Pseudoscalar decay constants from Wtm-Lattice-QCD with $\mathbf{N_f = 2 + 1 + 1}$ dynamical quark flavours

Andreas Nube[1]

[1]HU-Berlin & DESY-Zeuthen

Blockkurs Graduiertenkolleg Herbst 2011, 4. Oktober

Gliederung

Why?

- QCD widely accepted theory of strong interaction within SM
- verifications of QCD by comparison of **theoretically** calculated and **experimentally** measured quantities are of high interest
- simple mesonic observables(masses/decay constants) can be determined to high precision theoretically on the lattice and experimentally
- f_{D_s} is specifically interesting because of inconsistencies (theory/experiment) in the past

The former f_{D_s} puzzle

Experiment

f_{D_s} measured from $D_s \to l^+\nu$ decays & SM decay rate:

$$\Gamma(D_s^+ \to l^+\nu) = \frac{G_F^2}{8\pi} f_{D_s}^2 m_l^2 M_{D_s^+} \left(1 - \frac{m_l^2}{M_{D_s^+}^2}\right)^2 |V_{cs}|^2 \qquad (1)$$

CLEO-c 2009: $f_{D_s} = (\mathbf{259.7} \pm 7.8 \pm 3.4)$MeV (most precise)

Theory $N_f = 2 + 1$

theoretical definition in QCD from matrix element (w/o QED):

$$\langle 0|A_\mu|D_s^-(p)\rangle = f_{D_s} p_\mu \qquad (2)$$

HPQCD & UKQCD: $f_{D_s} = (\mathbf{241.0} \pm 3.0)$MeV (most precise 2007)

$\to$ **2.4σ Discrepancy**

HPQCD 2010: $f_{D_s} = (\mathbf{248.0} \pm 2.5)$MeV (most precise 2010)

$\to$ **1.6σ Discrepancy**

Gliederung

Motivation

- new simulation techniques have been developed enabling us to perform realistic lattice-QCD simulations with (u,d)- and (c,s)-doublet
- estimates of f_{D_s} are possible including the effect of a **dynamic** charm quark
- clarify agreement (discrepancy?) of f_{D_s} between theory and experiment
- estimates of $|V_{cs}|$ can be obtained from f_{D_s} and $\Gamma(D_s^{\pm} \longrightarrow l^{\pm}\nu)$
    - check CKM-matrix for unitarity violations (possible source of new physics)

Gliederung

$N_f = 2+1+1$ Wilson twisted mass lattice QCD

Fermion Action:

$$S_l^F = a^4 \sum_x \overline{\chi}^l(x)[D_W + m_0 + i\mu_l\gamma_5\tau^3]\chi^l(x) \quad (3)$$

$$S_h^F = a^4 \sum_x \overline{\chi}^h(x)[D_W + m_0 + i\mu_\sigma\gamma_5\tau^1 + \tau^3\mu_\delta]\chi^h(x) \quad (4)$$

Boson Action: Iwasaki gauge action (depending only on $\beta \approx \frac{1}{g_0^2}$)

Features:

- automatic $\mathcal{O}(a)$ improvement of masses and decay constants (and many other observables)
- computationally cheap ($\approx D^W$)
- strictly positive fermion determinant → easier simulation
- action breaks **parity** and **flavour** symmetry at $\mathcal{O}(a^2)$

$\mathcal{O}$(a) improvement

tree level lattice propagator of light doublet:

$$\tilde{G}(p) = \frac{-i\gamma_\mu \hat{p}_\mu + \mathcal{M}(p) - i\mu_l \gamma_5 \tau^3}{\hat{p}_\mu^2 + \mathcal{M}(p)^2 + \mu_l^2} \quad (5)$$

with $\mathcal{M} = m_q + \frac{r}{2} a\tilde{p}_\mu^2$, $\hat{p}_\mu = \frac{1}{a}\sin(ap_\mu)$ and $\tilde{p}_\mu = \frac{2}{a}\sin(\frac{ap_\mu}{2})$

small-a-scaling of denominator in (5):

$p^2 + m_q^2 + am_q rp^2 + \mu_l^2$

- $\mathcal{O}(am_q)$ -terms also present in relevant observables
- $\Rightarrow$ $\mathcal{O}(a)$ effects vanish if $m_q = 0$
- still non-zero quark mass for $\mu_l > 0$

Valence Action

problem of flavour mixing can be circumvented by using a different action in the valence sector:

$$S^F_{h,\text{valence}} = a^4 \sum_x \overline{\chi}^{\text{val}}(x) \left[D_W + m_0\right] \chi^{\text{val}}(x) \qquad (6)$$

$$+ \; a^4 \sum_x \overline{\chi}^{\text{val}}(x) i\gamma_5 \begin{pmatrix} \mu_1 & 0 \\ 0 & -\mu_2 \end{pmatrix} \chi^{\text{val}}(x) \qquad (7)$$

- no flavour mixing by definition
- μ_1 and μ_2 can be adjusted to in order to represent any meson

Valence Action

problem of flavour mixing can be circumvented by using a different action in the valence sector:

$$S^F_{h,\text{valence}} = a^4 \sum_x \overline{\chi}^{\text{val}}(x) \left[D_W + m_0 \right] \chi^{\text{val}}(x) \quad (6)$$

$$+ \; a^4 \sum_x \overline{\chi}^{\text{val}}(x) i\gamma_5 \begin{pmatrix} \mu_1 & 0 \\ 0 & -\mu_2 \end{pmatrix} \chi^{\text{val}}(x) \quad (7)$$

Meson	$\mu_1 =$	$\mu_2 =$
$\pi^\pm$	m_u	m_d
$K^\pm$	m_u	m_s
$D^\pm$	m_c	m_d
$D_s^\pm$	m_c	m_s

Masses and Decay Constants

can be determined from matrix elements $\langle \mathcal{A}^\dagger(x)\mathcal{B}(0)\rangle$

$$\mathcal{A}/\mathcal{B} = \begin{cases} P^a(\mu_1,\mu_2) & = & \overline{\chi}^{\text{val}}\gamma_5\frac{\tau^a}{2}\chi^{\text{val}} \\ A^a_\mu(\mu_1,\mu_2) & = & \overline{\chi}^{\text{val}}\gamma_\mu\gamma_5\frac{\tau^a}{2}\chi^{\text{val}} \end{cases} \tag{8}$$

PCAC: $\partial_\mu A^a_\mu = 2m_q P^a,\ a = 1,2$

$$\Rightarrow m_q^{pcac} = \frac{\sum_{\vec{x}}\langle \partial_0 A^a_0(x) P^a(0)\rangle}{2\sum_{\vec{x}}\langle P^a(x) P^a(0)\rangle} \quad , \quad \mu_{1,2} = \mu_l \tag{9}$$

Pseudoscalar meson masses $M_{(\mu_1,\mu_2)}$ and decay constants $f_{(\mu_1,\mu_2)}$ can be extracted from the correlator:

$$C(t) = \sum_{\vec{x}} \langle P^a(\mu_1,\mu_2)(t,\vec{x})^\dagger P^a(\mu_1,\mu_2)(0,0)\rangle \tag{10}$$

$$\sim f_{(\mu_1,\mu_2)} \times \cosh\left[M_{(\mu_1,\mu_2)}(t - T/2)\right] \tag{11}$$

(μ_1 & μ_2 chosen suitably)

Chiral Perturbation Theory (χPT)

Keep finite size effects under control $\Rightarrow$ Pion Compton wave length has to be smaller than spatial lattice extend L :
$m_\pi^{-1} \ll L \quad \Rightarrow \quad m_\pi^{\text{simulation}} > m_\pi^{\text{physical}}$
Solution: Chiral Perturbation theory provides extrapolation formulae for decay constants in dependence of the Pion mass

$$
\begin{aligned}
f_{\pi^\pm}(m_\pi) &= f_0\left(1 - 2\xi_{ll}\ln\xi_{ll} + b\xi_{ll}\right), \quad \xi_{ll} = \left(\frac{m_\pi}{4\pi f_0}\right)^2 \\
f_{K^\pm}(m_\pi) &= (f_0^{(K)} + f_m^{(K)}\xi_{ss}) \times \mid \quad \xi_{ss} = \left(\frac{M_{(m_s,m_s)}}{4\pi f_0}\right)^2 \\
&\times \; (1 - \frac{3}{4}\xi_{ll}\ln\xi_{ll} + (b_0^{(K)} + b_m^{(K)}\xi_{ss})\xi_{ll}),
\end{aligned}
$$

- perform simulations with several Pion masses
- fit the data to the formulae above
- perform extrapolation the “physical point” at which $m_\pi = 139.6 MeV$ and read off value for f_K

Gliederung

Tuning of Parameters

- **achieving** $\mathcal{O}(a)$**-improvement**:
  $m_q^{pcac} \longrightarrow 0$ by adjusting m_0
- scale a set by β

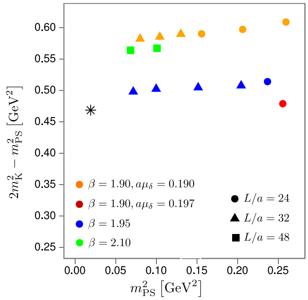
β	a
1.90	0.0861fm
1.95	0.0781fm
2.10	0.0607fm

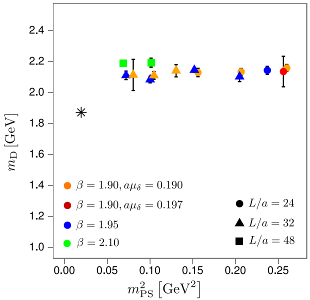
- Pion mass determined by μ_l (lightest pion mass $\approx 230 MeV$)
- adjust the Kaon and D-Meson mass to their experimental values by tuning μ_σ and μ_δ

Setup Parameters

Ensemble	β	$\kappa = \frac{1}{8+2m_q}$	$a\mu_q$	$a\mu_\sigma$	$a\mu_\delta$
A30.32	1.90	0.1632720	0.0030	0.150	0.190
A40.32	1.90	0.1632700	0.0040	0.150	0.190
A50.32	1.90	0.1632670	0.0050	0.150	0.190
A60.24	1.90	0.1632650	0.0060	0.150	0.190
A80.24	1.90	0.1632600	0.0080	0.150	0.190
A100.24	1.90	0.1632550	0.0100	0.150	0.190
A100.24s	1.90	0.1631960	0.0100	0.150	0.197
B35.32	1.95	0.1612400	0.0035	0.135	0.170
B55.32	1.95	0.1612360	0.0055	0.135	0.170
B75.32	1.95	0.1612320	0.0075	0.135	0.170
B85.24	1.95	0.1612312	0.0085	0.135	0.170
D15.48	2.10	0.1563610	0.0015	0.120	0.1385
D20.48	2.10	0.1563570	0.0020	0.120	0.1385
D30.48	2.10	0.1563550	0.0030	0.120	0.1385

Tuning to experimental Kaon and D-meson Mass





$K^{\pm}$ (strange quarkonium) tuning result

$D^{\pm}$ tuning result

Results of the Chiral f_K fit

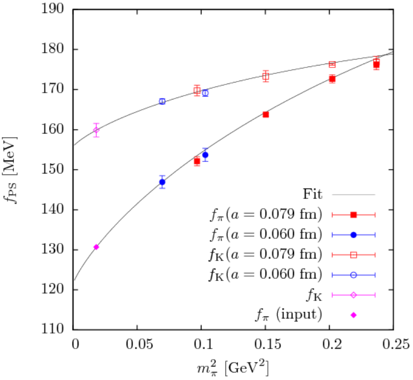


Figure: *chi*PT f_K & f_π fit result

preliminary results (only statistical errors):

- $f_K = 160(2)$MeV
- $f_K/f_\pi = 1.224(13)$
- $|V_{us}| = 0.220(2)$

conclusions

- first determination of f_K,f_D and f_{D_S} with dynamical up, down, strange and charm
- result for f_K is in agreement with $N_f = 2/N_f = 2+1$ determinations
- tuning has been performed, Kaon and D-meson masses slightly higher than their experimental value

outlook

- investigate $\mathcal{O}(am_c)$ discretisation effects
- study influence of dynamic strange and charm quark masses on $f_{K/D/D_S}$ and extend set of ensembles if necessary
- apply χPT formulae to f_D and f_{D_s}

Bibliography

- P. Naik *et al.* [CLEO Collaboration], "Measurement of the Pseudoscalar Decay Constant f_{D_s} Using $D_s^+ \to \tau^+\nu$, $\tau^+ \to \rho^+\overline{\nu}$ Decays," Phys. Rev. **D80** (2009) 112004. [arXiv:0910.3602 [hep-ex]].
- C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na, J. Shigemitsu, Phys. Rev. **D82** (2010) 114504. [arXiv:1008.4018 [hep-lat]].
- F. Farchioni, G. Herdoiza, K. Jansen, A. Nube, M. Petschlies, C. Urbach, PoS **LATTICE2010** (2010) 128. [arXiv:1012.0200 [hep-lat]].
- R. Baron, B. Blossier, P. Boucaud, J. Carbonell, A. Deuzeman, V. Drach, F. Farchioni, V. Gimenez *et al.*, PoS **LATTICE2010** (2010) 123. [arXiv:1101.0518 [hep-lat]].