Nucleon Matrix Elements from Lattice QCD

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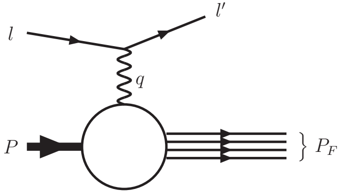
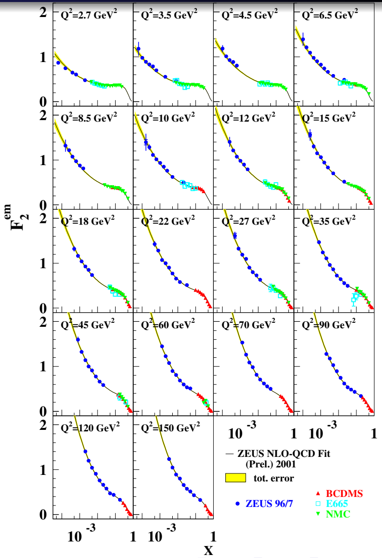
Outline

Motivation

- Experiment:
  precise measurements of unpol. structure functions in a wide kinematic regime
- Lattice:
  cannot access structure functions directly, but moments thereof are related to matrix elements of local operators
- in particular interested in (moments of) PDFs (important in processes with hadrons in initial state and large momentum tranfers)





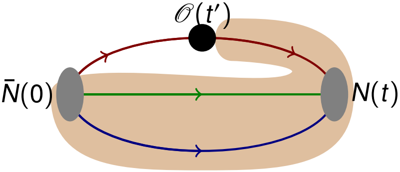
Basics I

- lattice calculations: u and d quark degenerate
  $\Rightarrow$ proton/neutron = nucleon
- save computational effort: pion masses higher than physical one
  $\Rightarrow$ extrapolation needed ($\exists$ chiral pert. theory), “chiral limit”
- continuum limit $a \rightarrow 0$ usually less problematic
  (in particular in our setup, leading effect is $\mathscr{O}(a^2)$)
- state of the art: 2 to 4 dynamical quark flavors in the simulations

Basics II

Nucleon matrix elements ↔ asymptotic limit of lattice nucleon 3-point function involving local operator

$$\langle N|\,\mathscr{O}\,|N\rangle = \lim_{t,t'\to\infty} \frac{\langle J_N(t)\big|\,\mathscr{O}(t')\,\big|J_{\bar{N}}(0)\rangle}{\langle J_N|\,J_{\bar{N}}\rangle}$$



- $J_{\bar{N}}$ (J_N): lattice nucleon creation (annihilation) operator
- $\mathscr{O}$ in most cases quark bilinear $\sim \bar{q}\Gamma q$
- for feasibility reasons: t fixed, (generally not very large)
  ⇒ basically unstudied systematic effect

general problem: $J_{\bar{N}}$ creates all states with same quantum numbers as nucleon
⇒ also excited states
main contributions to (nucleon) 3-point function

$$\frac{\langle J_N(t) |\, \mathcal{O}(t')\, | J_{\bar{N}}(0)\rangle}{\langle J_N(t) |\, J_{\bar{N}}(0)\rangle} \sim \quad \mathcal{O}^{(0,0)} \tag{1}$$

$$+\ \mathcal{O}^{(1,0)}\, \frac{J_N^{(1)}}{J_N^{(0)}} \qquad \exp\left(-\Delta m t'\right) \tag{2}$$

$$+\ \mathcal{O}^{(0,1)}\, \frac{J_{\bar{N}}^{(1)}}{J_{\bar{N}}^{(0)}} \qquad \exp\left[-\Delta m(t-t')\right] \tag{3}$$

$$+\ \mathcal{O}^{(1,1)}\, \frac{J_N^{(1)}}{J_N^{(0)}}\, \frac{J_{\bar{N}}^{(1)}}{J_{\bar{N}}^{(0)}}\, \exp\left(-\Delta m t\right) \tag{4}$$

$J_{\bar{N}}^{(i)}$: overlap with i$^{\text{th}}$ exited state, $\langle 0|N|i\rangle$
Δm : mass difference between ground state and first excited state

- notice that t (source-sink separation) is usually fixed
- (4) independent of t' ⇒ no handle unless several t available

Observables

- $\langle x \rangle_{u-d}$ (“momentum fraction”), relation to local operator via

$$\langle N(p,s)| \underbrace{\overline{q}\gamma^{\{\mu} iD^{\nu\}}\tau^3 q}_{O^{\mu\nu}}, |N(p,s)\rangle \Big|_{\mu^2} = 2\,\langle x \rangle_{u-d,\mu^2}\, p^{\{\mu} p^{\nu\}}$$

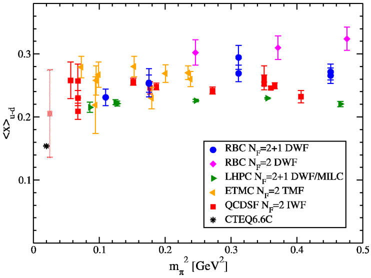
$$\langle x \rangle_{q,\mu^2} = \int_{-1}^{1} dx\; xq(x,\mu^2) = \int_{0}^{1} dx\; x\left\{q(x,\mu^2) + \overline{q}(x,\mu^2)\right\}$$

- nucleon axial charge g_A (“charge pions couple to”) → neutron decay

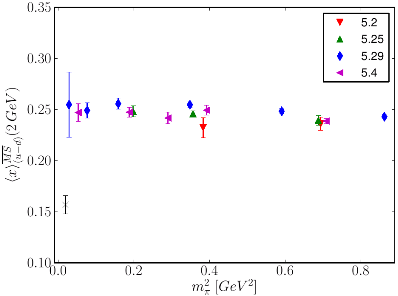
$$\langle N(p,s)|\overline{q}\gamma_\mu\gamma_5\tau^3 q|N(p,s)\rangle = 2g_A s_\mu$$

Comparison between lattice and experiment

summary plot by D. Renner,
Lattice 2009

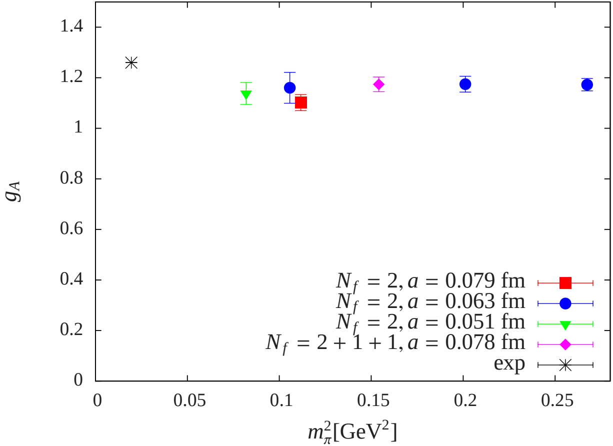


QCDSF results, J. Zanotti,
T(r)opical QCD 2010

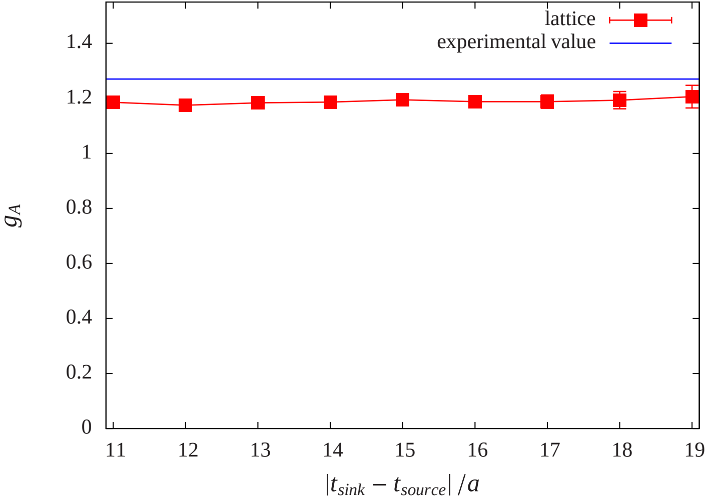


- similar situation for g_A, but difference smaller
- chiral limit?
- ETMC: more realistic setup with $N_f = 2 + 1 + 1$
  lower pion masses ($\lesssim$ 300 MeV) not available yet
- investigate excited state contamination

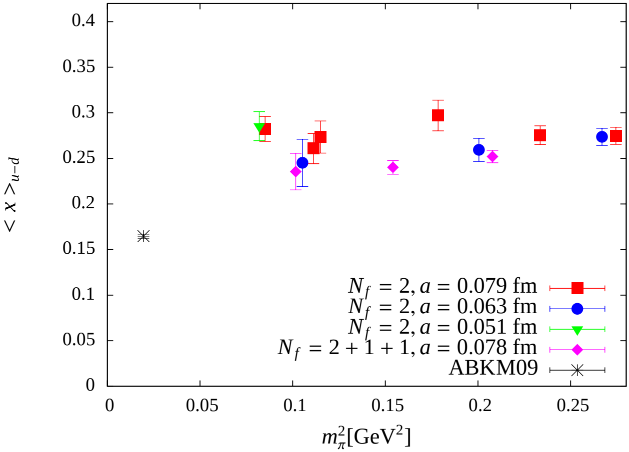
$N_f = 2$ and $N_f = 2+1+1$ results for g_A



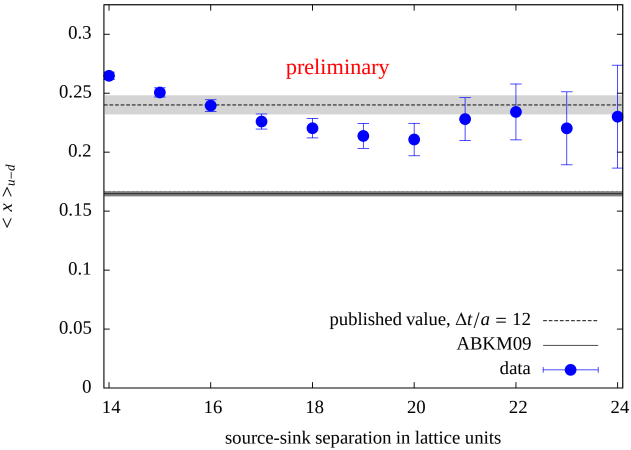
source-sink separation dependence of g_A



$N_f = 2$ and $N_f = 2+1+1$ results for $\langle x \rangle_{u-d}$



source-sink separation dependence of $\langle x\rangle_{u-d}$



Summary and Outlook

- calculations of g_A and $\langle x \rangle_{u-d}$ for $N_f = 2 + 1 + 1$
- non-perturbative renormalization factor (not covered in this talk)
- agreement with $N_f = 2$ results
- high statistics run revealing excited state contamination:
  g_A: contamination negligible
  $\langle x \rangle_{u-d}$: results can change (preliminary!)
- future: other systematic effects ⇒ ratios of matrix elements, lower pion masses