Neutrino masses, mixing and oscillations

S.M. Bilenky
JINR(Dubna), TUM(Munich)

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Observation of neutrino oscillations in solar, atmospheric, reactor and accelerator neutrino experiments is one of the most important recent discovery in particle physics

The observation of neutrino oscillations means that

- Neutrinos have small but different from zero masses.
- Fields of neutrinos with definite masses enter into charged and neutral currents in a mixed form

Brief history of neutrino mass

Idea of neutrino was proposed by W. Pauli on December 4th 1930

At that time nuclei were considered as bound states of protons and electrons

Two problems in the framework of this assumptions

I. β-decay: $(A, Z) \to (A, Z+1) + e^{-}$

Two particle decay. Monochromatic electron must be produced. In experiment continuous β-spectrum was observed

II. Spins of some nuclei. ${}^{7}N_{14} = (14p + 7e) \rightarrow \mathrm{half\ integer\ spin}$

From molecular spectra : ${}^{7}N_{14}$ satisfy Bose-Einstein statistics; spin must be integer

Pauli came to idea that only existence of a new particle could solve these nuclear problems

In order to come to continuous β-spectrum, β-decay of nuclei must be three-particle decay

Additional particle must not be visible in an experiment

It must have spin $1/2$ and be constituent of nuclei (problem of spin can be solved)

Thus, Pauli assumed that exist a neutral, spin $1/2$, particle with interaction which is much weaker that the interaction of photon.

Pauli called a new particle neutron

Pauli considered a new particle as a particle with mass (constituent). From Pauli letter

"The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 of the proton mass".

In 1932 neutron was discovered by Chadwick

Soon after this discovery Heisenberg, Majorana, Ivanenko came to correct idea of proton-neutron structure of nuclei

No problem of spin. For example, ${}^7N_{14} = (7p + 7n)$, integral spin

What about β-decay and continues β spectrum?

The problem β-decay of nuclei which are bound states of protons and neutrons was solved by E. Fermi in 1933-34

F. Fermi accepted Pauli hypothesis of the existence of a new light particle (much lighter than neutron) which E. Fermi proposed to call neutrino (from Italian, *neutral, small*)

Fermi assumed that (e, ν) pair is produced in the quantum transition of neutron to proton

$$n \to p + e + \nu$$

From analogy with electrodynamics Fermi proposed the first Hamiltonian which provides this transition

$$\mathcal{H}_I = G_F \bar{p} \gamma^\alpha n \; \bar{e} \gamma_\alpha \nu + \mathrm{h.c.}$$

G_F is a constant of the dimension M^{-2}. Only vector interaction

Neutrino mass in Fermi theory? Fermi considered neutrino mass as unknown parameter and proposed a method of measuring of the neutrino mass via investigation of the β-spectrum

$$Q = E + E_\nu$$

(E is electron kinetic energy, Q is energy release)

The region $(Q - E) \simeq m_\nu$ is sensitive to m_ν

Convent decay for measuring of neutrino mass

$$^3H \to ^3He + e^- + \bar{\nu}$$

(relatively small energy release $Q \simeq 18.6\ \mathrm{KeV}$, superallowed transition (NME is a constant), $t_{1/2} \simeq 12.3\ \mathrm{years}$ etc)

β-spectrum is given by the phase space

$$\frac{d\Gamma}{dE} = C|M|^2 p(E+m_e)(Q-E)\sqrt{(Q-E)^2 - m_\beta^2}\, F(E)$$

The first tritium experiment was performed by Hanna and Pontecorvo and S. Curran et al (1949) It was found the bound

$$m_\beta \leq 500\ \mathrm{eV}$$

In 1957-58 violation of parity P (and C) was discovered in β-decay and other weak processes

β-decay of polarized nucleus (Wu et al experiment)

$$w_{\vec{P}}(\vec{p}) = w_0(1 + \alpha\ \vec{P}\cdot\vec{k}) = w_0(1 + \alpha P\cos\theta)$$

$\vec{k} = \frac{\vec{p}}{p}$, α is the asymmetry parameter

The pseudoscalar $\alpha\ \vec{P}\cdot\vec{k}$ is due to interference of P-conserving and P-violating parts of the Hamiltonian

In Wu et al experiment it was obtained $\alpha \simeq -0.7$. Large violation of parity

Two-component neutrino theory. Landau, Lee and Yang, Salam (1957)

Large violation of parity is connected with neutrino and neutrino mass

Dirac equation $(i\gamma^{\alpha}\partial_{\alpha} - m)\nu(x) = 0$

Left-handed (right-handed) components $\nu_{L,R}(x) = \frac{1 \mp \gamma_5}{2}\nu(x)$

$i\gamma^{\alpha}\partial_{\alpha}\nu_L(x) - m\ \nu_R(x) = 0, \quad i\gamma^{\alpha}\partial_{\alpha}\nu_R(x) - m\ \nu_L(x) = 0$

Equation are coupled because of mass m

In 1958 from tritium experiments $m_{\nu} < 200\ \mathrm{eV}$, much smaller than Pauli suggestion

Landau, Lee and Yang, Salam assumed $m_{\nu} = 0$

In this case equations are decoupled

$i\gamma^{\alpha}\partial_{\alpha}\nu_{L,R}(x) = 0$

For the neutrino field $\nu_L(x)$ $(\mathrm{or}\ \ \nu_R(x))$ can be chosen.

This choice is the two-component neutrino theory

The general β-decay Hamiltonian

$$\mathcal{H}_I = \sum_i G_i \, \bar{p} \; O_i n \; \bar{e} \; O^i \tfrac{1}{2}(1 \mp \gamma_5)\nu + \mathrm{h.c.}$$

$$O \to 1, \, \gamma_\alpha, \, \sigma_{\alpha\,\beta}, \, \gamma_\alpha\gamma_5, \, \gamma_5$$

Large violation of parity (in agreement with the Wu et al experiment)

Important prediction

If neutrino field is $\nu_L(x)$, neutrino is left-handed ($h = -1$) and antineutrino is right-handed ($h = +1$). In the case of $\nu_R(x)$ neutrino is right-handed and antineutrino is left-handed

This follows from the fact that for the massless neutrino

$$\gamma_5 \; u^r(p) = r \; u^r(p)$$

Neutrino helicity was measured in spectacular Goldhaber et al experiment (1958)

$$
\begin{array}{llc}
e^- + {}^{152}\mathrm{Eu} \to \nu + & {}^{152}\mathrm{Sm}^* & \\
 & \downarrow & \\
 & {}^{152}\mathrm{Sm} & + \gamma
\end{array}
$$

Spins of ${}^{152}\mathrm{Eu}$ and ${}^{152}\mathrm{Sm}$ are equal to zero and ${}^{152}Sm^*$ is equal to one

Measurement of the circular polarization of γ allows to determine neutrino helicity

From Goldhaber et al experiment followed that the neutrino has negative helicity

Two-component neutrino theory with neutrino field $\nu_L(x)$ was confirmed

After this success of the two-component theory during many years physicists believed than neutrinos are massless particles ($V - A$ theory and original Standard model were build for massless two-component neutrinos)

The first physicist who started to think about a possibility of small neutrino masses was B. Pontecorvo (1957-58)

He believed in analogy between weak interaction of hadrons and leptons and looked for analogy of $K^0 \leftrightarrows \bar{K}^0$ oscillations in the lepton sector

In such a way B. Pontecorvo came to an idea of neutrino oscillations

"If the two-component neutrino theory turn out to be incorrect (which at present seems to be rather improbable) and if the conservation law of neutrino charge would not apply, then in principle neutrino $\rightleftarrows$ antineutrino transitions could take place in vacuum."

At that time only one type of neutrino was known

By analogy with $K^0 - \bar{K}^0$ B. Pontecorvo assumed

$$|\nu_L\rangle = \frac{1}{\sqrt{2}}(|\nu_{1L}\rangle + |\nu_{2L}\rangle), \quad |\bar{\nu}_L\rangle = \frac{1}{\sqrt{2}}(|\nu_{1L}\rangle - |\nu_{2L}\rangle)$$

where ν_1 and ν_2 are Majorana neutrino with masses m_1 and m_2 "neutrino and antineutrino are *mixed particles*, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2"

In the first paper on neutrino oscillations (1958) B. Pontecorvo wrote ...the number of events $\bar{\nu} + p \to e^+ + n$ with reactor antineutrino would be smaller than the expected number. "It would be extremely interesting to perform the Reins-Cowan experiment at different distances from reactor"

In 1962 Maki, Nakagawa and Sakata also came to an idea of massive neutrino

They used Nagoya model in which proton was considered as a bound state of neutrino and some vector boson B^+, "a new sort of matter". For MNS neutrino was a constituent and, correspondingly, massive (like neutrino Pauli). Following idea of Gell-Mann and Levy they assumed mixing

$$\nu_e = \nu_1 \cos\delta - \nu_2 \sin\delta, \quad \nu_\mu = \nu_1 \sin\delta + \nu_2 \cos\delta$$

In connection with the (first accelerator) Brookhaven experiment (1962) they qualitatively discussed "virtual transition $\nu_\mu \rightleftarrows \nu_e$"

To commemorate pioneer contribution to neutrino oscillations and neutrino mixing of Pontecorvo, Maki, Nakagawa and Sakata neutrino mixing matrix is called PMNS matrix

In eighties special reactor and accelerator experiments on the search for neutrino oscillations started. No indications. Model dependent evidence for oscillations from solar experiments was obtained Atmospheric neutrino anomaly was discovered

GOLDEN YEARS OF NEUTRINO OSCILLATIONS

1998 Super-Kamiokande discovery of neutrino oscillations in atmospheric experiment (zenith angle dependence of the number of ν_μ's)

2001 SNO Model independent proof of the transition of solar ν_e into ν_μ and ν_τ (ratio of the flux of ν_e 's to the total flux of ν_e, ν_μ and ν_τ is about $1/3$)

2002-2004 KamLAND reactor experiment (significant distortion of the spectrum of reactor $\bar{\nu}_e$'s)

All existing weak interaction data are perfectly described by the standard CC and NC interactions

The Standard CC lepton interaction

$$\mathcal{L}_{\mathcal{I}}^{\mathcal{CC}}(x) = -\frac{g}{2\sqrt{2}} j_{\alpha}^{CC}(x) W^{\alpha}(x) + \mathrm{h.c.}$$

$$j_{\alpha}^{CC}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\, \gamma_{\alpha}\, l_L(x)$$

CC interaction determines notions of flavor neutrino ν_l and antineutrinos $\bar{\nu}_l$

$\pi^+ \to l^+ + \nu_l$, $\pi^- \to l^- + \bar{\nu}_l$, $\nu_l + N \to l^- + X$ etc

From the measurement of the width of the decay $Z \to \nu_l + \bar{\nu}_l$ (LEP, CERN) follows that three flavor neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$ exist in nature ($N_{\nu_l} = 2.984 \pm 0.008$)

If neutrinos are massive and mixed a neutrino mass term enter into the Lagrangian

Mass terms of quarks and leptons are generated by the standard Higgs mechanism

Mass term of charged leptons

$$\mathcal{L}^{\rm leptons}(x) = -\bar{L}_L(x)\, M\, L_R(x) + \rm{h.c.}$$

$$L_{L,R} = \begin{pmatrix} e_{L,R} \\ \mu_{L.R} \\ \tau_{L,R} \end{pmatrix}$$

M is a 3×3 complex matrix. After the standard diagonalization

$\mathcal{L}^{\rm leptons}(x) = \sum_l^3 m_l\, \bar{l}(x)\, l(x)$

$l(x)$ is the Dirac field of l^- and l^+ with the mass m_i

No other possibilities for leptons. Charge is conserved and the mass term must be invariant under global gauge transformations

$l_{L,R}(x) \to e^{-i\Lambda} l(x)_{L,R}$

Neutrino charges are equal to zero. For neutrinos several possibilities of mass terms

I. DIRAC mass term.

$$\mathcal{L}^{\mathrm{D}}(x) = -\bar{\nu}_L(x)\, M^{\mathrm{D}}\, \nu_R(x) + \mathrm{h.c.}$$

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}$$

M^{D} is a 3×3 complex matrix

Any complex nonsingular matrix can be diagonalized by biunitary transformation

$$M = UmV^{\dagger}$$

$$U^{\dagger}U = 1, \quad V^{\dagger}V = 1, \quad m_{ik} = m_i\delta_{ik}$$

After diagonalization

$$\mathcal{L}^{\mathrm{D}}(x) = \sum_{i=1}^{3} m_i \, \bar{\nu}_i(x) \, \nu_i(x)$$

$\nu_i(x)$ is the field of neutrino with the mass m_i

Neutrino mixing

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \; \nu_{iL}(x)$$

The Dirac mass term is invariant under the global gauge transformations

$$\nu_i(x) \to e^{i\,\Lambda}\nu_i(x), \;\; l(x) \to e^{i\,\Lambda}\, l(x), \;\; q(x) \to q(x)$$

Λ is an arbitrary constant

The total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

$\nu_i(x)$ is the four-component Dirac field of neutrinos and antineutrinos with the same mass m_i and different lepton numbers

$$L(\nu_i) = 1, L(\bar{\nu}_i) = -1$$

The Dirac mass term can be generated by the standard Higgs mechanism

Quark, lepton and neutrino masses of the third family

$$m_t \simeq 1.7 \cdot 10^2 \ \mathrm{GeV}, \quad m_b \simeq 4.7 \ \mathrm{GeV}$$

$$m_3 \leq 2.3 \ 10^{-9} \ \mathrm{GeV}, \quad m_\tau \simeq 1.8 \ \mathrm{GeV}$$

It is very unlikely that neutrino masses and quark and lepton masses are of the same Higgs origin

Exist other mechanisms of the generation of the Dirac mass term (models with large extra dimensions, etc)

Other (apparently more likely) possibilities for neutrino mass term

Mass term is a sum of Lorentz-invariant products of left-handed and right-handed components

Can we build a neutrino mass term only from flavor fields ν_{lL}?

Conjugated neutrino field

$$\nu^c(x) = C\bar{\nu}^T(x)$$

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \quad C\gamma_5^T C^{-1} = \gamma_5 \quad C^T = -C$$

$$\gamma_5 \nu_{L.R} = \mp \nu_{L.R}$$

$$\bar{\nu}_L \gamma_5 = \bar{\nu}_L, \quad \gamma_5^T \bar{\nu}_L^T = \bar{\nu}_L^T, \quad \gamma_5 C\bar{\nu}_L^T = C\bar{\nu}_L^T$$

$(\nu_L)^c$ $((\nu_R)^c)$ is right(left) component

Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\,\bar{\nu}_L\, M_L(\nu_L)^c + \mathrm{h.c.}$$

M_L is symmetrical 3×3 complex matrix

$\bar{\nu}_L\, M_L(\nu_L)^c = \bar{\nu}_L\, M_L C\bar{\nu}_L^T = -\bar{\nu}_L\, M_L^T C^T \bar{\nu}_L^T = \bar{\nu}_L\, M_L^T(\nu_L)^c$

Symmetrical matrix is diagonalized by the following unitary transformation

$M = UmU^T, \quad U^\dagger U = 1, \quad m_{ik} = m_i\delta_{ik}$

$\mathcal{L}^{\mathrm{M}}(x) = -\frac{1}{2}\sum_{i=1}^{3} m_i\, \bar{\nu}_i(x)\, \nu_i(x)$

$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x)$ (i=1,2,3)

is Majorana field of neutrinos (no notion of neutrinos and antineutrinos)

There is no global gauge invariance, no conserve lepton number which could allow to distinguish neutrino and antineutrino

$\nu(x) = \int N_p[a_r(p)u^r(p)e^{-ipx} + a_r^\dagger(p)u^r(-p)e^{ipx}]d^3p$

$a_r(p)$ ($a_r^\dagger(p)$) is the operator of absorption (creation) of neutrino

$\nu_{lL} = \sum_{i=1}^{3} U_{li}\,\nu_{iL} \quad (l = e, \mu, \tau)$

The most general DIRAC AND MAJORANA MASS TERM

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\,\bar{\nu}_L\, M_L (\nu_L)^c - \bar{\nu}_L\, M^{\mathrm{D}}\, \nu_R - \frac{1}{2}\,\overline{(\nu_R)^c}\, M_R \nu_R + \mathrm{h.c.}$$

M_L and M_R are symmetrical 3×3 matrices

After the standard diagonalization

$$\mathcal{L}^{\mathrm{D+M}}(x) = -\frac{1}{2}\sum_{i=1}^{6} m_i\,\bar{\nu}_i(x)\,\nu_i(x)$$

$\nu_i(x) = \nu_i^c(x)$ is the Majorana field of neutrino with mass m_i

Mixing relations

$$\nu_{lL} = \sum_{i=1}^{6} U_{li}\,\nu_{iL}, \qquad (\nu_{lR})^c = \sum_{i=1}^{6} U_{\bar{l}i}\,\nu_{iL} \quad l = e, \mu, \tau$$

Active fields ν_{lL} and sterile fields $(\nu_{lR})^c$ are mixtures of the same left-handed components of the six Majorana fields

General conclusions

- Neutrino with definite masses ν_i can be Dirac particle (neutrino and antineutrino differ by a conserved lepton number) or Majorana particle (neutrino and antineutrino are identical)
- Number of neutrinos with definite masses can be larger than three. If this is the case sterile neutrinos exist

The problem of the nature of ν_i is the most fundamental one
The problem can be solved by the investigation of $0\nu\beta\beta$-decay

$$(A, Z) \to (A, Z+2) + e + e$$

Some indications in favor of existence of sterile neutrinos (LSND, MINIBooNE)

D+M mass term in the simplest case of one generation

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\; m_L \bar{\nu}_L (\nu_L)^c - m_D \bar{\nu}_L \nu_R - \frac{1}{2}\; m_R \overline{(\nu_R)^c} \nu_R + \mathrm{h.c.}$$

Assume that $m_{L,R}$ and m_D are real parameters
The mass term can be easily diagonalized

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \sum_{i=1,2} m_i \, \bar{\nu}_i \, \nu_i$$

$\nu_{1,2}$ are Majorana fields
Mixing relations

$$\nu_L = \cos\theta \;\nu_{1L} + \sin\theta \;\nu_{2L} \quad (\nu_R)^c = -\sin\theta \;\nu_{1L} + \cos\theta \;\nu_{2L}$$

$m_{1,2}$ and θ are given by

$$m_{1,2} = \left| \frac{1}{2} (m_R + m_L) \mp \frac{1}{2} \sqrt{(m_R - m_L)^2 + 4\, m_D^2} \right|$$

$$\tan 2\,\theta = \frac{2 m_D}{m_R - m_L}$$

Seesaw mechanism

1. $m_L = 0$
2. m_D is of the order of a mass of quark or lepton
3. Lepton number L is violated at a scale which is much larger than the electroweak scale $m_R \equiv M_R \gg m_D$

$$m_1 \simeq \frac{m_D^2}{M_R} \ll m_D, \quad m_2 \simeq M_R \gg m_D, \quad \theta \simeq \frac{m_D}{M_R} \ll 1$$

In the seesaw approach the smallness of neutrino masses is connected with violation of the total lepton number at a large scale given by M_R.

If $m_D \simeq m_t \simeq 170$ GeV and $m_1 \simeq 5 \cdot 10^{-2}$ we find

$$M_R \simeq \frac{m_D^2}{m_1} \simeq 10^{15} \text{ GeV}.$$

NEUTRINO OSCILLATIONS

Flavor fields in CC and NC are mixed

$$\nu_{lL} = \sum_i U_{li} \nu_{iL}$$

What is the state of the produced flavor neutrino?

Consider (in lab. system) a decay

$$a \to b + l^+ + \nu_l$$

In the case of the neutrino mixing the state of the final particles

$$|f\rangle = \sum_i |\nu_i\rangle |l^+\rangle |b\ \rangle \langle \nu_i\ l^+ b|S|a\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i and momentum $\vec{p}_i = p_i \vec{k}$ and $\langle b\ l^+ \nu_i|S|a\rangle$ is the matrix element of the process

$$a \to b + l^+ + \nu_i$$

In neutrino experiments $E(\gtrsim 1\ \mathrm{MeV}) \gg m_i(\lesssim 1\ \mathrm{eV})$

$$p_i = \sqrt{E_i^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$$

Difference of momenta of neutrino with different masses

$$|p_i - p_k| \simeq \frac{|\Delta m_{ki}^2|}{2E} = \frac{2\pi}{L_{\mathrm{osc}}^{ki}}$$

$$\Delta m_{ki}^2 = m_i^2 - m_k^2$$

Oscillation length

$$L_{\mathrm{osc}}^{ki} = 4\pi\ \frac{E}{|\Delta m_{ki}^2|} \simeq 2.5\ \frac{(E/\mathrm{MeV})}{|\Delta m_{ki}^2| c^4/\mathrm{eV}^2)}\ \mathrm{m}$$

For atmospheric and LBL accelerator neutrino experiments we have $L_{\mathrm{osc}}^{23} \simeq 10^3$ km

For reactor KamLAND experiment we have $L_{\mathrm{osc}}^{12} \simeq 10^2$ km

QM uncertainty of neutrino momenta

$(\Delta p)_{\mathrm{QM}} \simeq \frac{1}{d}$, d characterizes the QM size of the source

$$L^{12}_{\rm osc} \gg d, \quad L^{23}_{\rm osc} \gg d, \quad |p_i - p_k| \ll (\Delta p)_{\rm QM}$$

Difference of momenta of neutrinos with different masses is much smaller than QM uncertainty of momenta. Production of neutrinos with different masses can not be resolved in a production process

Lepton part of the matrix element

$$U^*_{li}\bar{u}_L(p_i)\gamma_\alpha u(-p_l) \simeq U^*_{li}\bar{u}_L(p)\gamma_\alpha u(-p_l)$$

$$\langle \nu_i\ l^+ b|S|a\rangle \simeq U^*_{li}\ \langle \nu_l\ l^+ b|S|a\rangle_{SM}$$

$\langle \nu_l\ l^+ b|S|a\rangle_{SM}$ is the SM matrix element of the process of production of flavor neutrino ν_l in the decay $a \to b + l^+ + \nu_l$

The final state

$$|f\rangle = |\nu_l\rangle |l^+\rangle |b\ \rangle \langle \nu_l\ l^+ b|S|a\rangle_{SM}$$

The state of the flavor neutrino ν_l

$$|\nu_l\rangle = \sum_i U^*_{li}\ |\nu_i\rangle \quad (l = e, \mu, \tau)$$

Flavor neutrino states

I. do not depend on the production process

II. are orthogonal and normalized

$$\langle \nu_{l'} | \nu_l \rangle = \delta_{l'l}$$

III. are characterized by the momentum

The evolution of states in QFT

$$i \frac{\partial \, |\Psi(t)\rangle}{\partial t} = H \, |\Psi(t)\rangle, \quad |\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle$$

H is the total Hamiltonian and t is the parameter which characterize the evolution

If at $t = 0$ ν_l is produced

$$|\nu_l\rangle_t = e^{-iHt} |\nu_l\rangle = \sum_i |\nu_i\rangle e^{-iE_i t} \; U_{li}^* = \sum_{l'} |\nu_{l'}\rangle \sum_i U_{l'i} e^{-iE_i t} \; U_{li}^*$$

$$E_i \simeq E + \frac{m_i^2}{2E}$$

Neutrinos are detected via observation of weak CC and NC processes. Let us consider

$$\nu_{l'} + N \to l' + X$$

Because neutrino masses can not be resolved in weak processes

$$\langle l' X|S|\nu_{l'} N\rangle \simeq \langle l' X|S|\nu_{l'} N\rangle_{SM}$$

To the chain $a \to b + l^{+} + \nu_{l}, \quad \nu_{l} \to \nu_{l'}, \quad \nu_{l'} + N \to l' + X$ corresponds *factorized product*

$$\langle l'\ X|S|\nu_{l'}\ N\rangle_{SM}\ \left(\sum_{i} U_{l'i}\ e^{-iE_i t}\ U^{*}_{li}\right)\ \langle b\ l^{+}\nu_{l}|S|a\rangle_{SM}$$

Factorization property is based on the smallness of the neutrino masses and on the Heisenberg uncertainty relation

Neutrino transition probability

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i}\ e^{-iE_i t}\ U_{li}^*|^2 = |\sum_i U_{l'i}\ e^{-i\Delta m_{ji}^2 \frac{L}{2E}}\ U_{li}^*|^2 =$$

$$|\sum_{i\neq j} U_{l'i}\ (e^{-i\Delta m_{ji}^2 \frac{L}{2E}} - 1)\ U_{li}^* + \delta_{l'l}|^2$$

We took into account that $t \simeq L$

There exist other approaches to neutrino oscillations

For example, plane wave approach

$$P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i} e^{-i(p_i - p_j)\cdot x} U_{li}^*|^2$$

The phase difference

$$(p_i - p_j)\cdot x = (E_i - E_j)t - (p_i - p_j)L = [(E_i - E_j) - (p_i - p_j)L]L = \Delta m_{ji}^2 \frac{L}{2E}$$

The same result

The simplest case: two neutrino oscillations

$$P(\nu_l \to \nu_{l'}) = |U_{l'2}\ (e^{-i\Delta m^2_{12}\frac{L}{2E}} - 1)\ U^*_{l2} + \delta_{l'l}|^2$$

$$U = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix}$$

$$l' \neq l$$

Transition probability

$$P(\nu_l \to \nu_{l'}) = 2|U_{l'2}|^2|U_{l2}|^2(1 - \cos\Delta m^2_{12}\frac{L}{2E}) =$$
$$\frac{1}{2}\sin^2 2\theta_{12}(1 - \cos\Delta m^2_{12}\frac{L}{2E}) = P(\nu_{l'} \to \nu_l)$$

Survival probability

$$P(\nu_l \to \nu_l) = 1 - P(\nu_l \to \nu_{l'}) =$$
$$1 - \frac{1}{2}\sin^2 2\theta_{12}(1 - \cos\Delta m^2_{12}\frac{L}{2E}) = P(\nu_{l'} \to \nu_{l'})$$

Oscillation length

$$P(\nu_l \to \nu_l) = 1 - \frac{1}{2}\sin^2 2\theta_{12}(1 - \cos\frac{L}{L_{\rm osc}})$$

$$L_{\rm osc} = 4\pi\ \frac{E}{\Delta m^2_{12}} \simeq 2.5\ \frac{(E/{\rm MeV})}{(\Delta m^2_{12}/{\rm eV}^2)}\ {\rm m}$$

characterizes a distance where oscillations can be observed

Accelerator LBL experiments: $E \simeq 1$ GeV, $\Delta m^2_{23} \simeq 2.5 \cdot 10^{-3}{\rm eV}^2$, $L^{23}_{\rm osc} \simeq 10^3$ km

Reactor KamLAND experiment; $E \simeq 3$ MeV, $\Delta m^2_{12} \simeq 8 \cdot 10^{-5}{\rm eV}^2$, $L^{12}_{\rm osc} \simeq 10^2$ km

Three neutrino oscillations

The unitary 3×3 PMNS mixing matrix is characterized by three mixing angles and one CP phase. It can be obtained by three Euler rotations

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In the Majorana case additional phase matrix enter

$$U_M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

The Majorana phases $\alpha_{1,2}$ do not enter into transition probability

In the case of three neutrino mixing six parameters: $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}, \theta_{23}, \theta_{13}, \delta$ enter into transition probability

From experimental data follows

$\Delta m^2_{12} \simeq \frac{1}{30} \Delta m^2_{23}, \quad \sin^2\theta_{13} \leq 4 \cdot 10^{-2}$

Leading approximation works very well

Atmospheric, LBL accelerator region of $\frac{L}{E}$ $(\frac{\Delta m^2_{23} L}{2E} \gtrsim 1)$

contribution of Δm^2_{12} can be neglected

$$P(\nu_l \to \nu_{l'}) = |U_{l'3}\ (e^{i \Delta m^2_{23} \frac{L}{2E}} - 1)\ U^*_{l3} + \delta_{l'l}|^2$$

$U_{e3} = \sin\theta_{13} e^{-i\delta} \to 0,\ l, l' = \mu, \tau$

Dominant transitions $\nu_\mu \leftrightarrows \nu_\tau$

Survival probability

$$P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu) = 1 - \frac{1}{2}\sin^2 2\theta_{23}(1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

From the analysis of the LBL accelerator neutrino MINOS data

$$\Delta m^2_{23} = (2.43 \pm 0.13) \cdot 10^{-3}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{23} > 0.90$$

Good agreement with atmospheric neutrino data

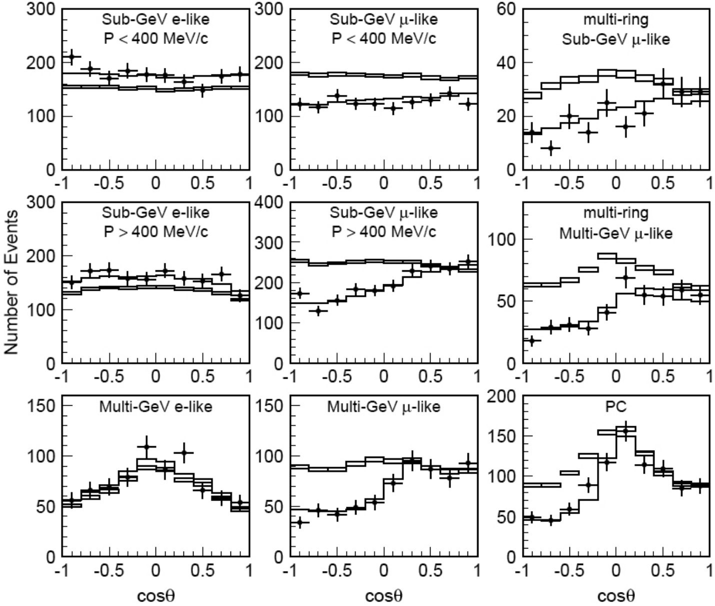
In the reactor KamLAND region of $\frac{L}{E}$ ($\frac{\Delta m^2_{12} L}{2E} \gtrsim 1$) we have

$$P(\nu_e \to \nu_e) = ||U_{e1}|^2\ (e^{i\Delta m^2_{12}\frac{L}{2E}} - 1)\ + 1|^2 =$$
$$1 - \frac{1}{2}\ \sin^2 2\theta_{12}\ (1 - \cos\Delta m^2_{12}\frac{L}{2E}) = P(\bar{\nu}_e \to \bar{\nu}_e)$$

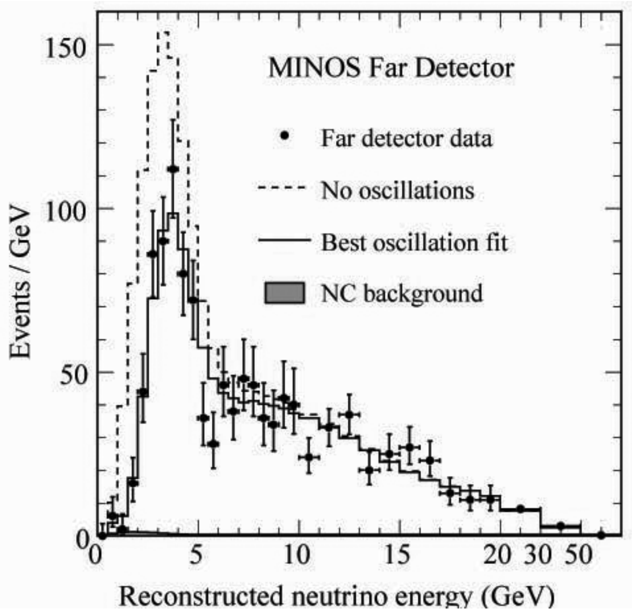
From the global analysis of the reactor KamLAND and solar data

$$\Delta m^2_{12} = (7.50^{+0.19}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2, \quad \tan^2\theta_{12} = 0.452^{+0.035}_{-0.032}$$

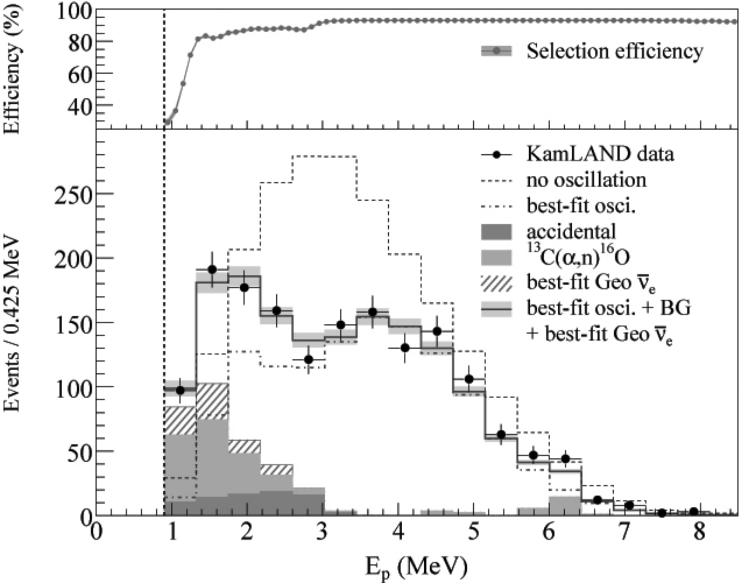
From reactor CHOOZ data $\sin^2\theta_{13} \leq 4 \cdot 10^{-2}$



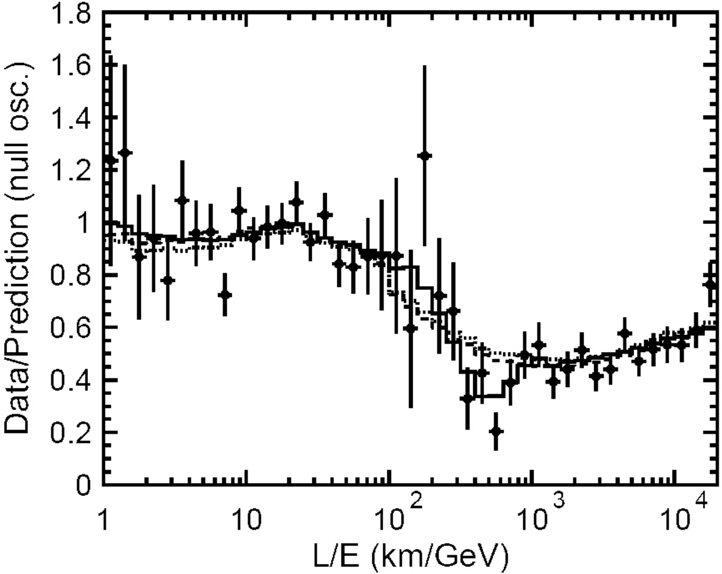
Sub-GeV e-like
P < 400 MeV/c
Sub-GeV μ-like
P < 400 MeV/c
multi-ring
Sub-GeV μ-like
Sub-GeV e-like
P > 400 MeV/c
Sub-GeV μ-like
P > 400 MeV/c
multi-ring
Multi-GeV μ-like
Multi-GeV e-like
Multi-GeV μ-like
PC
Number of Events
cosθ



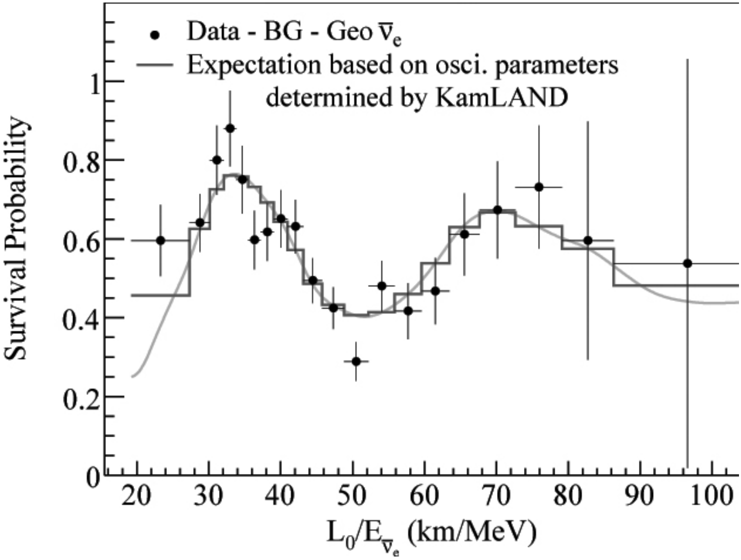
MINOS Far Detector
Far detector data
No oscillations
Best oscillation fit
NC background
Events / GeV
150
100
50
0
0
5
10
15
20
30
50
Reconstructed neutrino energy (GeV)



Efficiency (%)
100
80
60
40
Selection efficiency
KamLAND data
no oscillation
best-fit osci.
accidental
$^{13}C(\alpha,n)^{16}O$
best-fit Geo $\bar{\nu}_e$
best-fit osci. + BG
+ best-fit Geo $\bar{\nu}_e$
Events / 0.425 MeV
250
200
150
100
50
0
0
1
2
3
4
5
6
7
8
E_p (MeV)



Data/Prediction (null osc.)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
10
10 2
10 3
10 4
L/E (km/GeV)



Data - BG - Geo $\overline{\nu}_e$
Expectation based on osci. parameters determined by KamLAND
Survival Probability
0
0.2
0.4
0.6
0.8
1
20
30
40
50
60
70
80
90
100
$L_0/E_{\overline{\nu}_e}$ (km/MeV)

PRESENT STATUS OF NEUTRINO MASSES AND MIXING

Four neutrino oscillation parameters are known with accuracies (3-10)%. Upper bound on $\sin^2\theta_{13}$. No information about CP phase δ

From TROITSK and MAINZ tritium experiments on the measurement of the absolute value of the neutrino mass

$$m_\beta < 2.3 \text{ eV}$$

From cosmology $\sum_i m_i < (0.6 - 1.0)$ eV

In future accelerator T2K, reactor Double CHOOZ, RENO and Daya Bay experiments sensitivities to $\sin^2\theta_{13}$ will be at least one order of magnitude better than in the CHOOZ experiment

If the value of the parameter $\sin^2\theta_{13}$ will be measured, in the experiments of the next generation CP violation in the lepton sector and neutrino mass spectrum will be studied

New Facilities (Super beam, β-beam, Neutrino factory) under R&D

NEUTRINO MASSES ARE DIFFERENT FROM ZERO BUT VERY SMALL

Much smaller than masses of quarks and leptons

New beyond the SM physics?

The most plausible and popular mechanism of neutrino mass generation is SEESAW MECHANISM

A beyond the SM physics generate non-renormalizable effective Lagrangians

In the EW region we have

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{n} \frac{c_n}{\Lambda^n} \mathcal{L}^{\mathrm{eff}}_{4+n}$$

The large parameter Λ has dimension M and characterizes a scale of a new physics

The only dimension five effective Lagrangian has the form (Weinberg)

$$\mathcal{L}_5^{\text{eff}} = -\frac{1}{\Lambda} \sum_{l',l,i} \overline{L}_{l'L} \tilde{H} X_{l'l} C \tilde{H}^T (\overline{L}_{lL})^T + \text{h.c.}.$$

$$L_{lL} = \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix} \qquad H = \begin{pmatrix} H^{(+)} \\ H^{(0)} \end{pmatrix} \qquad \tilde{H} = i\tau_2 H^*$$

The Lagrangian $\mathcal{L}_5^{\text{eff}}$ does not conserve the total lepton number L

After electroweak symmetry breaking

$$\tilde{H} = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \qquad v = (\sqrt{2} G_F)^{-1/2} \simeq 246 \text{ GeV}$$

(parameter v (Higgs vacuum expectation value) characterizes scale of the electroweak breaking)

The left-handed Majorana mass term is generated

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \sum_{l'l} \bar{\nu}_{l'L} M^{L}_{l'l} \; C \bar{\nu}^{T}_{lL} + \mathrm{h.c.}$$

$$M^{L}_{l'l} = \frac{v^2}{\Lambda} X_{l'l}$$

Performing the standard diagonalization of the mass term
$(X = U x U^{T})$ we have
$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2} \sum_{i} m_i \bar{\nu}_i \nu_i$
$\nu_i = \nu_i^c$ is the field of Majorana neutrino with mass m_i
Neutrino masses and mixing

$$m_i = \frac{v^2}{\Lambda} x_i \quad \nu_{lL} = \sum_{i} U_{li} \nu_{iL}$$

Neutrino masses are determined by the seesaw factor

$$\frac{v^2}{\Lambda} = \frac{(\mathrm{EW\ scale})^2}{\mathrm{scale\ of\ new\ physics}}$$

We can estimate $\Lambda \simeq (10^{14} - 10^{15})$ GeV

The effective Lagrangian $\mathcal{L}_5^{\mathrm{eff}}$ can be generated by the following Yukawa interaction of heavy Majorana leptons N_i, singlets of the $SU_L(2) \times U(1)$ group, with lepton and Higgs doublets

$$\mathcal{L}_{\mathcal{I}} = -\sqrt{2} \sum_{i,l} Y_{li} \overline{L}_{lL} N_{iR} \tilde{H} + \mathrm{h.c.}.$$

At electroweak energies for the processes with virtual N_i this interaction generates the effective Lagrangian $\mathcal{L}_5^{\mathrm{eff}}$ with $X_{l'l} = \sum_i Y_{l'i} \frac{\Lambda}{M_i} Y_{li}$

$M_i \sim \Lambda$ is the mass of the heavy lepton N_i

Left-handed Majorana mass term can be generated not only by the interaction $\mathcal{L}_{\mathcal{I}}$ (type II seesaw)

Small Majorana neutrino masses are the only signature of a beyond the SM physics at a very large GUT scale where the total lepton number L is violated (original seesaw)

How can we test this idea?

First of all we need to prove that neutrinos with definite mass ν_i are Majorana particles?

This can not be done in neutrino oscillation experiments. We need to observe processes in which the total lepton number is violated

If neutrinos with definite masses are Majorana particles, some processes with virtual neutrinos in which lepton number is violated are allowed

The most sensitive to small neutrino masses process is $0\nu\beta\beta$-decay

$$(A,Z) \to (A,Z+2) + e^- + e^-$$

Neutrino propagator enters into the matrix element of $0\nu\beta\beta$-decay the form

$$\sum U_{ei}^2 \frac{1-\gamma_5}{2} \frac{\gamma\cdot + m_i}{p^2 - m_i^2} \frac{1-\gamma_5}{2} \simeq m_{\beta\beta} \frac{1}{p^2} \frac{1-\gamma_5}{2}$$

The probability of the $0\nu\beta\beta$-decay is extremely small

I. It is second order in the Fermi constant process

II. Additional suppression factor $m_{\beta\beta}\frac{1}{p^2}$ due to $V-A$ structure of currents ($|m_{\beta\beta}| \leq 1$ eV and $\bar{p}^2 \simeq 10^2\mathrm{MeV}^2$)

Half-life is given by the expression

$$\frac{1}{T^{0\,\nu}_{1/2}(A,Z)} = |m_{\beta\beta}|^2\,|M(A,Z)|^2\,G^{0\,\nu}(E_0,Z)$$

$M(A,Z)$ is the nuclear matrix elements and $G^{0\,\nu}(E_0,Z)$ is known phase-space factor

Theoretical problems of calculation of NME (five models, all give different results)

From existing data

$$|m_{\beta\beta}| < (0.20-0.32)\,\mathrm{eV}(^{76}\mathrm{Ge}),\quad |\mathrm{m}_{\beta\beta}| < (0.19-0.68)\,\mathrm{eV}(^{130}\mathrm{Te})$$

Future experiments will be sensitive to $|m_{\beta\beta}| = \mathrm{a\ few}10^{-2}\ \mathrm{eV}$