Fundamental parameters of QCD from non-perturbative methods

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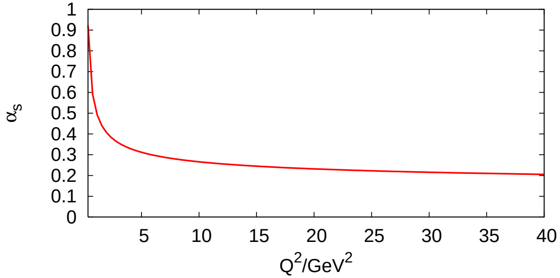
Quantum Chromodynamics

- Theory of strong interaction
- Two extremal regimes:
  - low energy: $\longrightarrow$ quarks confined into hadrons
  - high energy $\longrightarrow$ quarks essentially free: asymptotic freedom

QCD

→ At low energies: $\alpha_s \not\ll 1$

→ Perturbation theory methods fail



- Non-perturbative methods needed: lattice QCD

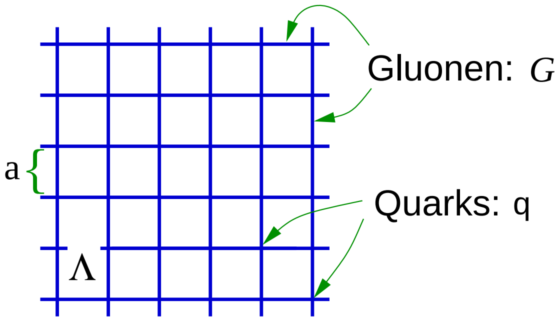
Lattice QCD

- Free fermion action

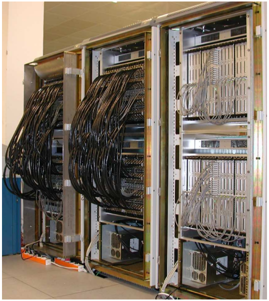
$$S_f[\psi, \bar{\psi}] = \int d^4x \; \bar{\psi}(x)[\gamma_\mu \partial_\mu + m]\psi(x)$$

- Discretization prescription:
  - x $\longrightarrow$ lattice
  - $\psi(x), \bar{\psi}(x) \;\longrightarrow\; \psi(n), \bar{\psi}(n)$
  - $\int d^4x \;\ldots \longrightarrow a^4 \sum_n \ldots$
  - $\partial_\mu \; \psi(x) \longrightarrow \frac{\psi(n+\hat{\mu}) - \psi(n-\hat{\mu})}{2a} + \mathcal{O}(a^2)$

QCD on a space-time lattice



Lattice simulations



- typical lattice sizes: ~ 3 *fm*
- 32ˆ3 *x* 64 lattice $\longrightarrow$ 2100000 points
- lattice spacings a: $0.05 - 0.1$ *fm*
- advanced algorithms
- large computer resources

Renormalization

- Lattice formulation: $\Lambda_{cut} \sim \frac{1}{a}$
- $\Lambda_{cut} \to \infty \implies a \to 0$
- Fundamental parameters of QCD:
  - quark masses: $m_i, \quad i = u, d, \dots$
  - Λ_{QCD} - parameter
- Finite a: bare masses, bare couplings, bare fields

  $\rightarrow$ HAVE TO BE RENORMALIZED!

Nonperturbative renormalization of QCD

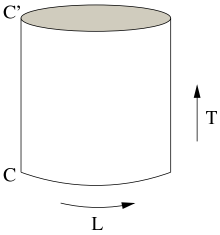
- Connection between the low energy sector and the perturbative regime
- Hadronic matrix elements of operators (high energies: $\bar{MS}$ scheme of dimensional regularization)
- Compute renormalization factors without directly relying on perturbation theory
  - Match the low energy sector with an intermediate NON-PERTURBATIVE renormalization scheme
  - Pass to the perturbative scheme ($\bar{MS}$) in the high energy region

Nonperturbative renormalization of QCD

- Conditions to be satisfied:
  - Compute $\alpha(\mu)$ at energy scales of $\mu \gtrsim 10 GeV$
    $\longrightarrow$ controlled connection to the perturbative regime!
  - Keep μ removed from the lattice cutoff $\frac{1}{a}$
    $\longrightarrow$ to avoid large discretization effects
  - Keep the box size L large compared to the confinement scale
    $\longrightarrow$ to avoid finite size effects in the simulations
- Summary: $L \gg \frac{1}{0.14 GeV} \gg \frac{1}{\mu} \sim \frac{1}{10 GeV} \gg a$
- Outcome: lattice $N \equiv \frac{L}{a} \gg 70$
- Possible to compute: lattices max $N \equiv \frac{L}{a} \sim 70$

Nonperturbative renormalization of QCD

- Solution: $\mu \equiv \frac{1}{L}$
- Finite size effect: physical observable
- Schrödinger Functional scheme:
  - QCD on a space-time cylinder L^3 x T
  - periodic b.c. in spatial direction
  - fixed (Dirichlet) b.c. in time direction
- [Lüscher, Weisz, Wolff], ALPHA Collaboration



General strategy

$$
\begin{array}{lcccc}
L_{\max} = \mathrm{O}(\tfrac{1}{2}\mathrm{fm}): & & \mathrm{HS} & \longrightarrow & \mathrm{SF}(q = 1/L_{\max}) \\
 & & & & \downarrow \\
 & & & & \mathrm{SF}(q = 2/L_{\max}) \\
 & & & & \downarrow \\
 & & & & \bullet \\
 & & & & \bullet \\
 & & & & \bullet \\
 & & & & \downarrow \\
 & & & & \mathrm{SF}(q = 2^n/L_{\max}) \\
 & & & & \mathrm{PT:} \quad \downarrow \\
 & \mathrm{jet-physics} & & \stackrel{\mathrm{PT}}{\longleftarrow} & \Lambda_{\mathrm{QCD}}, M
\end{array}
$$

Nonperturbative renormalization of QCD

- step scaling function:

$$\sigma(u) = \bar{g}^2(2L)|_{u=\bar{g}^2(2L),m_i=0}$$

- Describes a finite jump in the scale evolution (here: $L \rightarrow 2L$)
- Discrete form of β function
- Recursive construction of the running coupling at discrete length scale: $u_k = \bar{g}^2(2^{-k}L_{max})$
- Lattice effects of order a: extrapolated away by repeating the calculation for several values of $\frac{L}{a}$

Nonperturbative renormalization of QCD

Running of the coupling/mass, $N_f = 2$

[Della Morte et al (ALPHA Collab.),2004]

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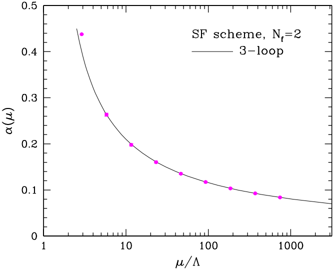

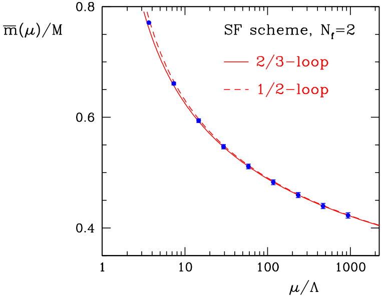

Summary:

- Lattice QCD: determination of fundamental QCD parameters from low energy hadron data
- Non-perturbative renormalization
- Schrödinger Funktional scheme
- Well suitable for numerical calculations
- Results obtained for: $N_f = 0$, $N_f = 2$

Outlook:

- apeNEXT/APEmille $\longrightarrow$ PC cluster, Blue Gene ...
- Extensions:
  - Include s, c quarks ($N_f = 4$)
  - Algorithmic improvements: solvers with deflation etc.
- Higher precission data