{I=2} = -\frac{m_\pi^2}{8\pi f_\pi^2}\left\{1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2}\left[3\ln\left(\frac{m_\pi^2}{f_\pi^2}\right) - 1 - l_{\pi\pi}^{I=2}(\mu = f_{\pi,phy})\right]\right\}$$



- this gives $m_\pi a_{\pi\pi}^{I=2}$ at physical limit and $l_{\pi\pi}^{I=2}(\mu)$ at a scale of $\mu = f_{\pi,phy}$
  - ⋆ ETMC: $m_\pi a_{\pi\pi}^{I=2} = -0.04385(47)$ $l_{\pi\pi}^{I=2}(\mu) = 4.7(1.4)$
  - ⋆ NPLQCD: $m_\pi a_{\pi\pi}^{I=2} = -0.04330(42)$ $l_{\pi\pi}^{I=2}(\mu) = 6.2(1.2)$
  - ⋆ CGL: $m_\pi a_{\pi\pi}^{I=2} = -0.0444(10)$
  - ⋆ E865 at BNL: $m_\pi a_{\pi\pi}^{I=2} = -0.0454(34)$

6 I=1 channel

- resonance $\rho(770)$ in $e^+e^- \to \pi^+\pi^-$ cross section, large width: Γ=149 MeV:



- cross section σ is contributed by l-th partial wave scattering phase δ_l^I:

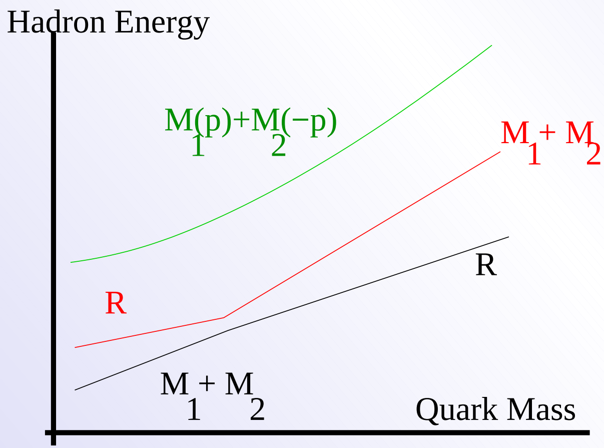
$$\sigma(\sqrt{s}) \propto \sum_l (2l+1)\sin^2(\delta_l^I(\sqrt{s}))$$

- $I=1$ channel, P-wave scattering phase δ_1^1 dominates the contribution to σ
- definition for resonance mass M_R:

$$\sigma(\sqrt{s})\Big|_{\sqrt{s}=M_R} = \sigma_{\max} \quad \text{or} \quad \delta_1^1(\sqrt{s})\Big|_{\sqrt{s}=M_R} = \frac{\pi}{2}$$

- twisted mass fermions:

$$m_\pi = 390,\ 310,\ 270\ \mathbf{MeV}\ ,\quad L/a = 24,\ 32,\ 32\ ,\quad a = 0.086\ \mathrm{fm}$$

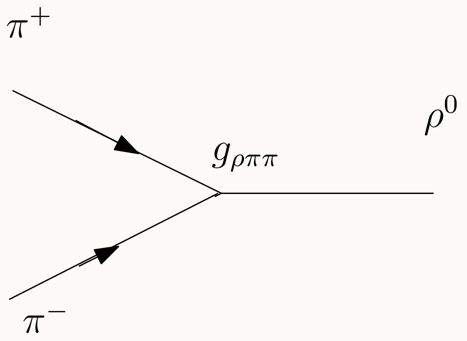


- $m_{\rho^0} > 2m_{\pi^+}$: threshold is open for $\rho^0 \to \pi^+\pi^-$
- elastic scattering region: $2m_\pi < \sqrt{s_L} < 4m_\pi$
- collect four (three) energy spectrums $E_{\pi\pi}^{I=1}$ for each m_π
- insert $\sqrt{s_L} = E_{\pi\pi}^{I=1}$ into Lüscher formula, evaluate scattering phase $\delta_1^1(\sqrt{s_L})$

- call effective range formula to describe the resonant behavior of δ_1^1

$$\tan\delta_1^1(k) = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k^3}{\sqrt{s}(M_R^2 - s)}\ , \quad k = \sqrt{s/4 - m_\pi^2}$$

- determine resonance mass M_R and coupling constant $g_{\rho\pi\pi}$



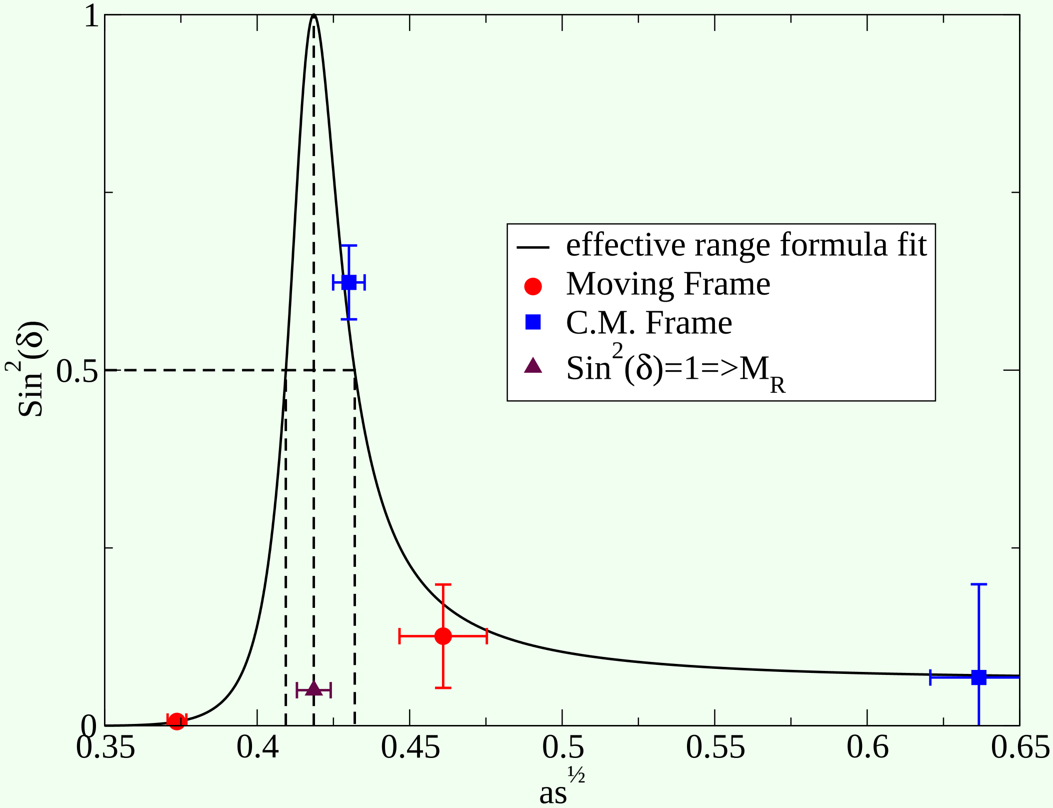
- decay width is given by $g_{\rho\pi\pi}$:

$$\Gamma = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{k^3}{M_R^2}\ , \quad k = \sqrt{M_R^2/4 - m_\pi^2}$$

- NLO χ-PT theory predicts: $g_{\rho\pi\pi}$ is almost m_π independent

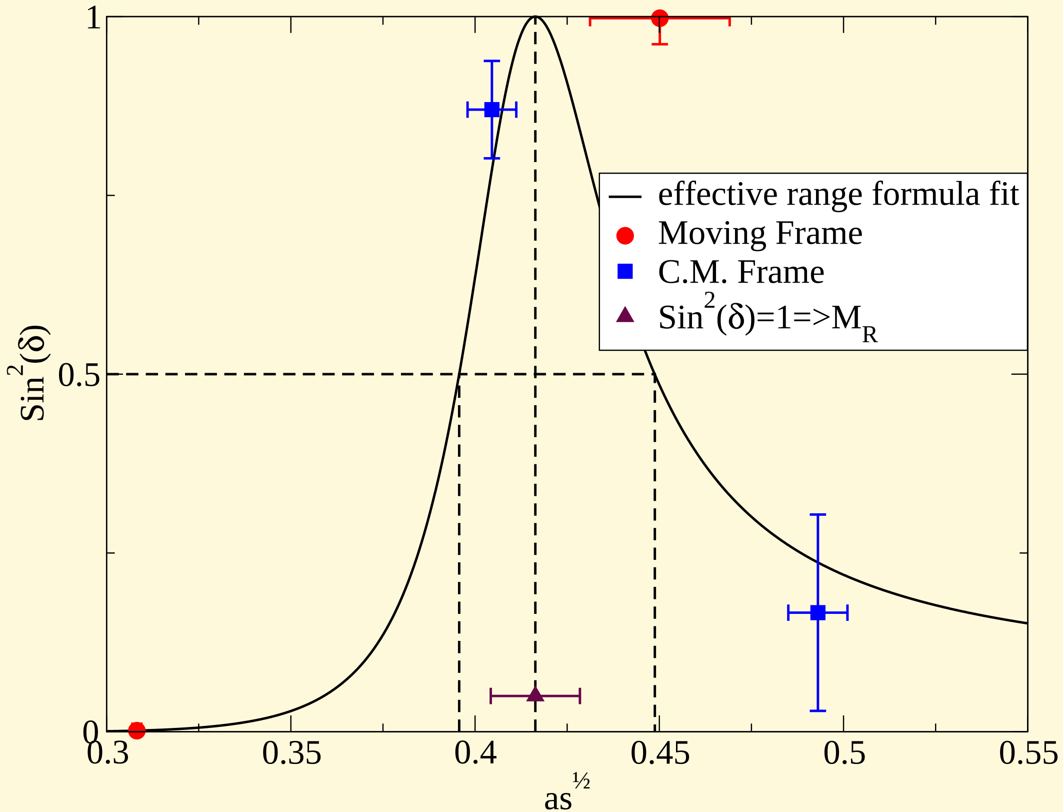
访问主页
标题页
第 12 页 18
返回
全屏显示
关闭
退出

- $m_{\pi^+} = 390$ MeV



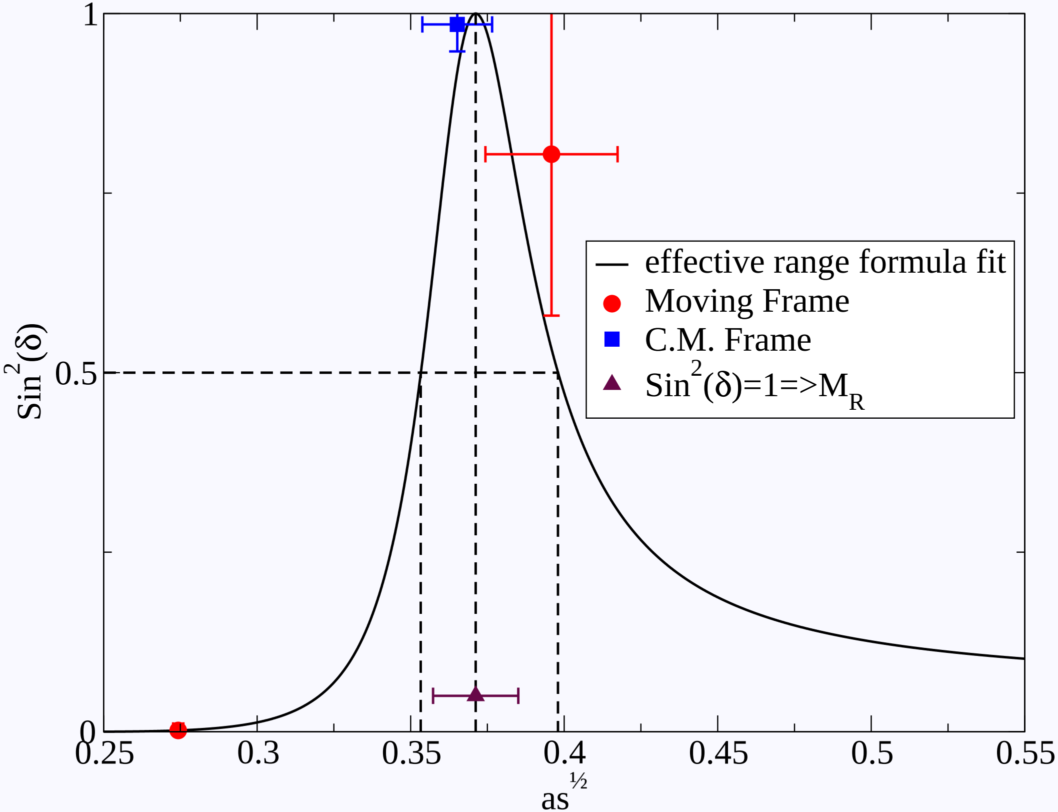
- $aM_R = 0.4186(56)\ , \quad g_{\rho\pi\pi} = 6.16(48)$

- $m_{\pi^+} = 310$ MeV



- $aM_R = 0.416(12)$, $g_{\rho\pi\pi} = 6.33(84)$

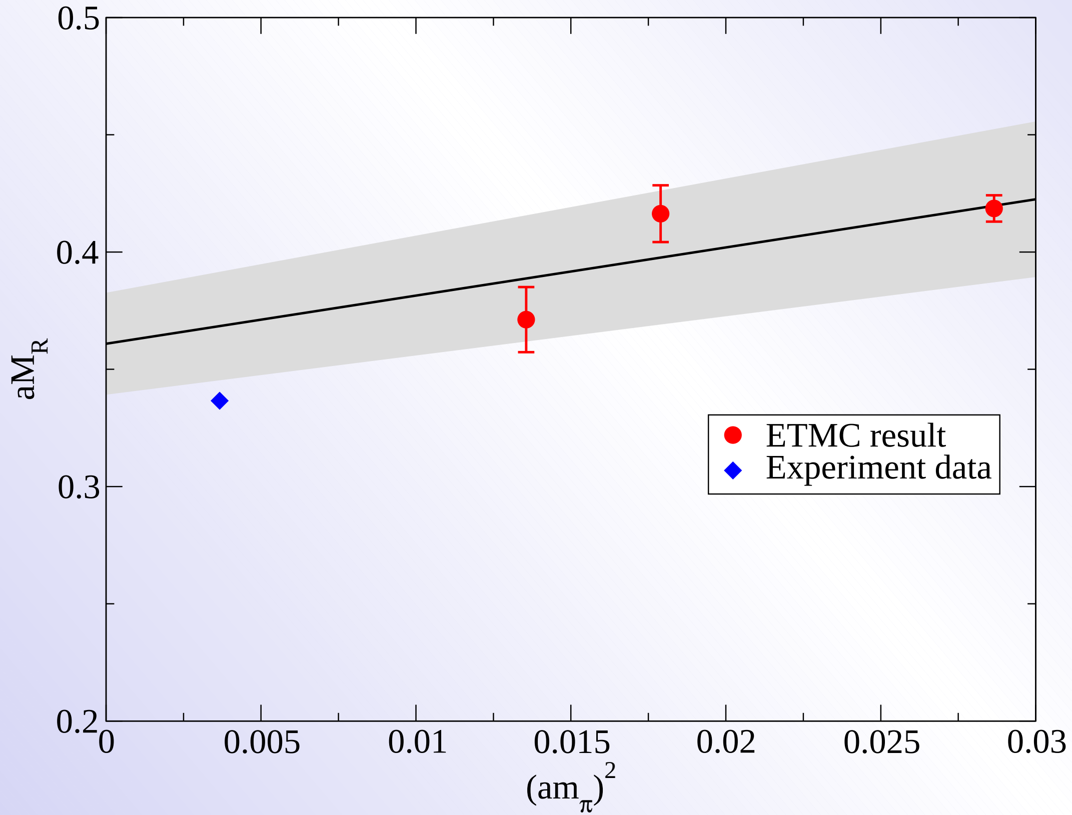
- $m_{\pi^+} = 270$ MeV



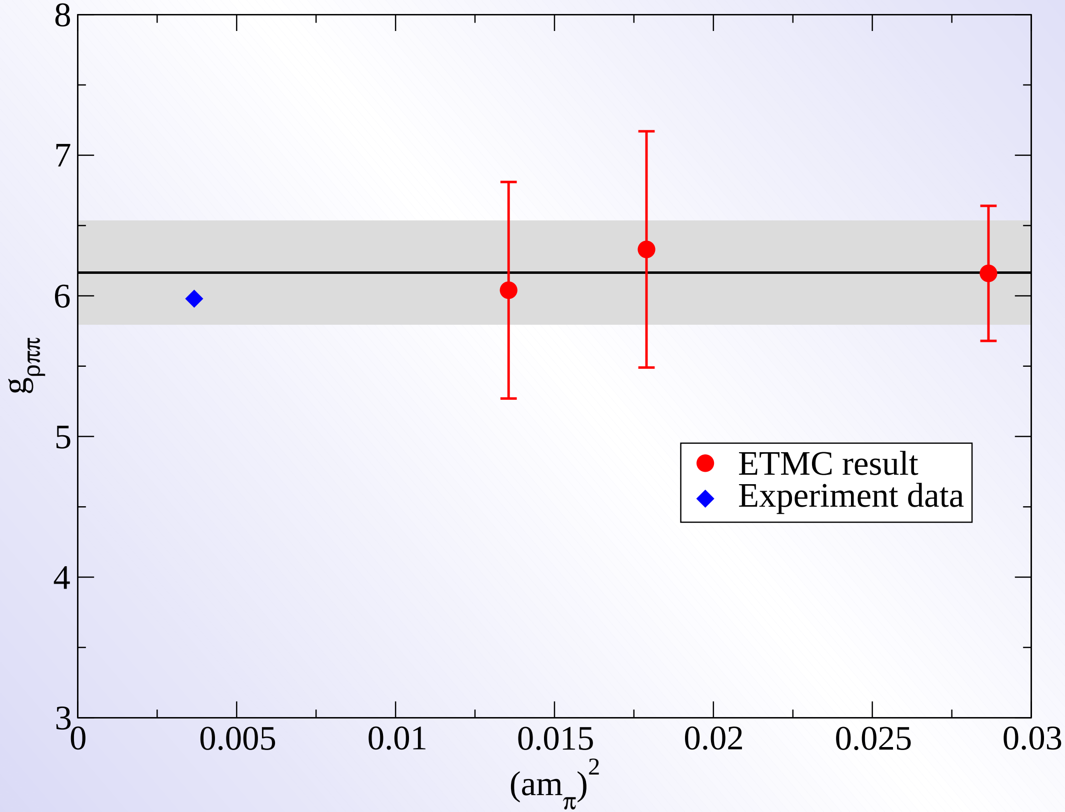
- $aM_R = 0.371(14)$, $g_{\rho\pi\pi} = 6.04(77)$

- chiral extrapolation for M_R

$$M_R = M_R^0 + c_1 m_\pi^2 + O(m_\pi^3)$$



• fit $g_{\rho\pi\pi}$ as a constant



• ETMC: $g_{\rho\pi\pi} = 6.17(37)$ Experiment: $g_{\rho\pi\pi} = 5.98(02)$

7 Conclusion

- in $I = 2$ channel, we probe the chiral dynamics of strong interaction
  - ⋆ smaller m_π is used: 270 MeV–480 MeV
  - ⋆ precise scattering length at physical limit:
    ETMC: $m_\pi a_{\pi\pi}^{I=2} = -0.04385(47)$
    NPLQCD: $m_\pi a_{\pi\pi}^{I=2} = -0.04330(42)$
    E865 at BNL: $m_\pi a_{\pi\pi}^{I=2} = -0.0454(34)$
- in $I = 1$ channel
  - ⋆ at all three pion masses, we get obvious indications of resonance
    - ∗ $m_\pi = 390$ **MeV**, $aM_R = 0.419(06)$, $g_{\rho\pi\pi} = 6.16(48)$
    - ∗ $m_\pi = 310$ **MeV**, $aM_R = 0.416(12)$, $g_{\rho\pi\pi} = 6.33(84)$
    - ∗ $m_\pi = 270$ **MeV**, $aM_R = 0.371(14)$, $g_{\rho\pi\pi} = 6.04(77)$
  - ⋆ chiral extrapolation for M_R and $g_{\rho\pi\pi}$
    - ∗ M_R: deviation from experiment is a little more than 1σ
      only three data, linear fit??
    - ∗ $g_{\rho\pi\pi}$: well consistent with experiment
      pion mass dependence of $g_{\rho\pi\pi}$ is small
- future improvement:
  - ⋆ calculate M_R and $g_{\rho\pi\pi}$ at smaller lattice spacing to check lattice artifacts
  - ⋆ extract scattering parameter in $I = 0$ channel