On the Way towards an Automated Tool for Multileg NLO Computations

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AG Phenomenology of Elementary Particles

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Motivation

Signal for new physics requires precise understanding of QCD:

High energy pp-scattering gives rise to multi-leg parton amplitudes

- Automation at tree-level solved, e.g. MadGraph, HELAC,…
- Important goal: Automation of NLO gluon amplitudes with “many” legs

Why NLO amplitudes?

Tree-level approximation is the classical approximation i.e. no quantum corrections

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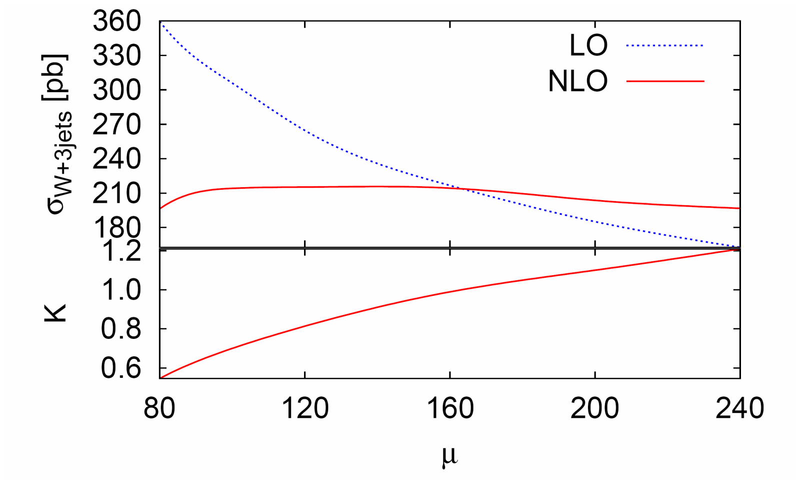
First quantum corrections at one-loop level



Inclusive W^++3 jet cross-section at the LHC and the K-factor defined as $K = \sigma_{NLO}/\sigma_{LO}$

One-loop Methods

[Passarino,Veltman1979]

Amplitude
= sum of all Feynman diagrams → Scalar one-loop integrals $\mathcal{I}_j$

$$\mathcal{A} = \sum_j c_j \mathcal{I}_j$$

One-loop Amplitude =
Determination of the coefficients c_j

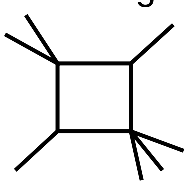
Scalar Integral Basis

Decomposition of an arbitrary one-loop amplitude

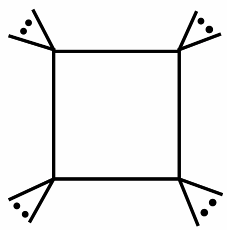
$$\mathcal{A}_N = \sum_{\{ijkl\}} d_{ijkl}\, \mathcal{I}^{(4)}_{ijkl} + \sum_{\{ijk\}} c_{ijk}\, \mathcal{I}^{(3)}_{ijk} + \sum_{\{ij\}} b_{ij}\, \mathcal{I}^{(2)}_{ij} + \sum_{\{i\}} a_i\, \mathcal{I}^{(1)}_{i}$$

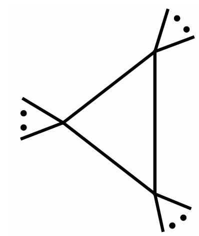
boxes | triangles | bubbles | tadpoles

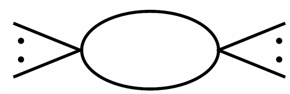
Indexing:

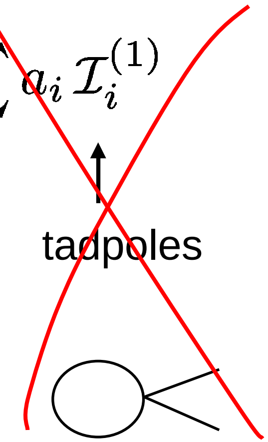


$\{ijkl\} = \{1347\}$









$$\mathcal{I}^{(4)}_{ijkl} = \int \mathrm{d}^d l \frac{1}{D_i D_j D_k D_l}$$

$$D_i = (p_i + l)^2 - m_i^2$$

No tadpoles in massless theories

Aim: get d, c and b

Scalar Integrand

Focus on the **integrand of the amplitudes**

[Ossola,Papadopoulos,Pittau2007]
[Ellis, Giele, Kunszt 2008]

$$\mathcal{A}_N = \int \mathrm{d}^4 l \, \mathcal{A}_N(l)$$

$$\mathcal{A}_N(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

Numerator: Loop-momentum independent part + spurious terms

$$\bar{d}(l) = d + \tilde{d}(l)$$

Loop-momentum independent part is the desired integral coefficient

$$\int \mathrm{d}^4 l \frac{d + \tilde{d}(l)}{D_i D_j D_k D_l} = d \int \mathrm{d}^4 l \frac{1}{D_i D_j D_k D_l} = d \, \mathcal{I}^{(4)}$$

Extraction of the Coefficients

Number of spurious terms is fixed and finite:

- Evaluate the integrand for different loop-momenta

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"system of equations"

↓

- Disentangle integral coefficients from spurious terms

Partial Fractioning the Integrand

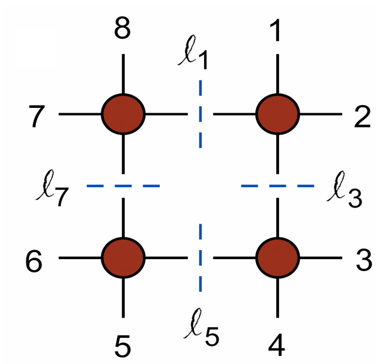
$$\mathcal{A}_N(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

- multiply with $D_i D_j D_k D_l$
- go on-shell: $D_i = D_j = D_k = D_l = 0$

$$\bar{d}_{ijkl}(l) = \mathcal{A}_N(l)\, D_i D_j D_k D_l$$

$$\equiv \mathcal{A}_1^{\text{tree}}(l)\mathcal{A}_2^{\text{tree}}(l)\mathcal{A}_3^{\text{tree}}(l)\mathcal{A}_4^{\text{tree}}(l)$$

$\equiv$ **Product of four tree amplitudes**



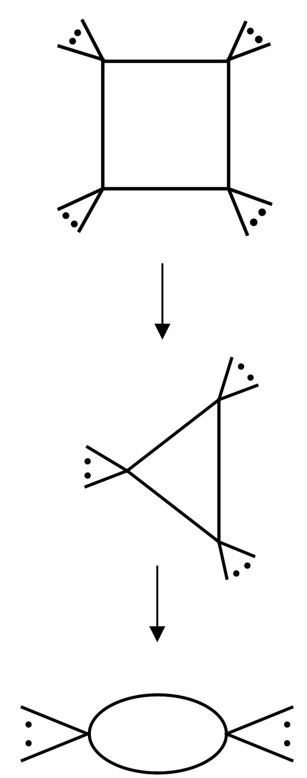
The loop-momentum must be constructed such that the propagators vanish (on-shell cut condition).

General Procedure

Boxes:

$$\bar{d}(l) = \mathcal{A}_1^{\rm tree}(l)\mathcal{A}_2^{\rm tree}(l)\mathcal{A}_3^{\rm tree}(l)\mathcal{A}_4^{\rm tree}(l)$$

$$\bar{d}(l) = d + \tilde{d}(l) \xrightarrow{\text{Disentanglement}} d$$



Triangles and bubbles: more involved

- boxes, triangles and bubbles have common cuts
  $\longrightarrow$ intricate subtraction procedure
- A lot of spurious terms

Triangles

$$\mathcal{A}_N(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

- multiply with $D_i D_j D_k$
- go on-shell: $D_i = D_j = D_k = 0$

$$\bar{c}_{ijk}(l) = \left[\mathcal{A}_N(l) - \sum_{l \neq \{i,j,k\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} \right] D_i D_j D_k$$

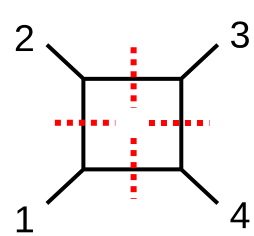
$$\equiv \mathcal{A}_1^{\text{tree}}(l) \mathcal{A}_2^{\text{tree}}(l) \mathcal{A}_3^{\text{tree}}(l) - \sum_{l \neq \{i,j,k\}} \frac{\bar{d}_{ijkl}(l)}{D_l}$$

$\equiv$ **Product of three tree amplitudes – box part**

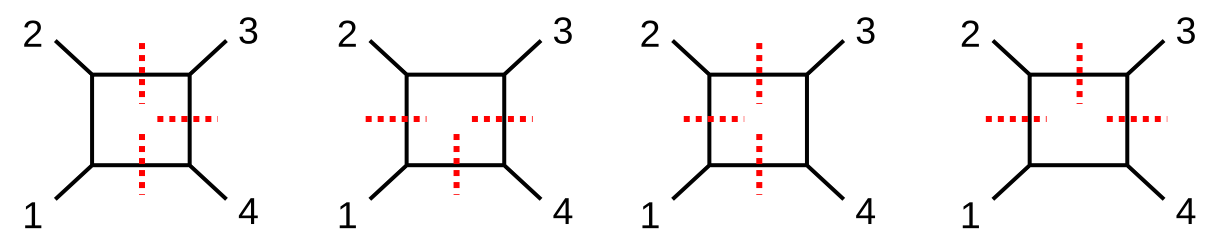
Boxes versus Triangles

Example with four external legs:

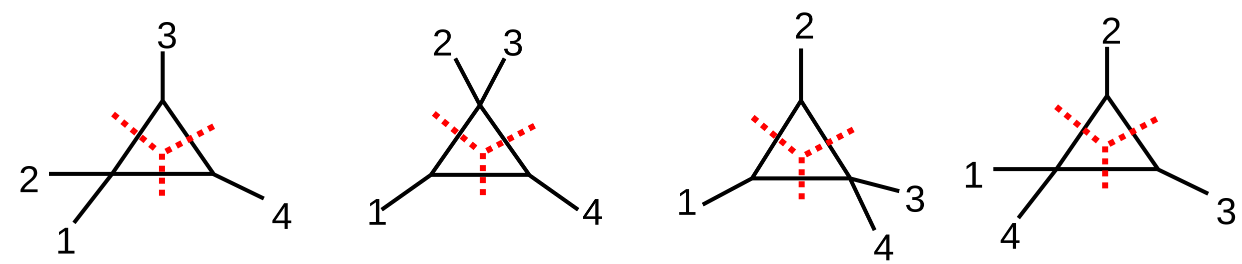
One box with
a quadruple cut:



Four boxes with
triple cuts:



Four triangles
with triple cuts:



Boxsubtraction: Remove all boxes with triple cuts to get the pure triangle part.

Numerical Implementation

- Coded in C++

- **On-shell:** loop-momentum parametrization with van Neerven-Vermaseren basis [van Neerven, Vermaseren 1984]

- **Disentanglement:** Discrete Fourier Projection (DFP)

- **Tree amplitudes:** recursion techniques [Berends, Giele 1987]

On-shell conditions in 4 Dimensions

Box case:

4 equations: $$D_i = D_j = D_k = D_l = 0$$

$\longrightarrow$ loop momentum entirely "frozen"

Triangle case:

3 equations: $$D_i = D_j = D_k = 0$$

$\longrightarrow$ loop-momentum and integrand depend on one free parameter t:

$$\bar{c}_{ijk}(t) = \mathcal{A}_1^{\text{tree}}(t)\mathcal{A}_2^{\text{tree}}(t)\mathcal{A}_3^{\text{tree}}(t) - \sum_{l \neq \{i,j,k\}} \frac{\bar{d}_{ijkl}(t)}{D_l}$$

Bubble case:

2 equations: $$D_i = D_j = 0$$

$\longrightarrow$ two free parameters t and y: $\bar{b} = \bar{b}(t, y)$

Discrete Fourier Projection I

Integrand is a complex valued power series in t with **finite** number of power terms:

$$\overline{c}(t) = \sum_{j=-p}^{p} c_j t^j$$

# of power terms = # of spurious terms + 1

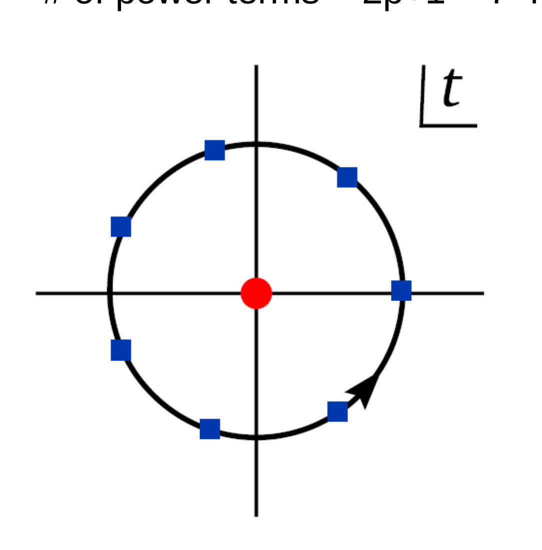
$$t_m = e^{2\pi\mathrm{i}\, m/(2p+1)}$$

$$\overline{c}(t_m) = \sum_{j=-p}^{p} c_j e^{2\pi\mathrm{i}\, mj/(2p+1)}$$

- multiply by $e^{2\pi\mathrm{i}\, ml/(2p+1)}$
- sum over *m*

$$c_l = \frac{1}{2p+1} \sum_{m=-p}^{p} e^{2\pi\mathrm{i}\, ml/(2p+1)} \overline{c}\left(t_m\right)$$

Triangles:
# of power terms = 2p+1 = 7 for p=3



Discrete Fourier Projection II

Mathematics behind the Fourier Projection:

- Complete and orthonormal function basis

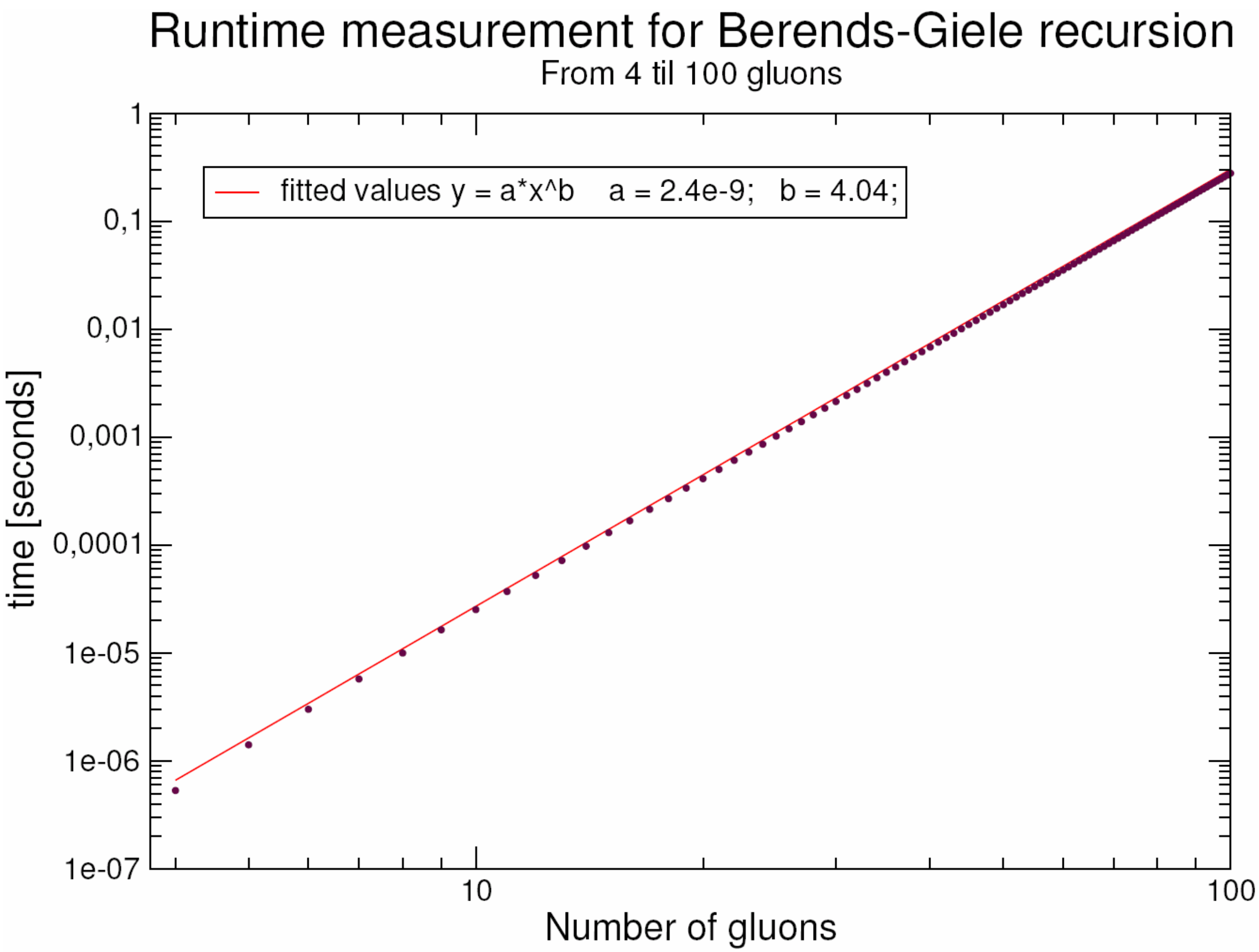
$$\frac{1}{2p+1}\sum_{j=-p}^{p} e^{\mathrm{i}2\pi(n-m)/(2p+1)} = \delta_{mn}$$

- Uses Z(N) as discrete subgroup of U(1)

Bubble case:

- double Fourier projection with two independent circles

Tree Amplitude Calculation



Very efficient calculation:
t < 0.3 seconds for 100 gluons

Results

Fully automated computation of the boxes, triangles and bubbles (cut-constructible parts) in pure gauge theory

estimate of the numerical precision via UV singularities:

- 1/e singularities of the bubbles canceled via gluonic beta function

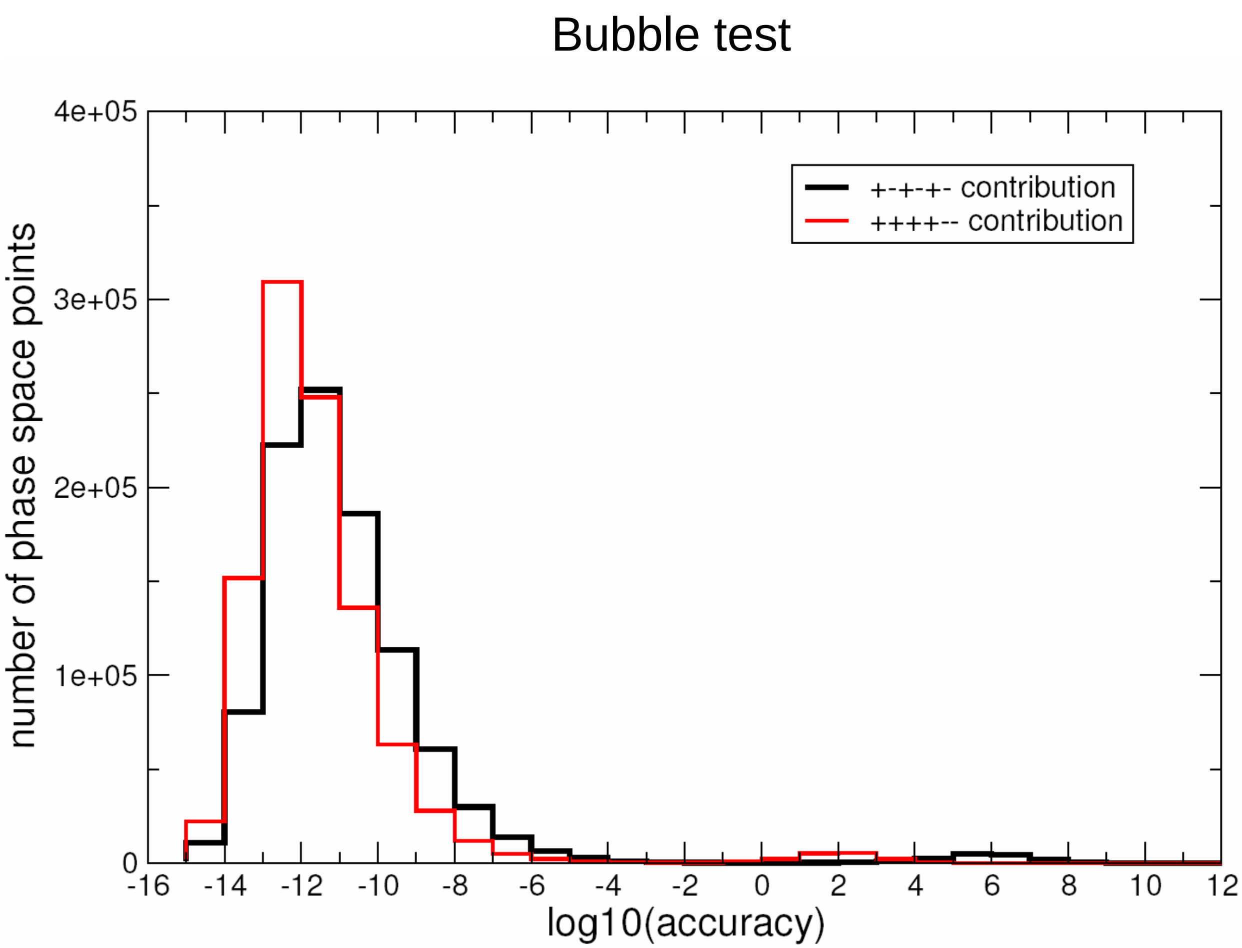
⟶ compare finite parts

$$\frac{1}{\mathcal{A}^{\text{tree}}} \sum_i b_i - \frac{11}{3} = 0$$

sum of all bubble coefficients

gluonic part of the beta function

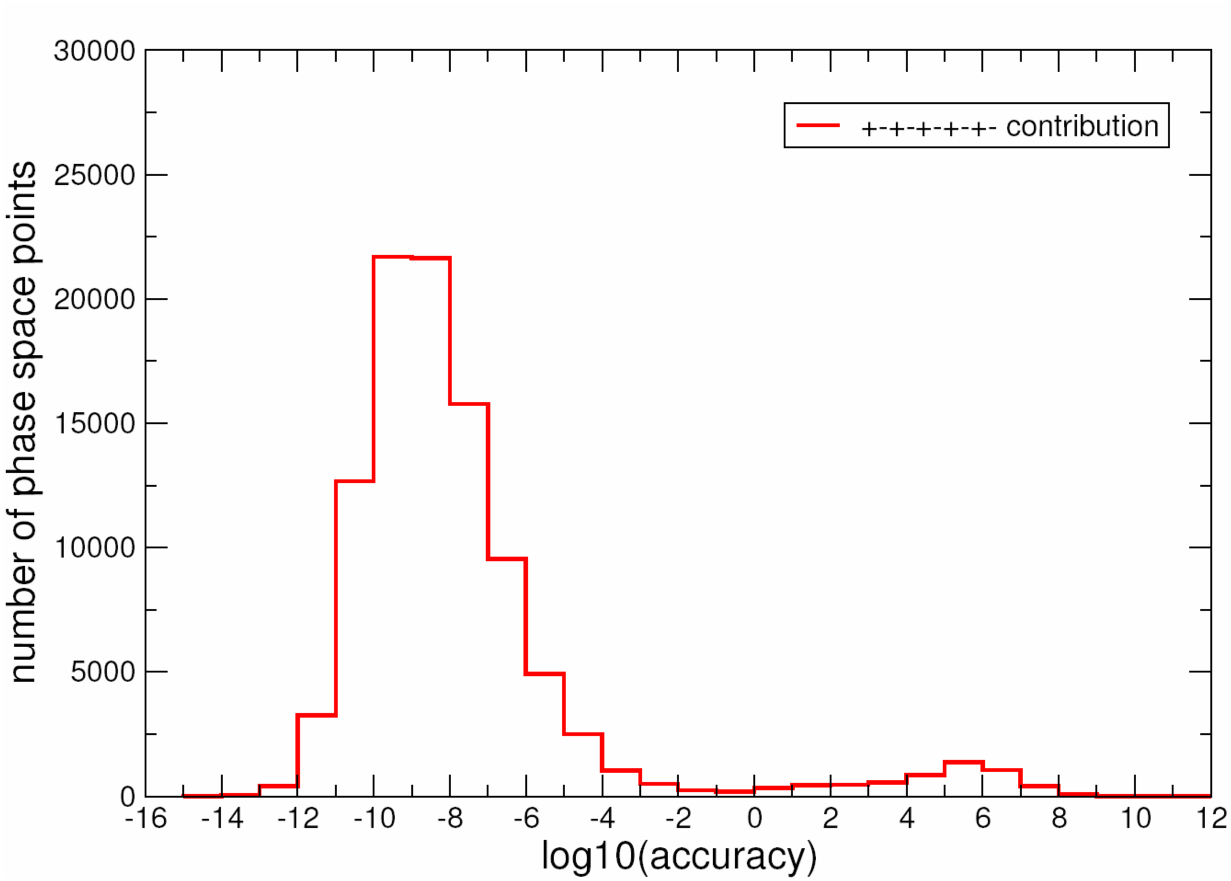
6-Gluon Amplitude



- 1e+6 phase space points
- very good accuracy, peaked at 1e-12
- works for different helicities
- small number of points failed test $\longrightarrow$ switch to multiple precision

10-gluon Amplitude

Bubble test



- 1e+5 phase space points
- accuracy peaked at 1e-9
- significant number of points failed the test ⟶ switch to multiple precision

Conclusion and Outlook

Conclusion

- Cut-constructible part works for all boxes, triangles and bubbles
- Good accuracy thanks to discrete Fourier Projection
- Speed can be improved

Outlook

- D-dimensional cuts for the rational terms