Neutrino Pendulum

Part I: General Introduction

Part II: 3-flavor Neutrino Mixing

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Graduate School Rathen

Part I: Coupled Pendula

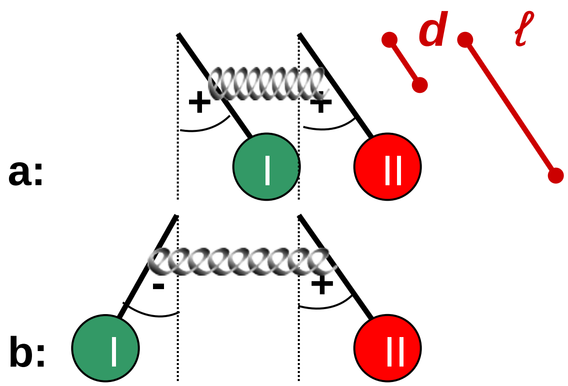
- Free Oscillation of one pendulum: $\omega_0^2 = \frac{g}{\ell}$
- 2 pendula with same length *ℓ*, mass *m* coupled by spring with strength *k*
- 2 Eigenmodes
  - Different eigenfrequencies = energies

    Mode a (II + I) with $\omega_a^2 = \omega_0^2$

    Mode b (II - I) with $\omega_b^2 = \omega_0^2 + \Delta\omega^2$
  - **Frequency (=energy) difference increases with stronger coupling**

$$\Delta\omega^2 = \frac{kd^2}{m\ell^2}$$

  - Coupling can be steered by varying ***k*** or ***d*** (*we'll vary d in the following*)



Two bases in Hilbert-space

flavor-basis

- eigenstates of flavor
- eigenstates of weak charge
- *particles take part in weak interactions as flavor-eigenstates*

- Examples:
  - $\bar{K}^0(s\,\bar{u})$ or $K^0(\bar{s}\,u)$
  - $\nu_e,\ \nu_\mu,\ \nu_\tau$

mass-basis

- eigenstates of mass
- well-defined lifetime
- *Particles propagate through space-time as mass-eigenstates*
  $|\upsilon(t)\rangle = |\upsilon\rangle e^{i(\vec{p}\vec{x}-Et)} e^{-\Gamma t}$

- Examples:
  - $K^0_L,\ K^0_S$
  - $\nu_1,\ \nu_2,\ \nu_3$

- Like coupled pendula, the coupling of particles leads to eigenstates with different masses and lifetimes, e.g. for linear combination of 2 states:

$$\nu_a = (\nu_\tau + \nu_\mu)/\sqrt{2} \quad \text{with} \quad m_a^2 = m_0^2$$

$$\nu_b = (\nu_\tau - \nu_\mu)/\sqrt{2} \quad \text{with} \quad m_b^2 = m_0^2 + \Delta m^2$$

Correspondences

pendulum	particles
Linear oscillation	complex phase rotation
Eigenmodes → fixed eigenfrequencies	**Mass eigenstates** → fixed phase frequencies
Frequency differences $\Delta\omega$ → different energies	Frequency differences $e^{i\Delta Et} \sim e^{i\Delta m^2 t}$ → different masses
One pendulum = lin. combination of eigenmodes	**Flavor eigenstate =** lin. combination of mass eigenstates
$\|amplitude^2\| \sim$ total energy in oscillation	$\|amplitude^2\| \sim$ detection probability
Beat-Frequency $\sim\Delta\omega$ of eigenmodes	**Flavor-Oscillation** $\sim \Delta m^2$ of mass eigenstates

Part II: Neutrino flavor pendulum

coupled pendula for demonstrating 3-flavor neutrino mixing as realized in nature

Idea: M.K.

built 2004 at Uni Bonn,
extended 2006 at TU Dresden
with variable mixing angles
and digital readout
http://neutrinopendel.tu-dresden.de

Copies in: Hamburg, Münster, DESY(Zeuthen), …

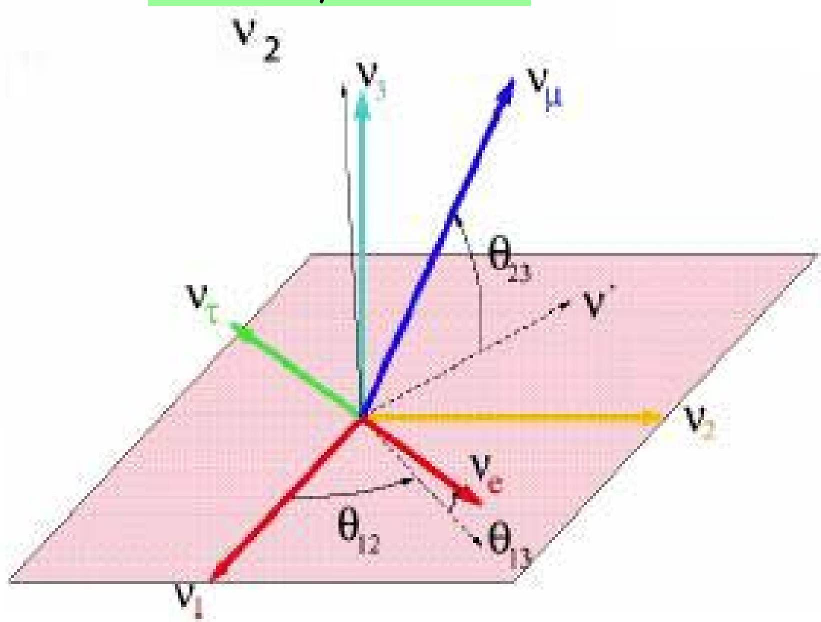
3-flavor neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\Theta_{atmos,\ beam}$ θ_{13}, δ $\Theta_{solar,\ reactor}$

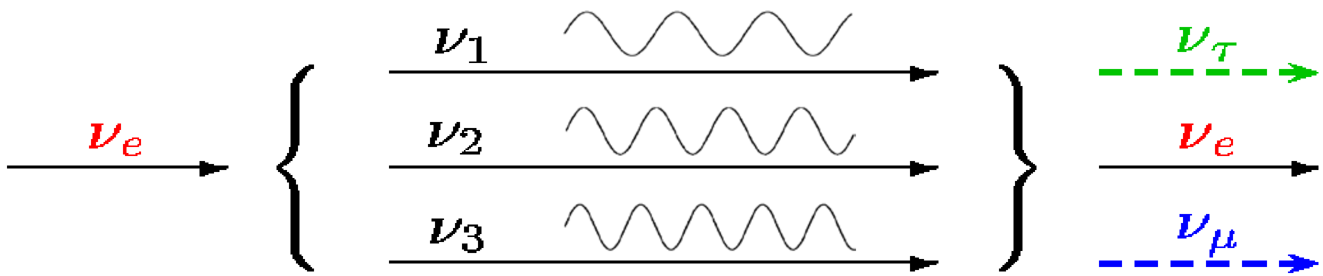
PMNS mixing matrix
(w/o Majorana Phases)

- 3 Mixing angles: $\theta_{12}, \theta_{23}, \theta_{13}$
- 1 CP-violating Dirac-Phase: δ
  (neglected in the following)

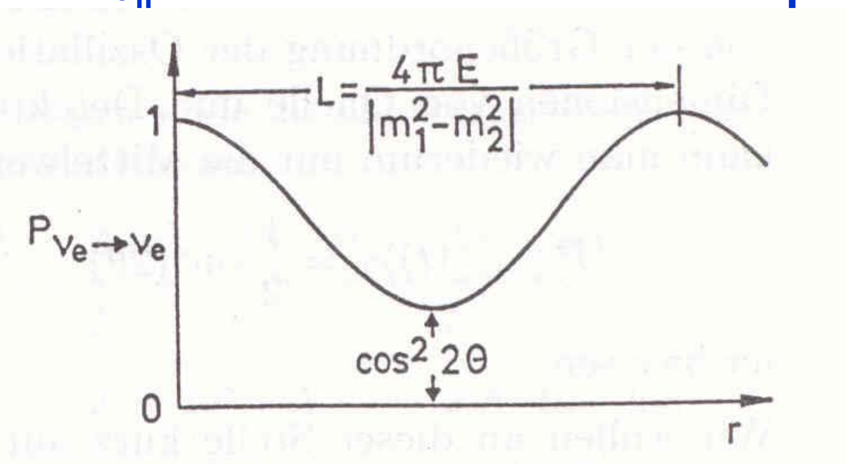


ν flavor-oscillations

- Each flavor (e.g. ν_e) is sum of mass eigenstates (ν_1, ν_2, ν_3)
- Each mass eigenstate with fixed p has a different phase frequency ω_i
  - $exp(i\omega_i t) = exp(iE_i t) = \exp(i(\sqrt{(p^2+m_i^2)}t) \sim exp(ipt+\mathbf{i m_i^2 t}/2p+\ldots)$



- **The differences $\Delta\omega_{ij} \sim |m_i^2 - m_j^2| =: \Delta m_{ij}^2$ lead to flavor oscillations**
  - $\Delta \mathbf{m}_{ij}^2$ determines the oscillation **period**
  - θ_{ij} determines the oscillation **amplitude**



$$L_{ij} = 2.5m \frac{E(MeV)}{\Delta m_{ij}^2(eV^2)}$$

Current values

cf. global fit Th.Schwetz et al., NJP 10 (2008)

$\Delta m^2_{23} = 2{,}4 \times 10^{-3}\ eV^2$	$\Delta m^2_{13} = 2{,}5 \times 10^{-3}\ eV^2$	$\Delta m^2_{12} = 0{,}08 \times 10^{-3}\ eV^2$
„fast“ oscillation		„slow“ oscillation
$L_{23} = 1\,km \times E(MeV)$		$L_{12} = 30\,km \times E(MeV)$
$\theta_{23} = 45° \pm 3°$	$\theta_{13} < 11°$ (90% CL)	$\theta_{12} = 33.5° \pm 1.5°$

$\theta_{atmos,\,beam}$ $\theta_{13},\,\delta$ $\theta_{solar,\,reactor}$

- consistent with so-called tri/bi-maximal mixing

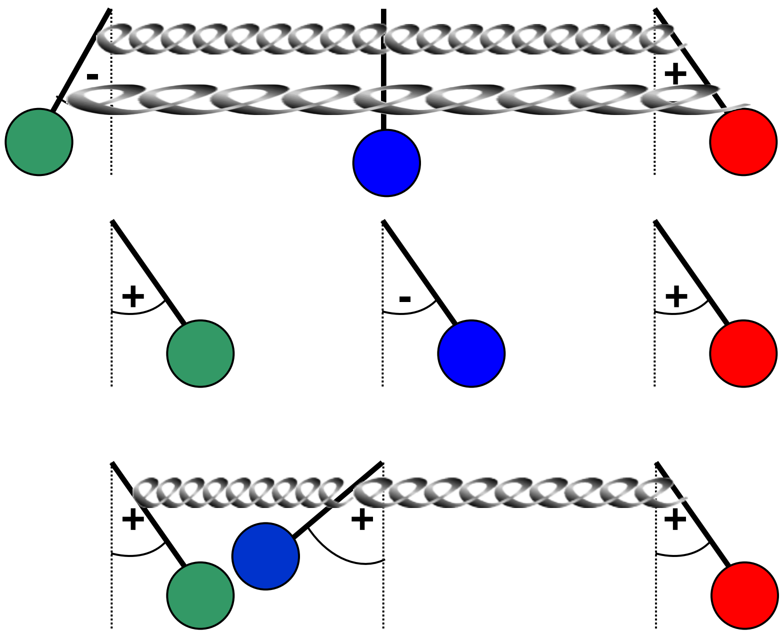
$\theta_{23} = 45°$ $\theta_{13} = 0°$ $\theta_{12} = 35.3°$

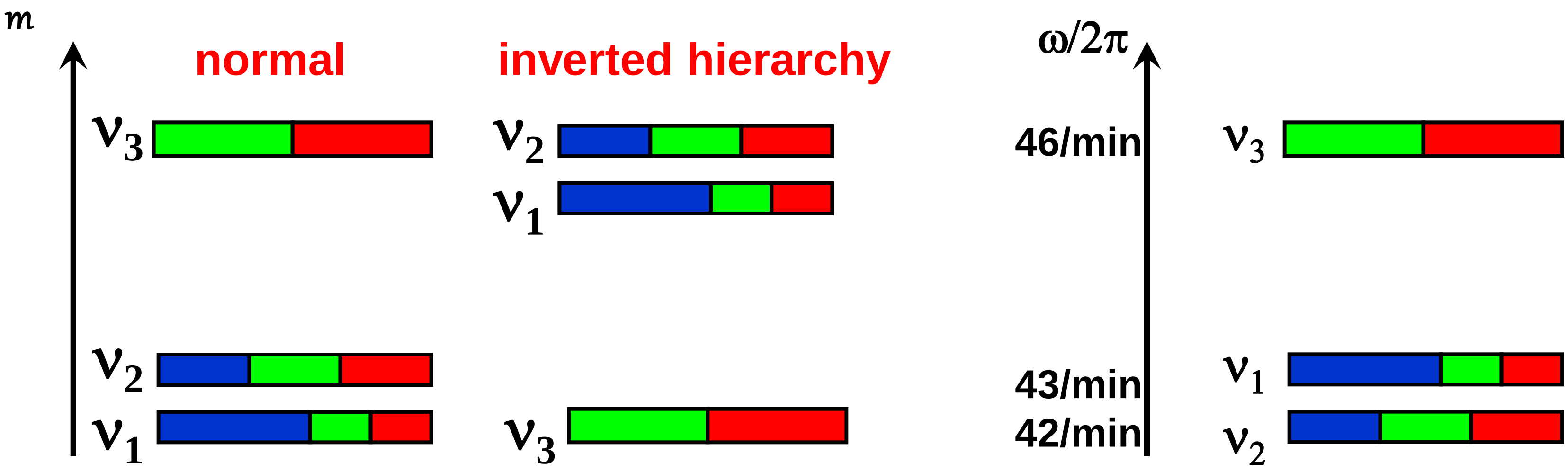
$$U_{PMNS} \approx \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, Scott '99,'02
Z.Xing,'02, He, Zee, '03, Koide '03
Chang, Kang, Kim '04, Kang'04

Realisation as coupled pendula

- $\nu_3 = (\quad - \nu_\mu + \nu_\tau)/\sqrt{2}$
- $\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$
- $\nu_1 = (2\nu_e + \nu_\mu + \nu_\tau)/\sqrt{6}$





The solar neutrino „deficit“

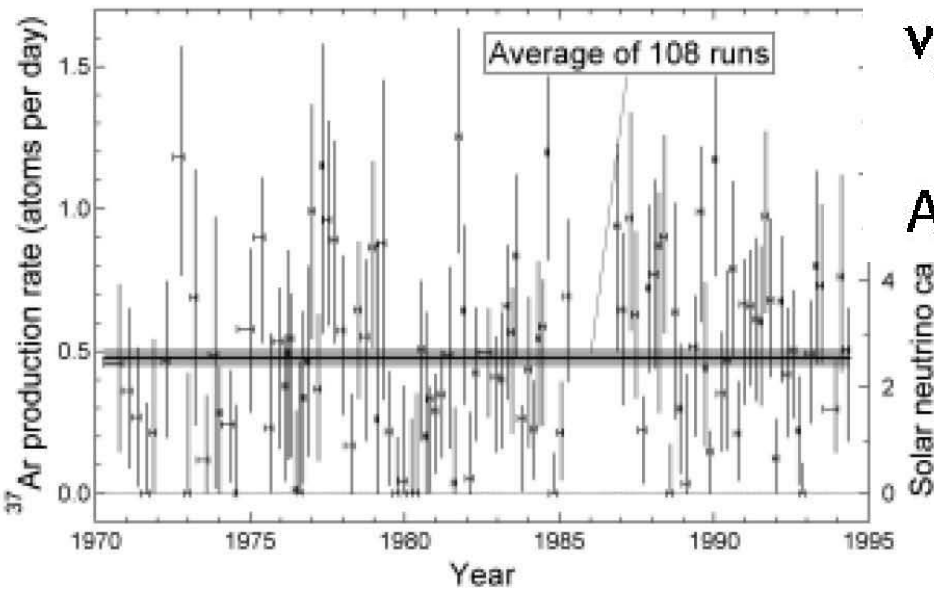
Ray Davis

Nobelpreis 2002

380000 l
Perchlorethylen
in der Homestake- Mine

$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$

Ausspülen des ^{37}Ar (0.5 Atome/Tag)



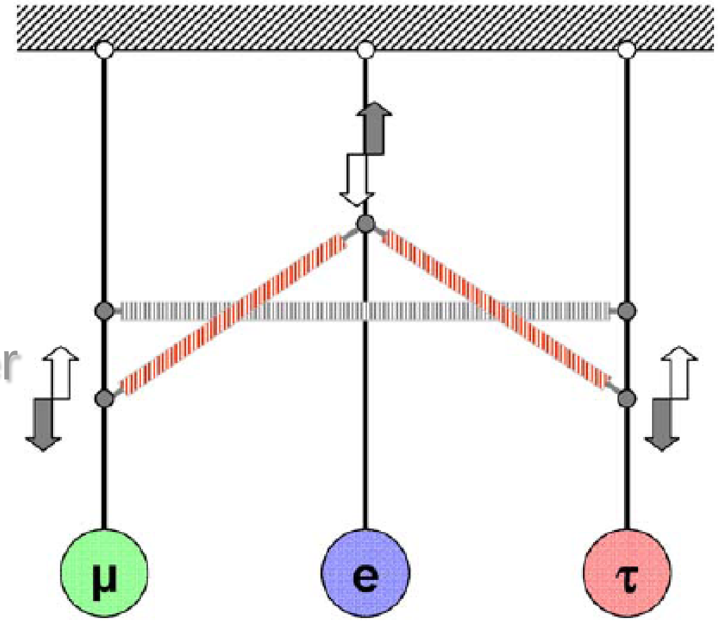
- Davis: only sensitive to ν_e
  Result: Only 32% of expected ν_e detected

Modify θ_{12}

- Modify fraction of ν_e in ν_1 and ν_2
- $\nu_2 = (-\nu_e + \nu_\mu + \nu_\tau)/\sqrt{3}$ no longer eigenmode

Possible range:
$20^o < \theta_{12} < 90^o$

θ_{12} smaller
θ_{12} larger



- *http://neutrinopendel.tu-dresden.de*
  (special high school thesis J. Pausch 2008)

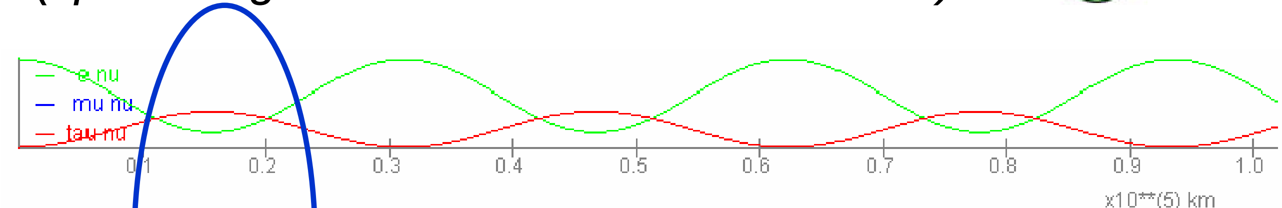


Abbildung 27: Sonnenneutrino-Oszillation mit θ_{12}=0,57rad (33°), Java-Applet.

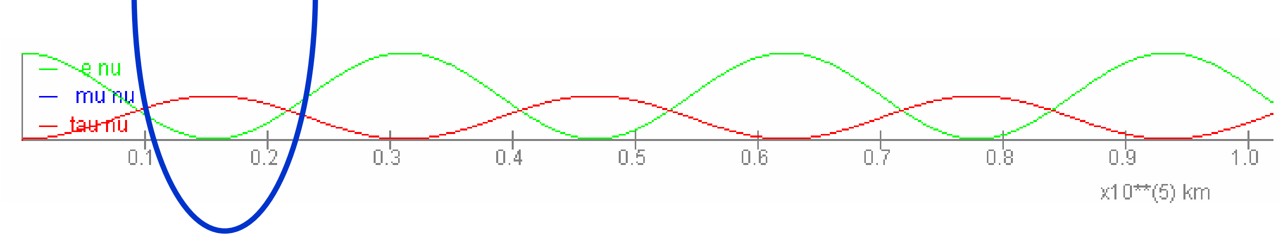
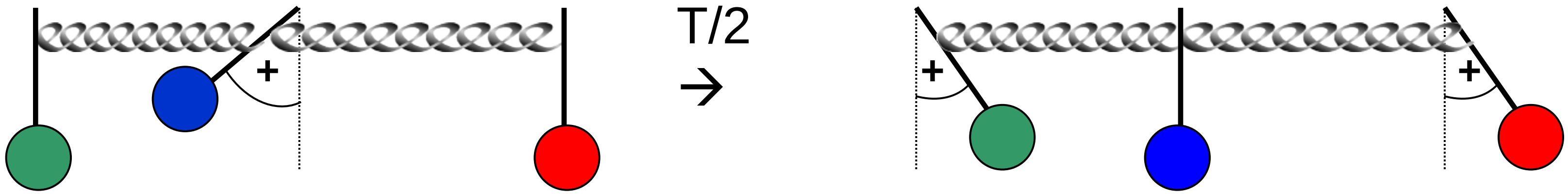


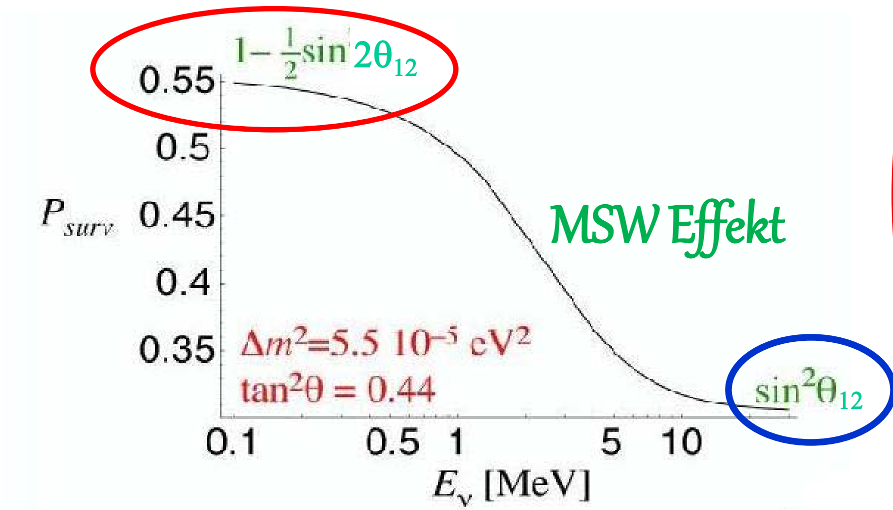
Abbildung 28: Sonnenneutrino-Oszillation mit θ_{12}=0,7854rad (45°), Java-Applet.

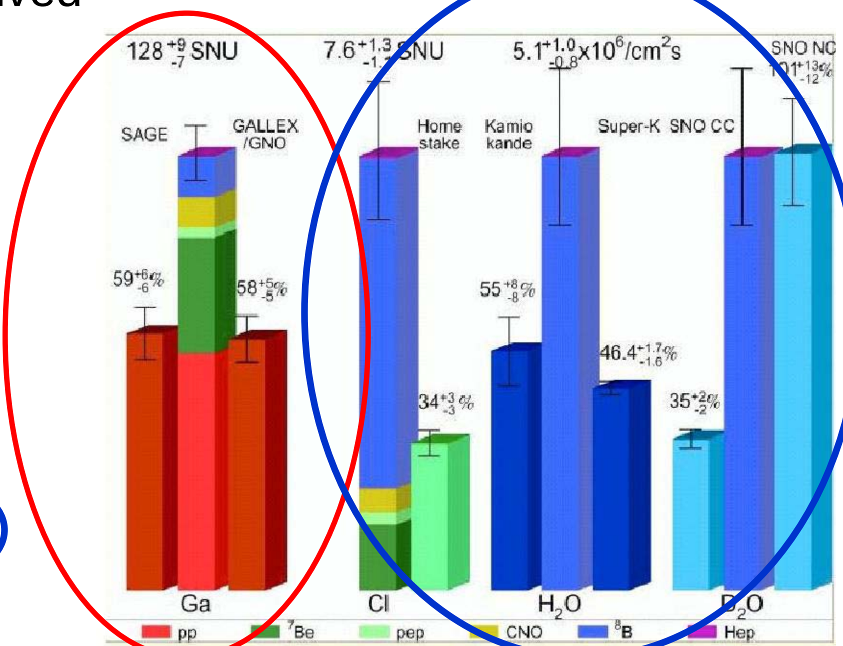
Need for enhancement (MSW effect)

- nuclear fusion: 100% ν_e leave the sun (w/o MSW effect)
  $4p \rightarrow {}^4He + 2e^+ + \mathbf{2}\ \nu_e + 27$ MeV
- "slow" oscillation via θ_{12} and small $\Delta \mathbf{m}^2_{12}$ (pendula: weak coupling)



- oscillation only to $(\nu_\tau + \nu_\mu)/\sqrt{2}$
- transition to $(\nu_\tau - \nu_\mu)/\sqrt{2}$ not possible, since ν_e not in ν_3
- $P(\nu_e \rightarrow \nu_e) > 50\%$ since just ν_1 and ν_2 involved
  → need for enhancement *(MSW effect)*



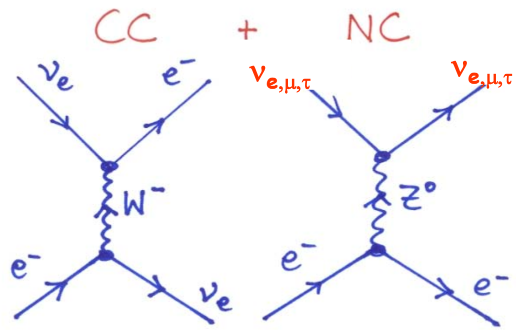


Problems solved 1985 by
MSW (Mikheyev–Smirnov–Wolfenstein) effect

- Historical Prejudice: mixing angle should be small
  - Problem: How to get large neutrino deficit w/ small mixing?
  - Today no problem: 2 mixing angles are large!

- Knowing about large θ_{12}, but having $\theta_{13} = 0$
  - Effective 2-flavor mixing!
    → min detection rate should be >= 50%
  - Problem: Observed rate of Homestake ~ 32% !

In matter there is an additional potential in the equation of motion for $\nu_e \rightarrow \nu_e$ scattering:

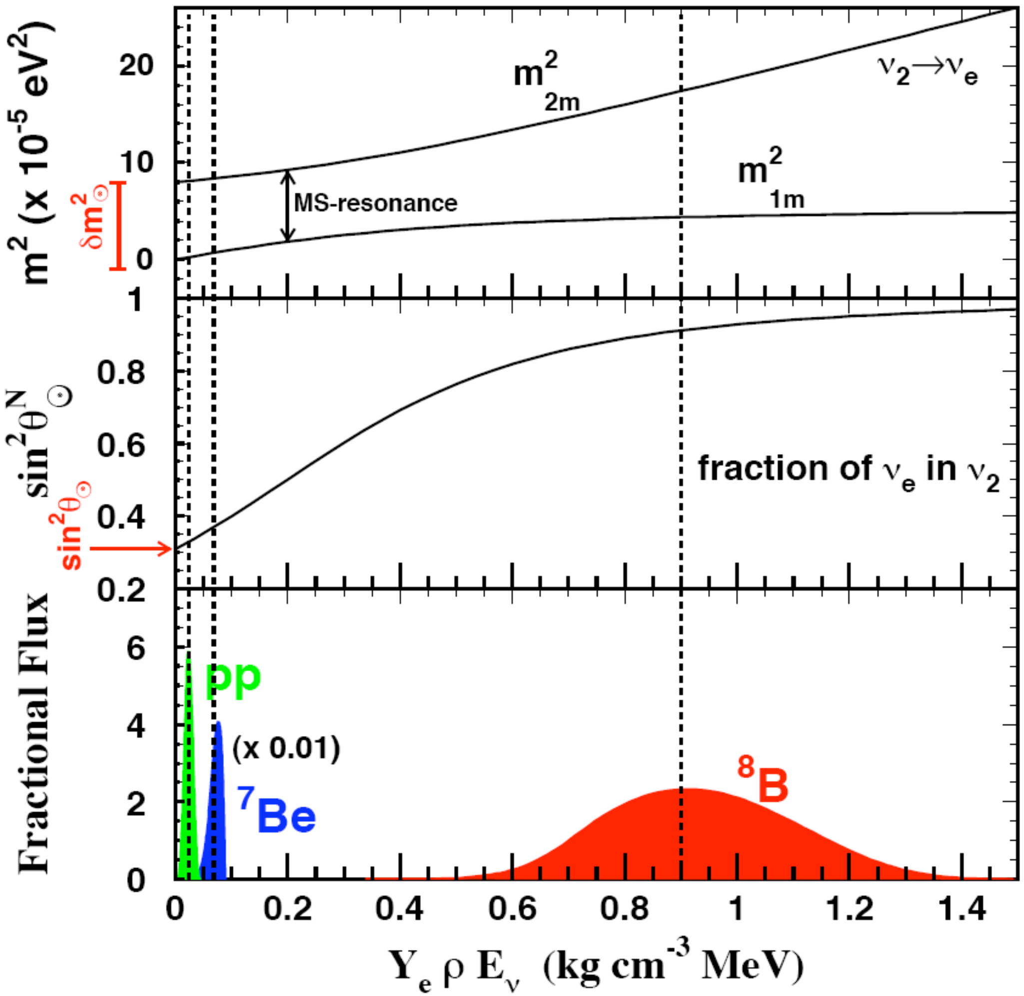


Matter:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta m^2\cos 2\theta + 2\sqrt{2}G_F N_e E & \Delta m^2 \sin 2\theta \\ \Delta m^2\sin 2\theta & \Delta m^2 \cos 2\theta - 2\sqrt{2}G_F N_e E\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

with $2\sqrt{2}G_F \underbrace{N_e}_{\frac{Y_e\rho}{m_p}} E = 1.53\cdot 10^{-7}\,\mathrm{eV}^2\left(\frac{Y_e\rho}{\mathrm{g/cm^3}}\cdot\frac{E}{\mathrm{MeV}}\right)$ center of Sun: $\frac{Y_e\rho}{\mathrm{g/cm^3}} \simeq 100$

Today's values for Solar ν



In Vacuum

$$\delta m^2_\odot = 8.0 \pm 0.4 \times 10^{-5}\ eV^2$$

$$\sin^2\theta_\odot = 0.31 \pm 0.03$$

Whereas for ^{8}B at center of Sun

$$\delta m^2_N = 14 \times 10^{-5}\ eV^2$$

$$\sin^2\theta^N_\odot = 0.91$$

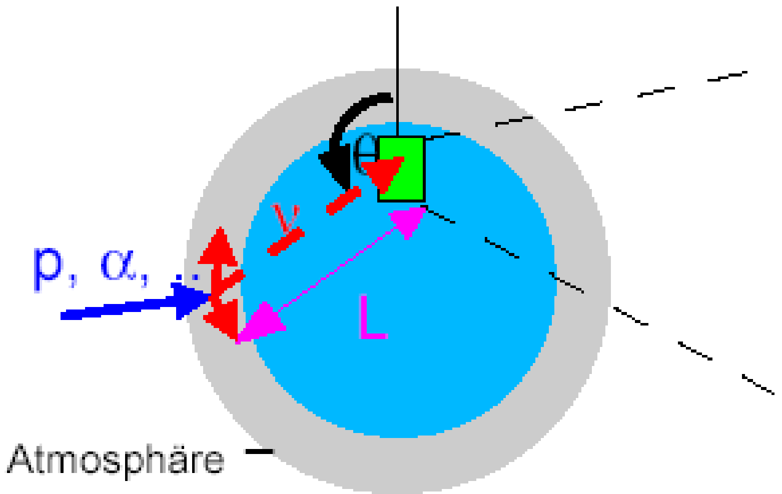
hep-ph/0601198

slide from Stephen Parke

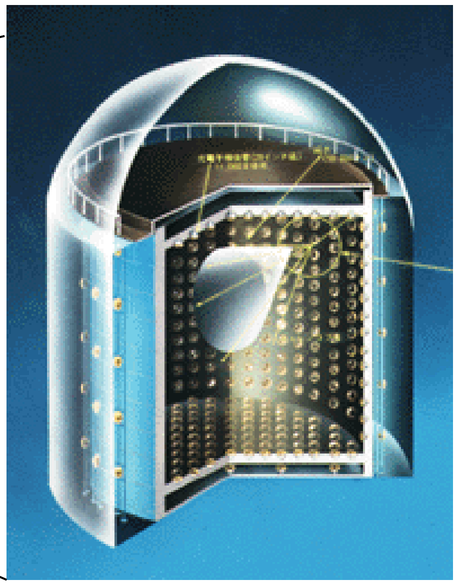
http://boudin.fnal.gov/AcLec/AcLecParke.html

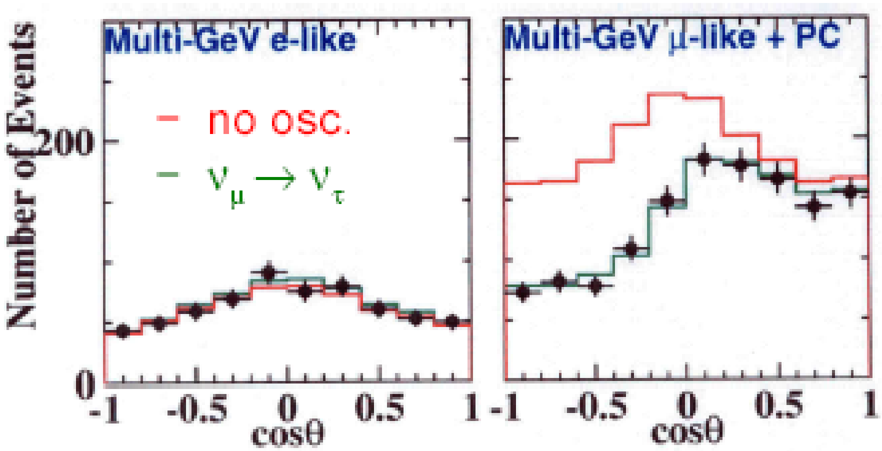
Atmospheric neutrinos

$\overset{(-)}{\nu}_\mu$ und $\overset{(-)}{\nu}_e$ aus π/K−Zerfällen



SuperKamiokande 2000:





(C. Mc Grew, NOON 2000, Dez.2000)

look at ν_e and ν_μ from air showers:

- no deficit for ν_e
- clear deficit for ν_μ
- fully compatible with $\nu_\mu \rightarrow \nu_\tau$

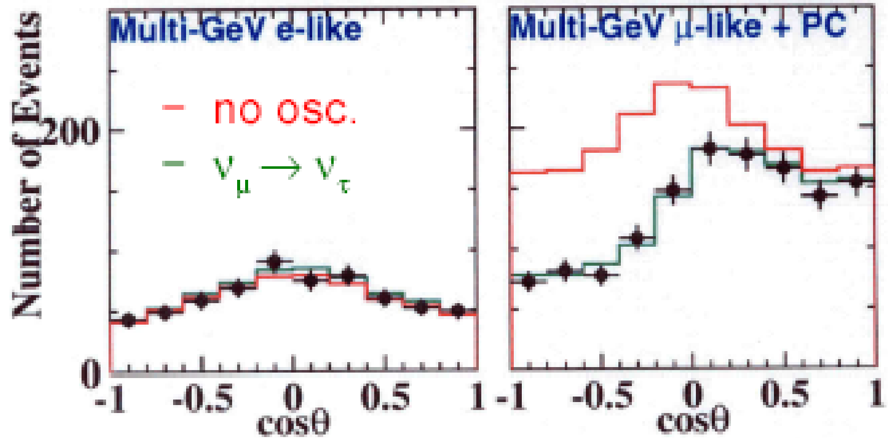
atmospheric neutrinos

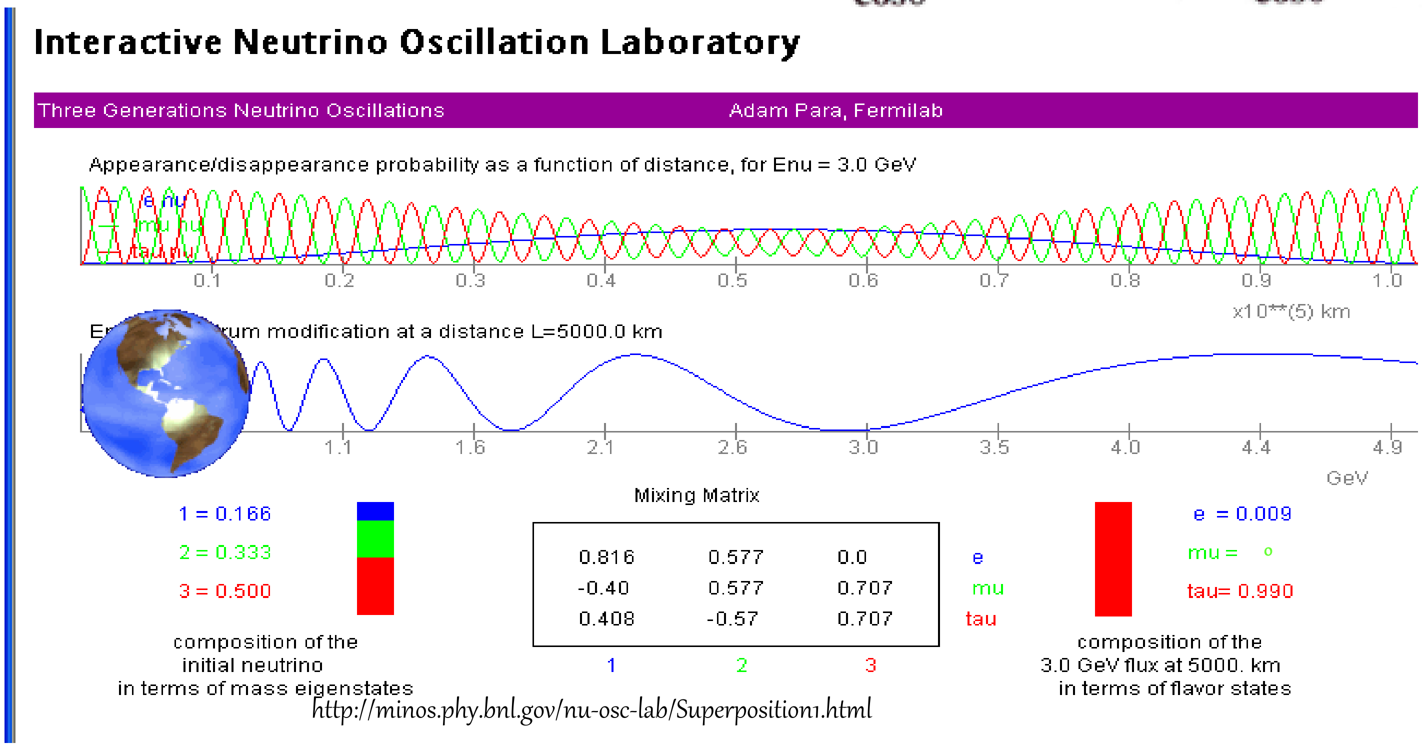
- SuperKamiokande 2000: described als $\nu_\mu \rightarrow \nu_\tau$
- pendula:

  ν_e : weak coupling to ν_μ, ν_τ

  ν_μ: weak coupling to ν_e

  strong coupling to ν_τ

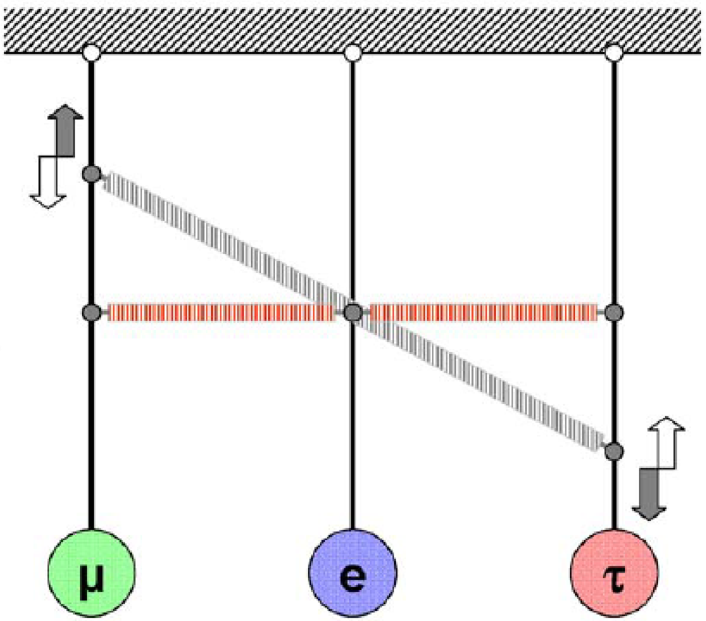




http://minos.phy.bnl.gov/nu-osc-lab/Superposition1.html

Modify θ_{23}

- Non-maximal mixing of ν_μ and ν_τ
- $\nu_3 = (\quad - \nu_\mu + \nu_\tau)/\sqrt{2}$ no longer eigenmode



Possible range:
$30^o < \theta_{23} < 60^o$

θ_{23} smaller
θ_{23} larger

- *http://neutrinopendel.tu-dresden.de*
(special high school thesis J. Pausch 2008)

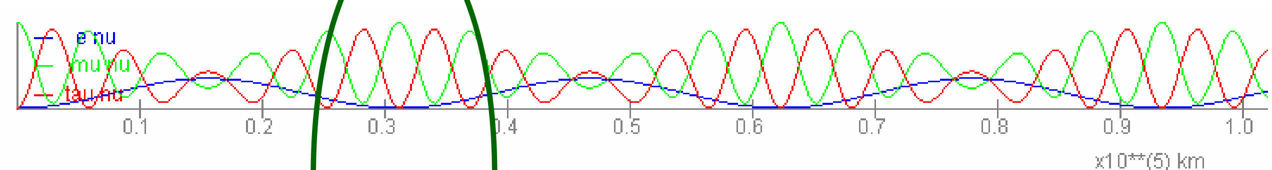


Abbildung 52: Atmosphärische Neutrino-Oszillation mit θ_{23} =0,9rad (51,6°), Java-Applet.

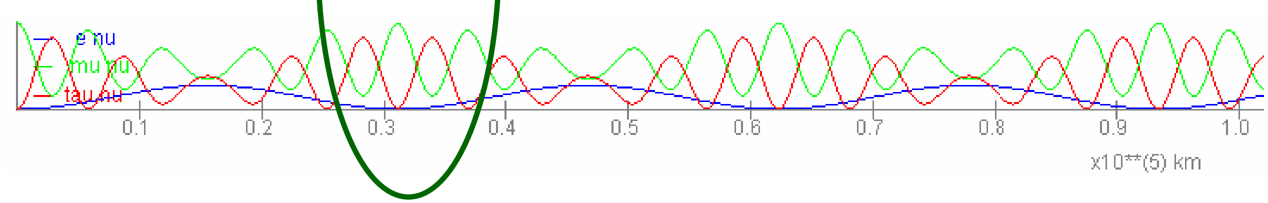
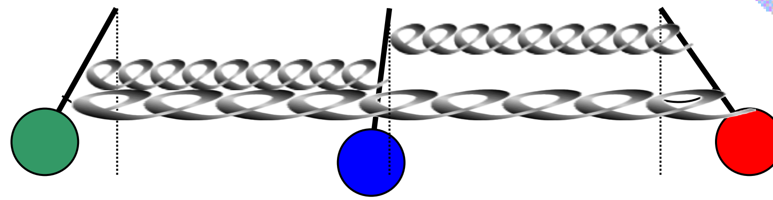


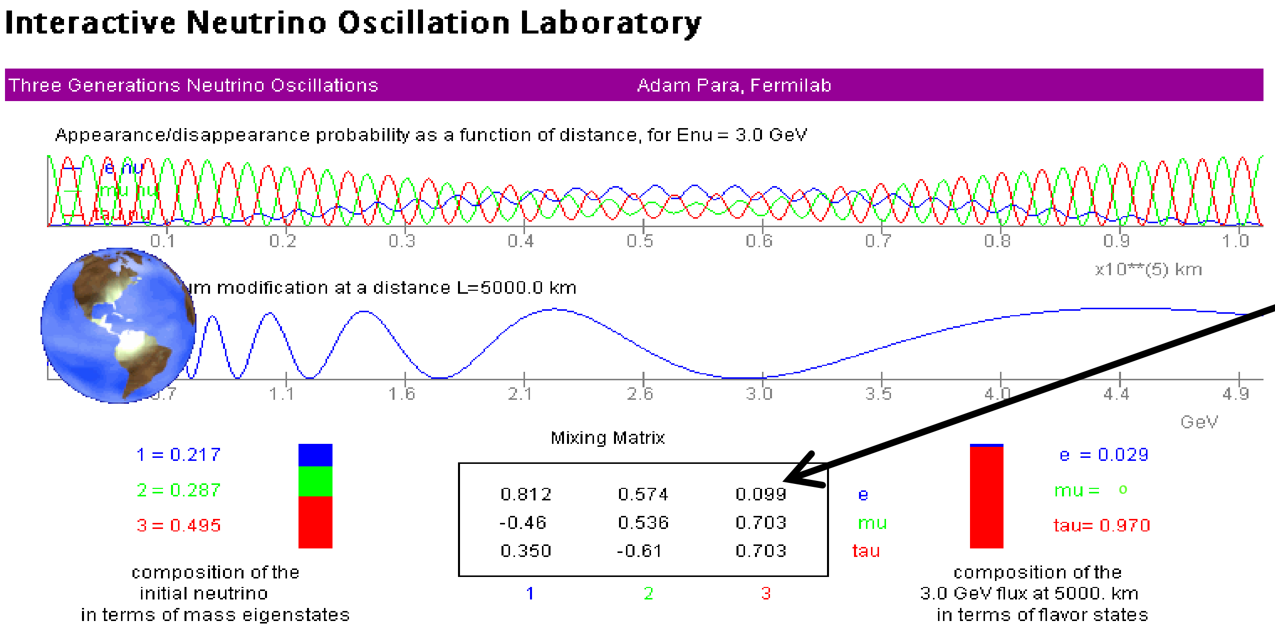
Abbildung 53: Atmosphärische Neutrino-Oszillation mit =0,98rad (56,2°), Java-Applet.

Impact of θ_{13}

- $\nu_3 = (\sin\theta_{13}\nu_e - \nu_\mu + \nu_\tau)/\sqrt{2.01}$



- reactor $\bar{\nu}_e \rightarrow \bar{\nu}_\tau + \bar{\nu}_\mu$ disappearance and atmospheric or beam $\nu_\mu \rightarrow \nu_e$ appearance
  - „slow“ directly via Δm_{12} (weak coupling)
  - „fast“ modulation via $\nu_\tau - \nu_\mu$ with Δm_{23} (strong coupling)



$\theta_{13} = 6^o$

$\sin\theta_{13} = 0.1$

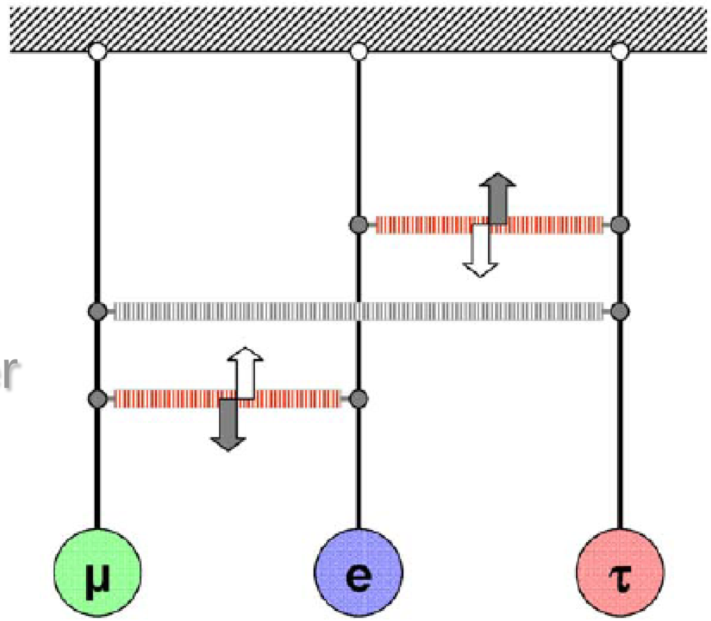
$\sin^2 2\theta_{13} = 0.04$

Modify θ_{13}

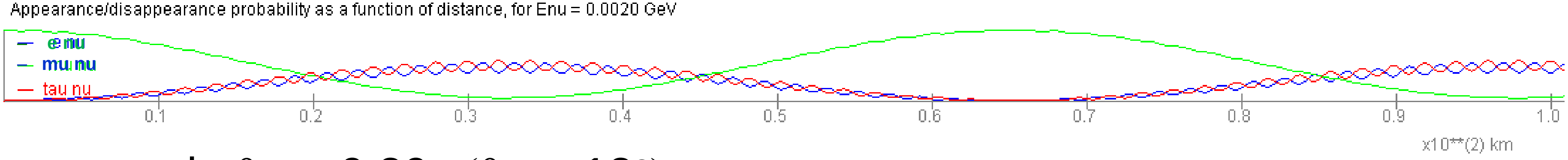
- ν_e present in $\nu_3 \sim (\sin\theta_{13}\nu_e - \nu_\mu + \nu_\tau)$
- ν_e can now excite $(\nu_\tau - \nu_\mu)$ mode, inducing fast $\nu_\tau - \nu_\mu$ modulation

Possible range:
$-6^o < \theta_{13} < 6^o$

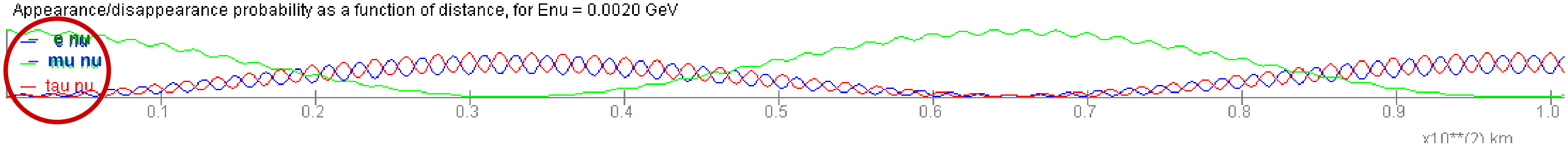
θ_{13} smaller
θ_{13} larger



- Reactor neutrinos (2 MeV)
  - $\sin\theta_{13} = 0.10 \quad (\theta_{13} = 6^o)$



  - $\sin\theta_{13} = 0.20 \quad (\theta_{13} = 12^o)$



Are neutrino pendula a perfect model?

- Few “features”
  - Need “creative” sign convention, leading to
  - imperfection for understanding sequence of masses
  - imperfection for $\theta_{23} \neq 45^o$
    - some $(\nu_\tau - \nu_\mu)$ present in ν_1 and ν_2
    - but $\nu_e \rightarrow (\nu_\tau - \nu_\mu)$ still not possible!
- Else perfect!

The END !