Two loop corrections for single top quark production

Mohammad Assadsolimani

HUMBOLDT-UNIVERSITÄT ZU BERLIN

15. October 2010

Contents

Introduction

single top quark production enables us

- *to study the nature of the weak interaction*
- *direct measurement of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element* V_{tb}

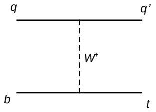
$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

unitarity and three family $\Rightarrow |V|_{tb}$

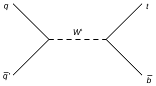
Introduction

The three main hadronic production modes for single top quark in the standard model:

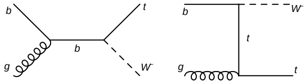
- *t channel*



- *s channel*



- *associated tW production*



Introduction

Predictions for single top-quark production cross sections at the Tevatron and the LHC. [Werner Bernreuther, arXiv:0805.1333v1]

cross section	*t channel*	*s channel*	*tW mode*
$\sigma^t_{\mathrm{Tevatron}}$	$1.15 \pm 0.07\ pb$	$0.54 \pm 0.04\ pb$	$0.14 \pm 0.03\ pb$
σ^t_{LHC}	$150 \pm 6\ pb$	$7.8 \pm 0.7\ pb$	$44 \pm 5\ pb$

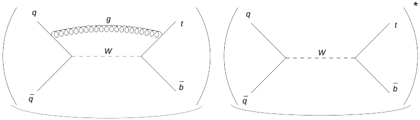
Introduction

Predictions for single top-quark production cross sections at the Tevatron and the LHC.[Werner Bernreuther, arXiv:0805.1333v1]

cross section	t channel	s channel	tW mode
$\sigma^t_{\mathrm{Tevatron}}$	$1.15 \pm 0.07\ pb$	$0.54 \pm 0.04\ pb$	$0.14 \pm 0.03\ pb$
σ^t_{LHC}	$150 \pm 6\ pb$	$7.8 \pm 0.7\ pb$	$44 \pm 5\ pb$

- *scale uncertainties $\Rightarrow$ errors*

The contributions of some diagrams like



$= 0$

$\neq 0$ $\neq 0$

disappear at NLO due to color.

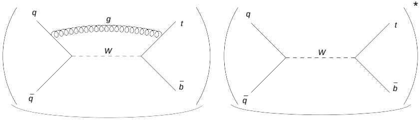
Introduction

Predictions for single top-quark production cross sections at the Tevatron and the LHC. [Werner Bernreuther, arXiv:0805.1333v1]

cross section	t channel	s channel	tW mode
$\sigma^t_{\mathrm{Tevatron}}$	$1.15 \pm 0.07\ pb$	$0.54 \pm 0.04\ pb$	$0.14 \pm 0.03\ pb$
σ^t_{LHC}	$150 \pm 6\ pb$	$7.8 \pm 0.7\ pb$	$44 \pm 5\ pb$

- *scale uncertainties $\Rightarrow$ errors*

The contributions of some diagrams like



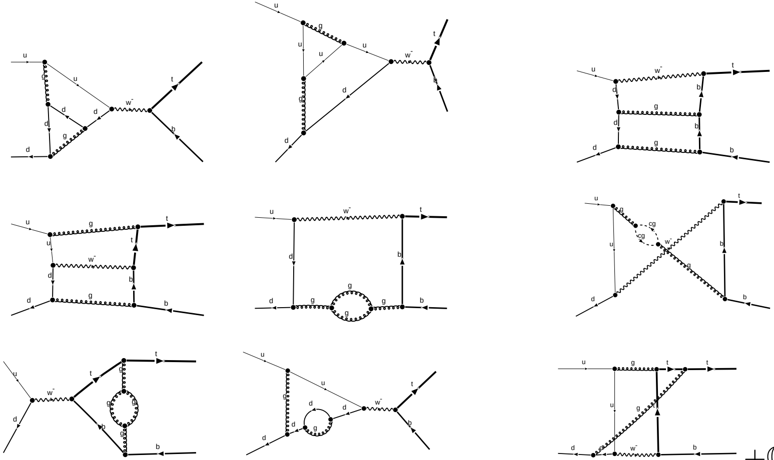
$= 0$

$\neq 0 \qquad \neq 0$

disappear at NLO due to color.

$\Rightarrow$ *NNLO calculation*

two-loop corrections to s channel



$+\mathbb{O}(100)$

Figure: some two loops corrections to s channel

Tensor reduction

In this corrections occur tensor integrals like

$$F(d,1,1,1,1,1,1,1)[k_1^\mu k_2^\nu] = \int d^d k_1 \int d^d k_2$$

$$\frac{k_1^\mu}{((-k_1)^2)^1((-k_2)^2)^1((p_1-k_1)^2)^1((-p_2+k_2)^2)^1((q_1-k_1)^2-m_t^2)^1}$$

$$\frac{k_2^\nu}{((p_1-k_1+k_2)^2-m_w^2)^1((p_1-q_2-k_1+k_2)^2-m_t^2)^1} \qquad (1)$$

Tensor reduction by Schwinger parametrization

$$F(d,\nu_1,\cdots,\nu_n) = \int d^d k_1 \int d^d k_2 \frac{1}{Q_1^{\nu_1}\cdots Q_n^{\nu_n}}$$

$$= \int d^d k_1 \int d^d k_2 \underbrace{(\prod_{i=1} \frac{1}{\Gamma(\nu_i)} \int_0^\infty dx_i x_i^{\nu_i-1})}_{=\int Dx} exp(\sum x_i k_i^2)$$

Tensor reduction

with the following substitutions

$$k_1^\mu \to l_1^\mu - \frac{-x_7}{a} l_2^\mu + X^\mu \qquad k_2^\mu \to l_2^\mu + Y^\mu$$

$$F(d,\nu_1,\cdots,\nu_n) = \int Dx \int d^d l_1 \int d^d l_2 exp\Big(a l_1^2 + \frac{p}{a} l_2^2 + \frac{Q}{p}\Big)$$

where

$$X^\mu = \frac{x_3 e^\mu - x_2 f^\mu}{p} \qquad Y^\mu = \frac{x_3 f^\mu - x_1 e^\mu}{p} \qquad p = ab + x_7^2$$

and

$$f^\mu = x_2 p_1^\mu + x_3 p_{12}^\mu + x_4 p_{123}^\mu \qquad e^\mu = x_5 p_{123}^\mu$$

$$a = x_1 + x_2 + x_3 + x_4 + x_7 \qquad b = x_5 + x_6 + x_7$$

$(\frac{1}{p})$ *acts as dimension shifter*

x_i *increases the power of propagator* i

Two-loop corrections

After tensor reduction :

$$F(d,1,1,1,1,1,1,1)[k_1^\mu k_2^\nu] \rightarrow \Big(F(2+d,1,1,2,1,1,3,1)[1]$$
$$+F(2+d,1,1,2,1,1,3,2)[1] \quad + \quad F(2+d,1,1,2,1,2,1,2)[1]$$
$$+F(2+d,1,1,2,1,2,1,3)[1] \quad + \quad F(2+d,1,1,2,1,2,2,1)[1] + \cdots \Big)$$

Tensor reduction $\Rightarrow$ *various scalar integrals with the same structure of the integrand with different distributions of powers of propagators.*

Integration by parts

Possible approaches

Integration by parts

Possible approaches

- *to solve each integral individual,*

Integration by parts

Possible approaches

- *to solve each integral individual,*
- *try to express all scalar integrals as a linear combination of some basic master integrals.*

Integration by parts (IBP)[Chetyrkin, Tkachov (1981)]

Integration by parts

Possible approaches

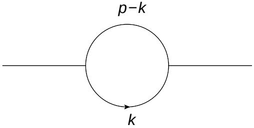
- *to solve each integral individual,*
- *try to express all scalar integrals as a linear combination of some basic master integrals.*

Integration by parts (IBP)[Chetyrkin, Tkachov (1981)]

$$\int d^d k_1 \cdots \int d^d k_l \frac{\partial}{\partial k_i^\mu} \frac{\{k_j^\mu, p_j\}}{p_1^{a_1} \cdots p_n^{a_n}} = 0$$

$\Rightarrow$ *relations between integrals*

Example



$$F(a_1, a_2) = \int \frac{d^d k}{(k^2)^{a_1}((p-k)^2)^{a_2}}$$

using,

$$\int \frac{\partial}{\partial k} k \frac{1}{(k^2)^{a_1}((p-k)^2)^{a_2}} = 0$$

and we obtain the following relation

$$[d - 2a_1 - a_2]F(a_1, a_2) - a_2 F(a_1 - 1, a_2 + 1) + a_2 p^2 F(a_1, a_2 + 1) = 0$$

Compact notation: $F(a_1 - 1, a_2 + 1) = 2^+ 1^- F(a_1, a_2)$.

Integration by parts

Setting $a_2 = 1$

$$F(a_1, 1) = -\frac{d - a_1 - 1}{(a_1 - 1)p^2} F(a_1 - 1, 1).$$

Integration by parts

Setting $a_2 = 1$

$$F(a_1, 1) = -\frac{d - a_1 - 1}{(a_1 - 1)p^2} F(a_1 - 1, 1).$$

$\Rightarrow$ *reducing the index* a_1 *to one*

$\Rightarrow$ *expressing any integral of the given family in terms of the only master integral* $I_1 = F(1, 1)$.

An efficient approache to solve IBP relations:Laporta's algorithm. In this algorithm

- *Introduce ordering of the integrals to reduce more difficult integrals to simpler ones*
- *evaluation for numeric indexes (not algebraic)*
- *solve systematically for the difficult integrals by a Gauss elimination-like algorithm*

Summary

- *NLO error estimate likely to be unreliable, new color structure in NNLO may give significant contributions to the calculation of cross section*
- *In this calculation occur tensor integrals, which are reduced to scalar integrals. For the evaluation of these scalar integrals we apply an algorithm named Laporta's algorithm.*