α_s, m_c and the muon $g-2$ from lattice QCD

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Content

Outline

Motivation

- heavy quark masses and strong coupling constant: fundamental parameters of the Standard Model, essential input parameters for processes involving heavy quarks
- sum rule approach in pQCD requires experimental input (low energy regions and near flavour thresholds)
- LQCD provides control over non-perturbative effects of strong interaction — alternative with less experimental input
$\Rightarrow$ combination of pQCD and LQCD: confrontation with experiment, precision test of the Standard Model

Low momentum expansion of Π in pQCD

- hadronic contributions to vacuum polarisation functions from charmed currents

$$q^2\,\Pi^p = i\int d^4x e^{iqx}\langle 0|\,T\{J^p(x)\,J^p(0)\}\,|0\rangle$$

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)\Pi^\delta + q_\mu q_\nu \Pi_L^\delta = i\int d^4x\,e^{iqx}\langle 0|\,T\{J_\mu^\delta(x)\,J_\nu^\delta(0)\}|0\rangle\,,$$

with $\delta = v, a$ $J^p = \bar\psi\gamma_5\psi$, $J_\mu^v = \bar\psi\gamma_\mu\psi$ and $J_\mu^a = \bar\psi\gamma_\mu\gamma_5\psi$

- low momentum region: expansion of $\Pi^{p,\,\delta}$ in $z = \frac{q^2}{4m_c^2(\mu)}$ in $\overline{MS}$ scheme

$$\Pi^{p,\delta}(q^2) = \frac{3}{16\pi^2}\sum_{k\geq -1}\bar C_k^{p,\delta} z^k\,,\quad \bar C_k = \sum_{m\geq 0}\left(\frac{\alpha_s}{\pi}\right)^m \bar C_k^{(m)}\left(\log\left(\frac{m_c^2(\mu)}{\mu^2}\right)\right)$$

- coefficients for (axial) vector and (pseudo-)scalar correlator available up to third order in α_s

Moments of current correlators in LQCD

- renormalised moments from charmed currents at zero spatial momentum:

$$C^{p,\delta}(t) = a^6 Z^2_{p,\delta} \sum_{\vec{x}} \langle J_c^{p,\delta}(\vec{x},t) J_c^{p,\delta}(\vec{0},0)\rangle$$

$$G_n^{p,\delta} = \sum_{t/a=-N_t/2+1}^{N_t/2-1} \left(\frac{t}{a}\right)^n C^{p,\delta}(t)\,,$$

- dimensional analysis implies

$$G_n^{p,\delta} = \frac{g_n^{p,\delta}(\alpha_s(\mu), m_c(\mu)/\mu)}{(am_c(\mu))^{n-2}} + \mathcal{O}((a\,m_c)^m)\,,$$

with $n = 2, 4, \ldots$

Matching pQCD and LQCD results

$$R_n = \begin{cases} G_2/G_2^{(0)} & \text{for } n=2 \\ \left(G_n/G_n^{(0)}\right)^{1/(n-2)} \left(\frac{am_{\eta_c}}{2am_{pole\ c}^{(0)}}\right) & \text{for } n \geq 4 \end{cases}$$

implying the relation to continuum quantities in the $\overline{MS}$ scheme

$$R_n = \begin{cases} r_2(\alpha_s, m_c/\mu) + \mathcal{O}((am_c)^m \alpha_s) & \text{for } n=2 \\ r_n(\alpha_s, m_c/\mu)\frac{m_{\eta_c}}{2\, m_c(\mu)} + \mathcal{O}((am_c)^m \alpha_s) & \text{for } n \geq 4 \end{cases}$$

r_n are related to coefficients of the polarisation functions $\Pi(q^2)$ via

$$r_{2k+2} = \left(\frac{\bar{C}_k}{\bar{C}_k^{(0)}}\right)^{\frac{1}{2k}} \quad \text{as 3rd degree polynomial in } \alpha_s$$

Matching pQCD and LQCD results

- obtain $m_c(\mu)$ as solution of equations:

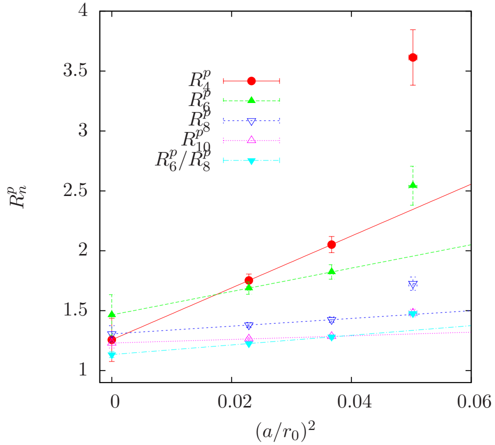
$$m_c(\mu) = \frac{m_{\eta_c}^{exp}}{2} \frac{r_n(\alpha_s(\mu), m_c(\mu)/\mu)}{R_n} \quad \text{for } n \geq 4$$

  with α_s fixed
- defining equations for α_s as solution of:

$$R_2 = r_2(\alpha_s(\mu), m_c(\mu)/\mu)$$

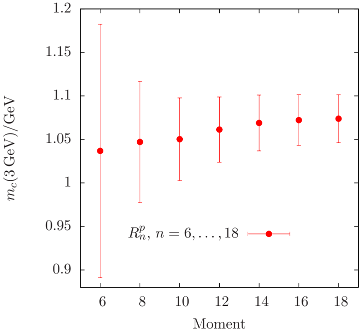
  for fixed $m_c(\mu)$
- scale μ corresponds to meson mass $m_{\eta_c} \approx 3\,\text{GeV}$

Preliminary results: continuum extrapolation of R_n^p



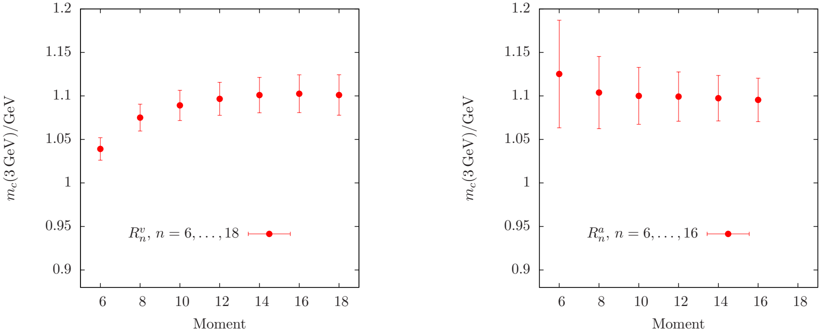
additional lattice spacing required: $(a/r_0)^2 = 0.015$ (available from ETMC)

Preliminary results: m_c from R^p



- fig. shows $m_c(\mu = 3\,\text{GeV})$ obtained from matching R_n^p to pQCD expansion for $n = 6, \ldots, 18$
- strong coupling set to $\alpha_{\overline{MS}}(n_f = 4, \mu = 3\,\text{GeV}) = 0.252(10)$ to extract m_c
- central values approx. 10% too large

Preliminary results m_c from R^v, R^a



Outlook

- moments from pseudoscalar, vector and axial vector currents give compatible results for m_c, but at present $\approx 10\%$ too large values and with large uncertainties
- check of continuum extrapolation with additional lattice spacing
- use advanced chiral perturbation theory motivated (combined) extrapolation $(a,\ m_{u,\,d}) \to (0,\ 0)$
- neglected vacuum polarisation effects due to *strange* and *charm* quark
- avoid renormalisation by other choices of current operators (e.g. scalar current) and ratios of moments
- analysis for α_s

Outline

1. Part I: Determination of α_s and m_c from lattice QCD

2. Part II: Hadronic contribution to muon $g-2$

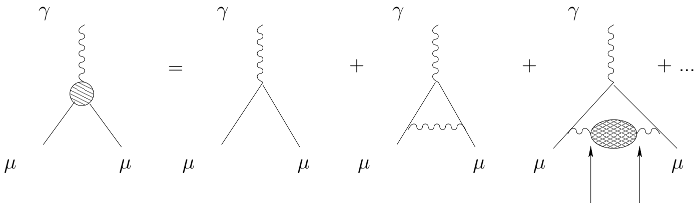
Motivation

- central object: muon's anomalous magnetic moment $a_\mu = \frac{g-2}{2}$
- measured and calculated with high precision

$$a_\mu^{exp} = 11659208.0(6.3) \times 10^{-10} \quad (0.54\text{ppm})$$
$$a_\mu^{th} = 11659179.3(6.8) \times 10^{-10} \quad (0.58\text{ppm})$$

- discrepancy between theory and experiment of $28.7(9.3) \times 10^{-10}$ or 3.1σ
- a_μ sensitive to physics beyond the SM: quantum fluctuations $\delta a_\mu \sim m_\mu^2/M_{NP}^2$ $(M_{NP} \gg m_\mu)$
- dominant part of theoretical uncertainty from $\mathcal{O}(\alpha^2)$ (LO) hadronic contributions
  $a_\mu^{had} = 692.1(5.6) \times 10^{-10}$
- pQCD: experimental input for low momentum and resonance region vs. LQCD: first principles

Continuum setup



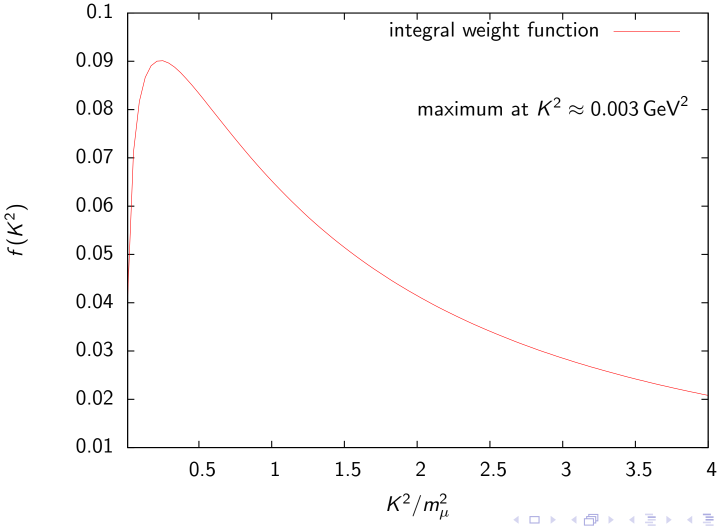
$$\Pi_{\mu\nu} = \langle J_\mu^{had} J_\nu^{had} \rangle$$

after loop integration:

$$a_\mu^{had} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dK^2}{K^2} f(K^2) \left(\Pi(K^2) - \Pi(0)\right)$$

- Π logarithmically divergent as $K^2 \to 0$, renormalized by subtraction
- $f(K^2)$ peaks in the region $K^2 \approx m_\mu^2 \Rightarrow$ dominant contribution from low momentum region

Continuum setup



Lattice implementation

- $\Pi_{\mu\nu}$ on the lattice from conserved electromagnetic current correlators
- point-split flavour-diagonal currents satisfy exact lattice Ward identity (at non-zero lattice spacing)

$$\nabla^*_\mu \langle J_\mu(x) J_\nu(y)\rangle_f = (\delta_{x-y-\nu} - \delta_{x-y}) C_\nu(y)$$

$$\hat{q}_\mu \Pi_{\mu\nu}(\hat{q}) = 0\,; \qquad \hat{q}_\mu = \frac{2}{a}\sin(a q_\mu/2)\,; \quad q_\mu = \frac{2\pi n_\mu}{L_\mu}$$

$$\Pi_{\mu\nu}(\hat{q}) = \left(\hat{q}_\mu \hat{q}_\nu - \delta_{\mu\nu}\hat{q}^2\right)\Pi(\hat{q}^2)$$

- no renormalization factor needed for conserved current

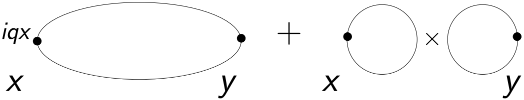
Lattice implementation

- leading corrections to $\mathrm{Re}\,(\Pi)$ automatically $\mathcal{O}(a^2)$ due to twisted mass formalism
- extrapolation $\hat{q}^2 \to 0$ crucial but smallest momentum accessible $\hat{q}^2 = (2/a\sin(\pi/T))^2 \approx 0.05\,\mathrm{GeV}^2$ (peak of weight function near $0.003\,\mathrm{GeV}^2$)
  $\Rightarrow$ theoretically sound fitting function required to model behaviour as $\hat{q}^2 \to 0$,
  e.g. motivated by chiral perturbation theory with photons

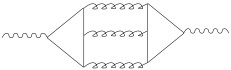
Lattice implementation

2 types of contributions to $\Pi_{\mu\nu}$: quark-connected and quark-disconnected

$$\Pi_{\mu\nu}(q) = \sum_x e^{iqx}$$

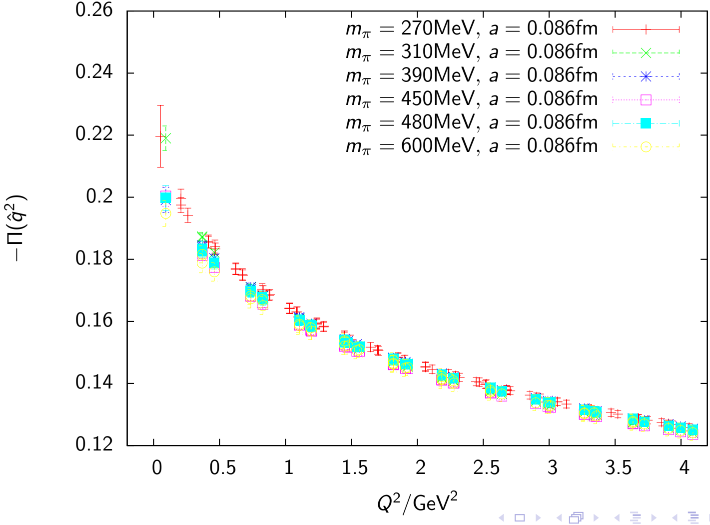


- quark-connected part computationally straightforward
- quark-disconnected part more demanding, but according to pQCD $\mathcal{O}(\alpha_s^3)$



$\Rightarrow$ neglected in previous calculations

Pion mass dependence of Π



Outlook Part II

- connected and disconnected part of Π can be measured for various pion masses and lattice spacings and volumes
- improved analysis of lattice artifacts based on hyper-cubic symmetry
- extraction of $\Pi(0)$ using different extrapolation methods (polynomial, χPT with photons)
- integration to extract a_μ^{had}
- extrapolations in lattice spacing and the pion mass

Thank you very much for your attention.