Drell-Yan Cross Section at NNLO via Mellin Space

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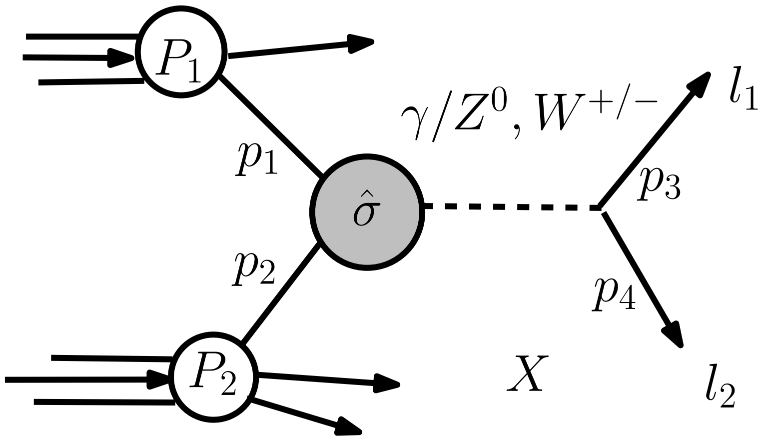
Motivation - Precision Physics at the LHC

- Search for physics beyond Standard Model at the LHC $\rightarrow$ precision knowledge of benchmark processes
  - Understanding the machine and detectors
  - Testing SM at new energies
- A general hadronic collision is a complex process involving different energy scales:
  hard scattering, parton distribution functions, parton showers, hadronization, underlying event

The Drell-Yan Process

Massive lepton pair production in hadron-hadron collision, $M^2_{l_1 l_2} >> 1$ GeV^2



- $M_{l_1 l_2} = Q^2 = (p_2 + p_3)^2$
- CM energy of hadrons $s = (P_1 + P_2)^2$
- $p_i = x_i P_i \qquad x_i \in (0, 1)$
- CM energy of partons $\hat{s} = (p_1 + p_2)^2 = s x_1 x_2$

Neutral Current	$pp \to \gamma^*/Z^0 \to l\bar{l}X$	$M_Z \sim 91.2$ GeV
Charged Current	$pp \to W^\pm \to l\nu X$	$M_W \sim 80.4$ GeV

- Very clean signal $\Rightarrow$ easy to measure experimentally

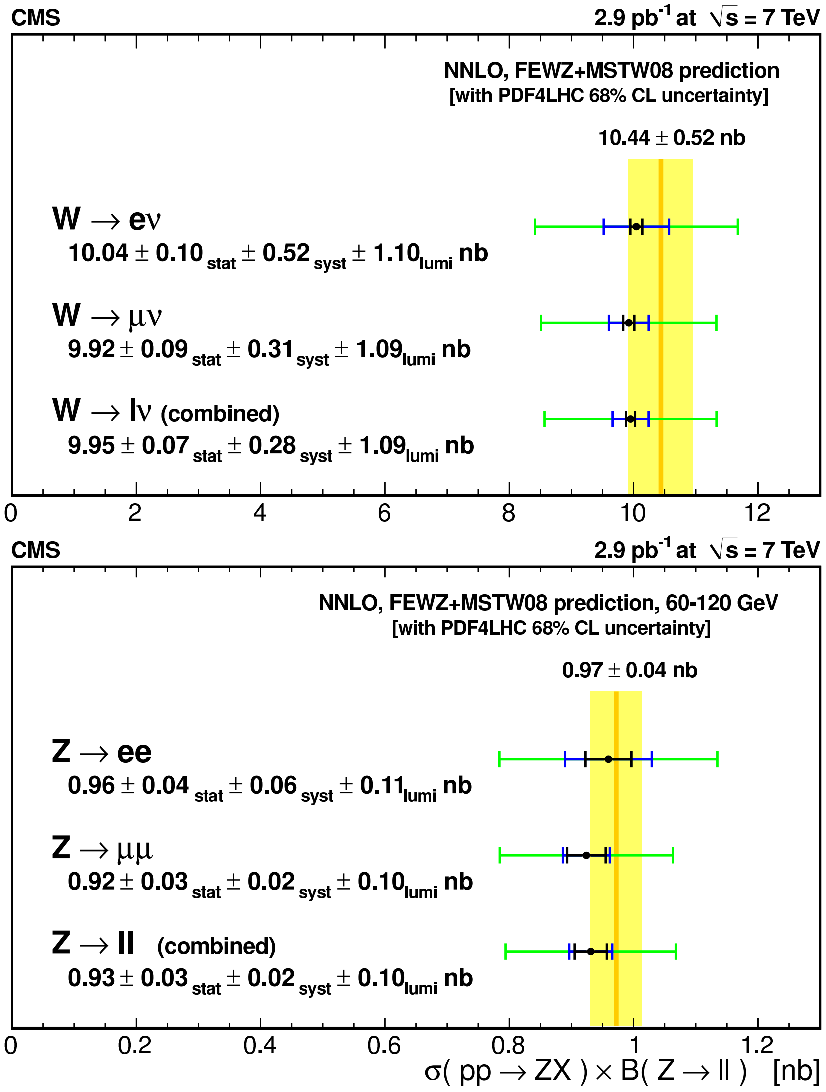
The Cross Section

Factorization theorem allows for separation of physics at different scales $\Rightarrow$

$$\sigma^V_{DY} = \sum_{a,b=q,\bar{q},g} \int_0^1 \mathrm{d}\, x_1 \int_0^1 \mathrm{d}\, x_2 \underbrace{f_a(x_1,\mu_f^2) f_b(x_2,\mu_f^2)}_{\text{PDFs}} \underbrace{\hat{\sigma}^V_{ab}(Q^2,\mu_f^2,x_1,x_2)}_{\text{Partonic cross section}}$$

- Partonic cross section calculable with perturbation theory
- Parton Distribution Functions (PDFs) non-perturbative, process independent, extracted from measurements (DIS, fixed target, DY@Tevatron)

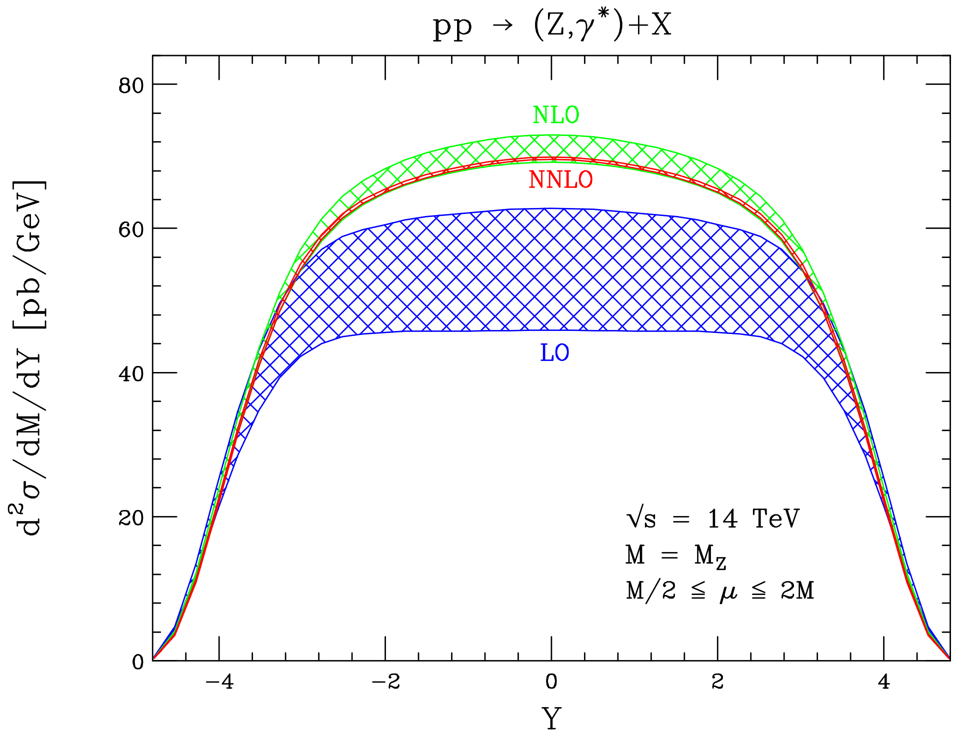
W and Z production at the LHC



CMS measurements at $\sqrt{s} = 7$ TeV and integrated luminosity $2.9\,\text{pb}^{-1}$

- Large cross sections
- Agreement between theory and data
- Experimental accuracy will soon reach the theoretical one
- Two main sources of theoretical uncertainties
  - Perturbative calculation
  - PDFs

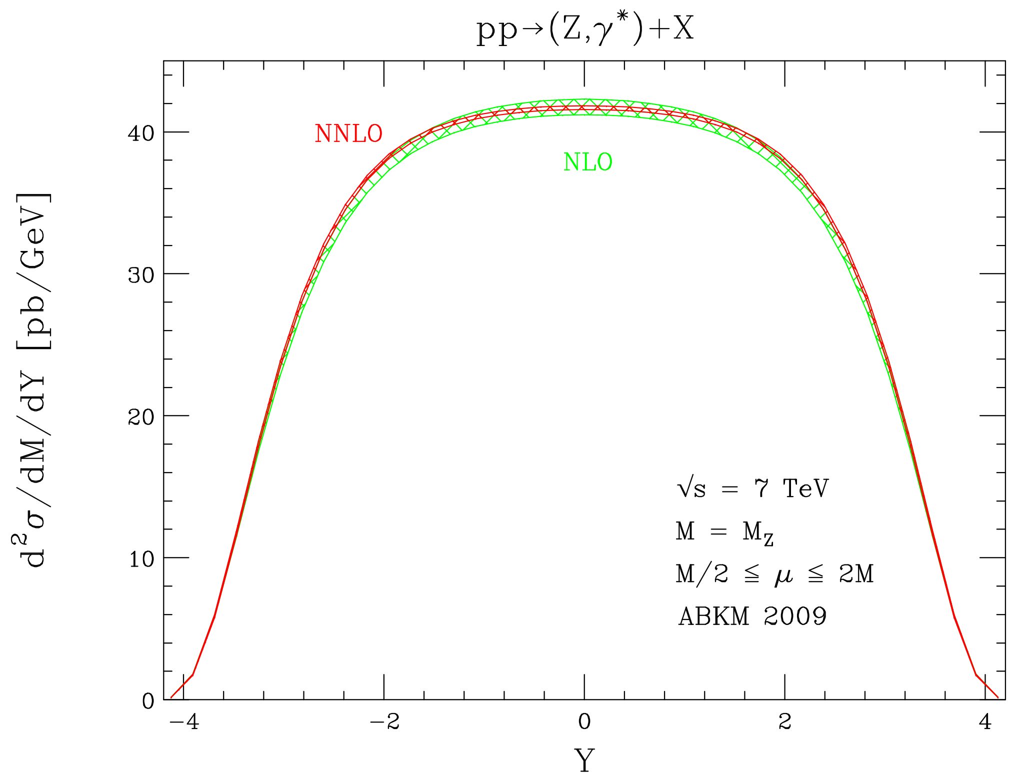
Higher order corrections



[Anastasiou, Dixon, Melnikov, Petriello 2004]

- QCD corrections known up to NNLO [Altarelli, Ellis, Martinelli 1979] [Hamberg, Matsuura, van Neerven 1990, Harlander, Kilgore 2002]
- LO - not predictive, large scale uncertainties
- NLO corrections are large (up to $\sim 30\%$ at the LHC)- need of NNLO
- NNLO - reduces significantly scale dependence, good convergence of PT
- EW corrections at NLO [Hollik, Wackeroth 1996]
- QCD+EW [Vicini et.al. 2009]

Higher order corrections



[Dixon,unpublished (2011)]

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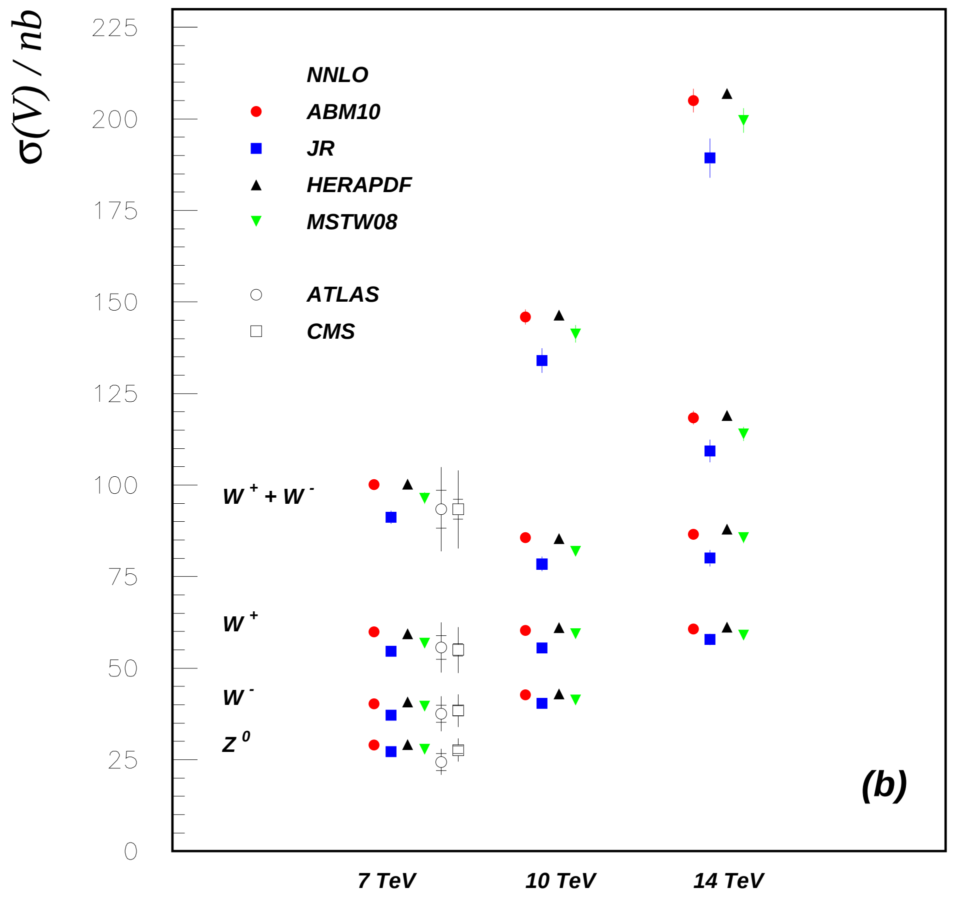
W and Z Production at the LHC



- PDFs are the main sources of theoretical uncertainties
- DY will soon provide more insight into PDFs $\rightarrow$ need fast and accurate way to evaluate the cross sections

Comparison of different predictions based on recent NNLO PDF analysis and the corresponding experimental data [Alekhin, Blumlein, Jimenez-Delgado, Moch, Reya 2010]

The Cross Section via Mellin Space

- Cross section (x-space): Two integrations

$$\frac{\sigma^V_{DY}(Q^2)}{\mathrm{d}\,Q^2} \sim \sum_{a,b} C^V_{ab} \int_0^1 \mathrm{d}\,x_1 \int_0^1 \mathrm{d}\,x_2 \overbrace{\Delta(x, x_1, x_2)}^{\text{Coefficient fcts}} f_a(x_1, Q^2) f_b(x_2, Q^2)$$

$$= \underbrace{\sum_{a,b} C^V_{ab}(f_a \otimes f_b \otimes \Delta)(x)}_{\text{Structure function } W(x,Q^2)} \qquad \otimes \equiv \text{Convolution} \qquad x = Q^2/s$$

- Mellin transform of coefficient functions and PDFs - cross section in N space

$$(f_a \otimes f_b \otimes \Delta)(x) \to f_a(N, Q^2) f_b(N, Q^2) \Delta(N)$$

  Convolution in x space $\to$ product in N-space

- Inverse Mellin transform in order to recover the original x space

The Mellin Transform and Inversion

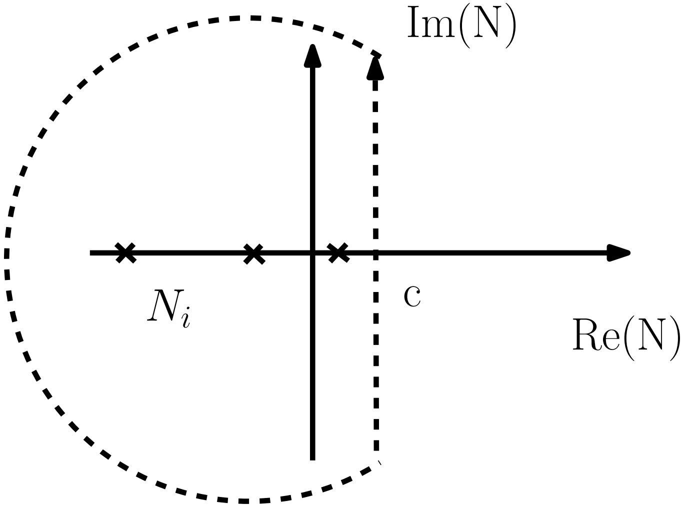
- The Mellin transform : $f(x) \to \tilde{f}(N)$

$$\mathbf{M}[f(x)] \;=\; \int_0^1 \mathrm{d}\,x\,x^{N-1} f(x) = \tilde{f}(N)$$

- The inverse Mellin Transform - integral over a complex plane

$$f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\,x^{-N}\tilde{f}(N)$$

  - All poles of $\tilde{f}(N)$ lie to the left from the contour c
  - Well suited for numerical integration



Ingredients

- Mellin transforms

  [Vermaseren 1998; Moch, Vermaseren, Vogt 2004; Blümlein, Kurth 1998 (2000); Blümlein, Ravindran 2005, . . .]

  - Coefficient functions
    - N space results calculated [Blümlein, Ravindran 2005]
    - Use of `harmpol` package [Remiddi, Vermaseren 2000]
  - Parton distribution functions
    - Analytic expression for initial parametrization $\rightarrow$ evolution in N space [`QCD-PEGASUS`, Vogt 2005]
    - Direct access to LHAPDF grids at required scale $\rightarrow$ interpolation
- Fast and accurate inversion
  - Analytic continuation of N space expressions to the complex plane
    - finite set of more complicated expression needs special treatment, e.g. [`ANCONT`, Blümlein 2000]
  - Suitable choice of contour
  - Integration using Gaussian quadrature

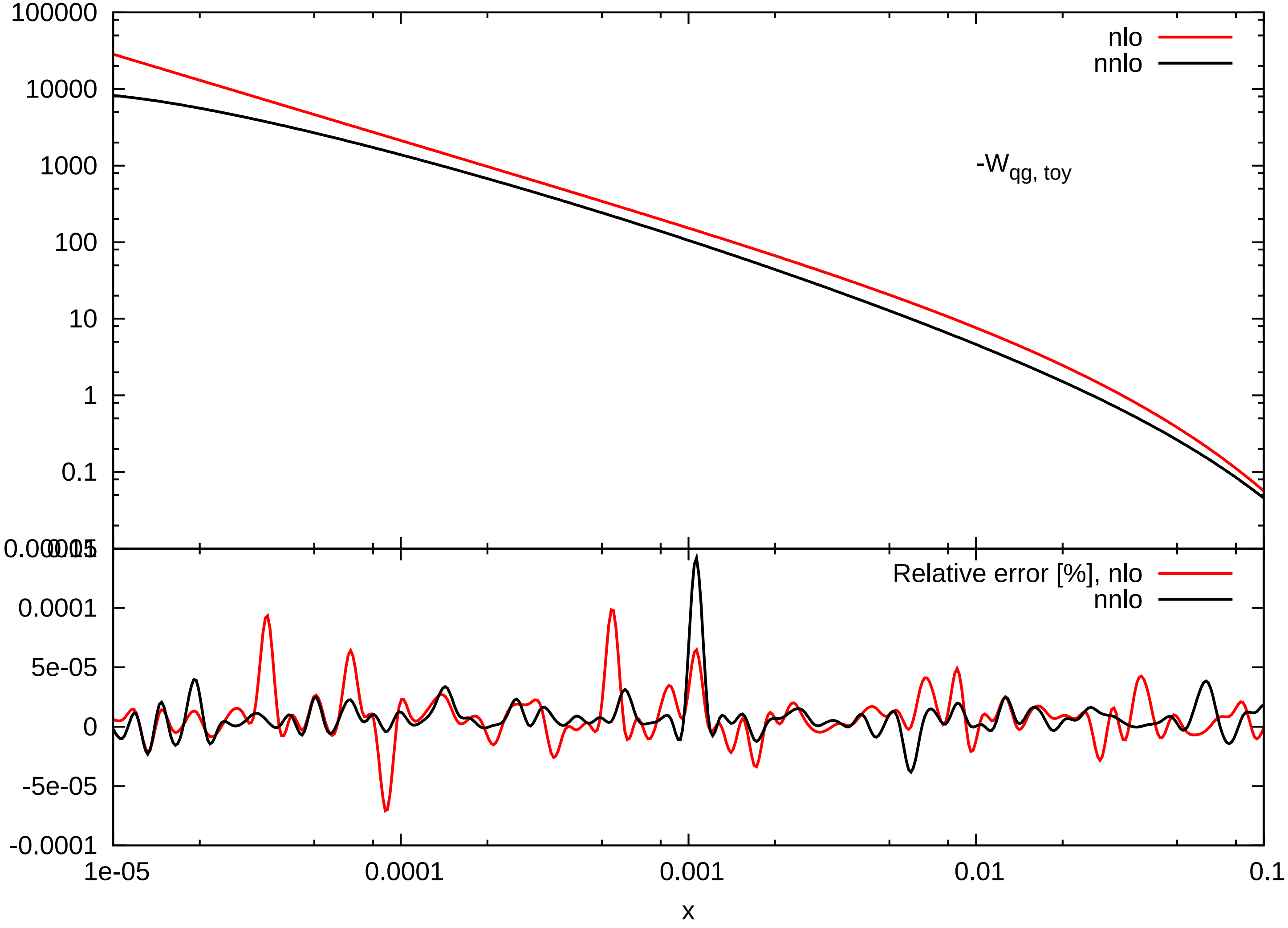
Checks

- Many programs on the market to provide checks
  - MCFM (NLO) [Campbell, Ellis]
  - ZWPROD [Hamberg, Matsuura, van Neerven 2002]
  - DYNNLO [Catani, Cieri, Ferrera, de Florian, Grazzini 2009; Catani, Grazzini 2007]
  - FEWZ [Melnikov, Petriello]
  - DYN (not public) [Blümlein,Ravindran 2005,Blümlein 2000]
- Calculation in a standard way (x space). As a start, use a toy PDF input

$$
\begin{aligned}
f(x,Q^2) &= ax^{b-1}(1-x)^c(1+dx^f+gx), \qquad a,\ldots,g\in\mathbb{R} \\
f(N,Q^2) &= a\Big[\beta(N+b,c+1)+d\beta(N+b+f,c+1) \\
&+ g\beta(N+b+1,c+1)\Big]
\end{aligned}
$$

β- Euler Beta function

QG contribution to the DY structure function



$$W^{\mathsf{n}^k\mathsf{lo}}(N)_{qg,toy} = \left(\frac{\alpha_s}{4\pi}\right)^k C_{qg}(N) f_{q,toy}(N) f_{g,toy}(N), \qquad W_{qg,toy} = \mathsf{IM}[W_{qg,toy}(N)]$$

Summary and Outlook

- Higher order corrections for Drell-Yan process are necessary for precise predictions
- Calculation in Mellin space is fast and accurate

- Done
  - c++ code calculating W and Z production up to NNLO checked against standard x space calculation with toy PDFs
  - Link to QCD-PEGASUS

- In Progress
  - Interface with LHAPDF grids
  - Checks for against other public codes
  - Implementation of DIS up to NNLO (easy)
  - Optimization of code and aim for public release