# On the Gluon Propagator at Finite Temperature Lattice QCD

## SU(3) Pure Gauge Investigation

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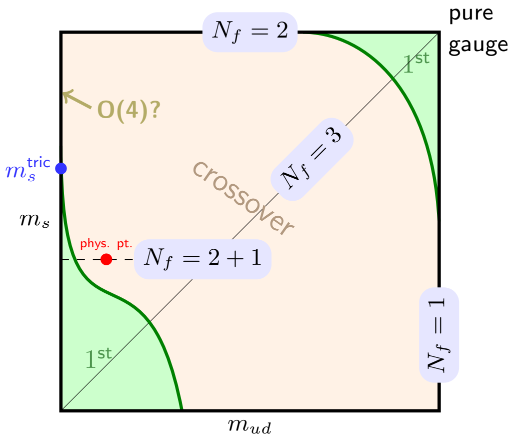
# Why QCD at Finite Temperature?

- The early universe under extreme conditions (density and temperature)
- Quark gluon plasma is produced at high energies in heavy ion collisions
- The equation of state
- Identify the nature of the phase transition and establish the phase diagram

# Nature of the Phase Transition of QCD




- Investigate the thermal transition by means of LQCD
- Here: $N_f = 0$ and vanishing chemical potential
- Expect a first order transition

# Motivation of this work

- Tiny signal of the crossover $\Longrightarrow$ intensive computational efforts to reach the realistic case (physical quark masses)
- The need of more observables sensitive to the phase transitions using LQCD
- Interest to the gluon propagator as input for the Dyson-Schwinger equations (DSE) $\rightarrow$ construction of new observables

  [C. Fischer, A. Maas, J.A. Müller, Eur. Phys. J. C(2010) 68:165-181]
- Study of the sensitivity of the gluon propagator in pure gauge theory at finite T (as we will see) $\Longrightarrow$ maybe the construction of a "new" order parameter $\Longrightarrow$ possibility to move from the confining regime to the perturbative one
- Study of the systematic lattice effects $\rightarrow$ finite volume, small lattice size and gauge fixing effects

# Brief Introduction to Lattice Wilson Formulation

- Periodic space-time lattice with lattice spacing $a$, cf. Ulli Wolf's talk
- Temperature dependence from $T \equiv 1/a \cdot n_\tau, \quad n_\tau \ll n_\sigma$
- Gauge field defined as link variables:

$$U_{x,\mu} = e^{ig_0 a A_\mu (x+\hat{\mu}/2)}$$

  where: $A(x)$ is the $SU(3)$ gauge field (the gluon field)
- Discretised gauge action for pure gauge theory ($N_c = 3$ for $SU(3)$ and $N_f = 0$)
- In the continuum limit, the Wilson action reduces to $SU(3)$ Yang Mills action $S_{YM}$

$$\begin{aligned} S_G(U) &= \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{N_c} \,\mathfrak{Re}\,\mathrm{tr}\, U_{x,\mu\nu}\right), \\ U_{x,\mu\nu} &\equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}, \qquad \beta \equiv 2N_c/g_0^2 \end{aligned}$$

$$S_{YM} = \int_0^\beta d\tau \int d^3x (-\frac{1}{4} tr F^a_{\mu\nu} F^{\mu\nu}_a),\; F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_0 f_{abc} A^b_\mu A^c_\nu$$

# Symmetries and Implications

- $S_G(U)$ invariant under periodic $SU(3)$ transformations and also global $Z(3)$ center symmetry:
  - $A(\vec{x},\tau) \longrightarrow s(\vec{x},\tau)(A_\mu(\vec{x},\tau) + i\partial_\mu)s(\vec{x},\tau)^\dagger$
  - $s(\vec{x},\beta) = s(\vec{x},0)$ , with , $s(\vec{x},\beta) = s(\vec{x},0)$, and, $s(\vec{x},\tau) \in SU(3)$

    $s(\vec{x},\beta) = zs(\vec{x},0)$ with $z \in Z(3)$
- Order parameter of the $Z(3)$ symmetry: Polyakov loop

$$< L >= \frac{1}{n_\sigma^3} \sum_{\vec{x}} L(\vec{x}),\ L(\vec{x}) = P\exp(ig_0 \int_0^{\frac{1}{T}} A_4(\vec{x},\tau)d\tau)$$

- Spontaneous breakdown of $Z(3)$ symmetry at $T > T_c \Longrightarrow$ signals deconfinement$\Longrightarrow < L >\neq 0$
- A good order parameter for Pure gauge theory (but not anymore in presence of fermions)$\Rightarrow$ looking for better order parameters
- Determination of $T_c \rightarrow$ use of the susceptibility: $\chi = n_\sigma^3(< L^2 > - < |L| >^2)$

# Finite T Gluon Propagator in Landau Gauge

- At zero temperature, the gluon propagator in momentum space:

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(k)\widetilde{A}^b_\nu(-k)\right\rangle \equiv \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu\, q_\nu}{q^2}\right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{L_\mu}\right), \qquad k_\mu \in (-L_\mu/2, L_\mu/2]$$

- At finite temperature, the gluon propagator $D(p)$ splits up into two structures:
    - Transverse propagator:
      $D_T \sim \langle\ \sum_{i=1}^{3} A^a_i(q)\ A^a_i(-q) - \frac{q_4^2}{\vec{q}^2}\ A^a_4(q)\ A^a_4(-q)\ \rangle$
    - Longitudinal propagator:
      $D_L \sim (1 + \frac{q_4^2}{\vec{q}^2})\ \langle\ A^a_4(q)\ A^a_4(-q)\ \rangle$
    - $q_4$ called the Matsubara frequency
    - Understand the behaviour of $D_L$ and $D_T$ with the temperature$\longrightarrow$expected different behaviour for $D_L$ and $D_T$

[C. Fischer, A. Maas, J.A. Müller, Eur. Phys. J. C(2010) 68:165-181],[A. Maas, A. Cucchieri, T. Mendez, hep-lat/0610006, A. Maas, Chin.J.Phys.34:1328-1330,2010]

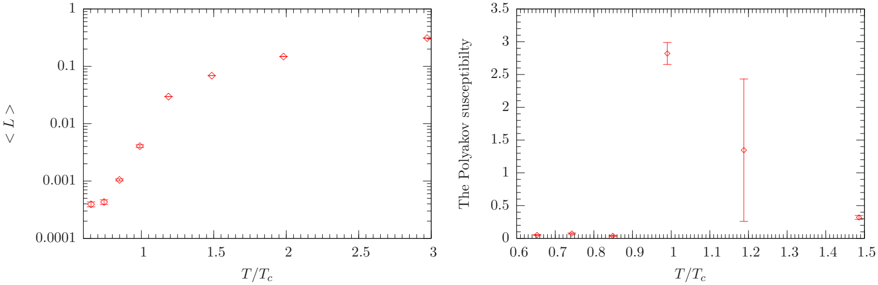
# Setup of the Simulations

- Temperature vs. volume
  We study quenched QCD (pure gauge QCD), $48^3 \times n_\tau$, $n_\tau = 4, 6, \ldots, 18$ varies
  Varying $n_\tau \Leftrightarrow$ varying the temperature
  the lattice spacing $a = a(\beta = 6.337) = 0.283 GeV^{-1}$ fixed,
  $T_c \longleftrightarrow n_\tau = 12$
- Gauge fixing iteratively to the Landau gauge
- Equivalent to minimize the gauge functional:

$$F_U(g) = \sum_{x,\mu} \left( 1 - \frac{1}{N_c} \, \mathfrak{Re} \, \mathrm{tr} \, U^g_{x\mu} \right)$$

$$|(\partial \mathcal{A})_x| = | \sum_{\mu=1}^{4} \left( \mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu} \right) | < \epsilon \quad \text{for all} \quad x$$

# Polyakov Loop and its Susceptibility (Preliminary Results)



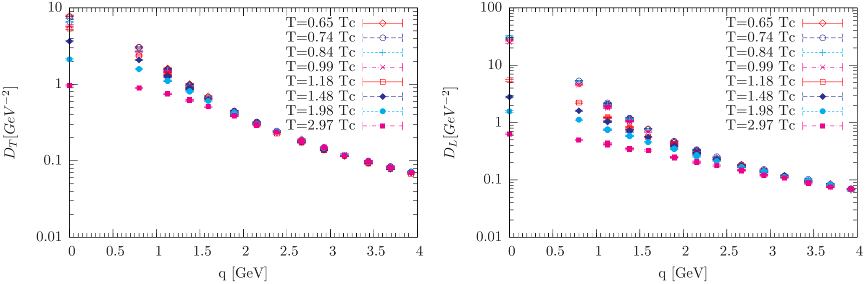

(a) The Polyakov loop (b) The susceptibility

Figure: Check of phase transition temperature in $SU(3)$ pure gauge theory

- Rise of the Polyakov loop around the transition temperature
- The susceptibility $\chi$ peaks around the phase transition temperature as expected

# Temperature Behaviour of Transverse and Longitudinal Propagators

- preselect only (nearly) diagonal momenta $\Longrightarrow$ to reduce the lattice artifacts
- The renormalisation point: 5 GEV , the Matsubara frequency $q_4 = 0$




(a) The transverse propagtor (b) The longitudinal propagator

Figure: Temperature behaviour of the transverse and longitudinal propagators

- Big jump in the values of $D_L$ when crossing $T_c$ $\Longrightarrow$ high sensitivity of $D_L$ to the phase transition
- $D_L$ $\rightarrow$ the same behaviour as an order parameter
- $D_T$ less sensitive

# Temperature Behaviour of the Transverse and Longitudinal Propagators at Low momenta

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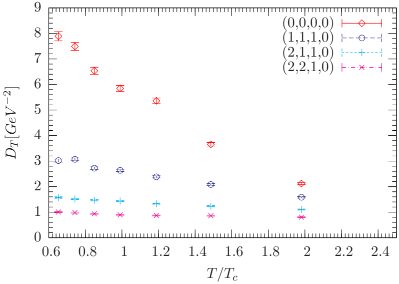
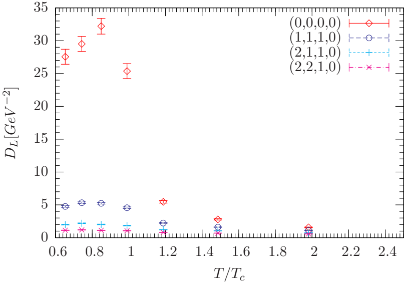



(a) The transverse propagtor




(b) The longitudinal propagator

Figure: Temperature behaviour of $D_T$ and $D_L$ at low momenta

- $D_L$ falls down stronger for lower momenta
- Clear sensitivity of $D_L \Longrightarrow$ possible connection between this behaviour and confinement
- $D_T$ behaves smoothly around $T_c$

# Summary/Outlook

- Results on the gluon propagator at finite T
  - Strong phase transition effect on the longitudinal gluon propagator around $T_c$
  - Less effect on the transverse gluon observed
- Outlook
  - Including fermions:Unquenched theory $(N_F = 2)$
  - Experimenting new order parameters and possible use of our gluon propagators
  - Study of finite volume and lattice spacing effects $\Longrightarrow$ scaling tests

# Danke für Ihre Aufmerksamkeit