QUANTUM OPTICS
Wintersemester 2008/2009

Blatt 2
zur Übung am 27. Oktober 2008

Exercise 1: Commutator relations

Based on the quantum mechanical operators of the quantum mechanical harmonic oscillator

$$\hat{q} = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(\hat{a} + \hat{a}^\dagger), \quad (1)$$

$$\hat{p} = -i\left(\frac{\hbar m\omega}{2}\right)^{1/2}(\hat{a} - \hat{a}^\dagger) \quad (2)$$

as well as their commutator $[\hat{q}, \hat{p}] = i\hbar$, show that

$$[\hat{a}, \hat{a}^\dagger] = 1.$$

Show also the following relations:

$$[\hat{a}, (\hat{a}^\dagger)^n] = n(\hat{a}^\dagger)^{n-1},$$

$$[\hat{a}^n, \hat{a}^\dagger] = n\hat{a}^{n-1}$$

Exercise 2: Coherent state

Construct the coherent state $|\alpha\rangle$ from the postulation that it is an eigen state of the annihilation operator $\hat{a}$, i.e. $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$.

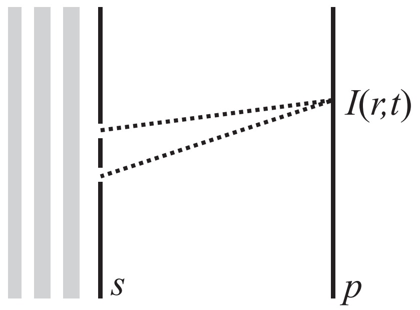
Exercise 3: Coherent state with an unknown phase

Let $\alpha = |\alpha| \exp(i\phi)$, with an unknown phase ϕ and uniformly distributed. Show that

$$\hat{\rho} = \frac{1}{2\pi}\int_0^{2\pi} |\alpha\rangle\langle\alpha| d\phi = \sum_n^\infty e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}|n\rangle\langle n|,$$

i.e., the phase ignorance washes out the off-diagonal elements.

Exercise 4: Quantum and classical interference



A single photon $|1\rangle$ is impinging on a double slit s. Behind the slit, the state is in a superposition between the two possible paths (index 1 and 2): $|\psi\rangle = 2^{-1/2}(\hat{a}_1{}^\dagger + \exp(i\phi)\hat{a}_2{}^\dagger)|0\rangle$. The phase $\phi = (k - \omega/c)\Delta s$ is given by the path difference. Show that the intensity at point $\mathbf{r}, t$ has the form

$$I(\mathbf{r}, t) = I_0(1 + \cos\phi).$$

Thus, the result is the same as in the classical case.

Exercise 5: Variance of a Schrödinger cat state

The variance V^2 of a photon field is given by

$$V^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2.$$

Let $|\Psi\rangle = c_1|\alpha\rangle + c_2|-\alpha\rangle$ be a superposition of the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$, with $|c_1|^2 + |c_2|^2 = 1$. Under which conditions is the statistic

a) sub-poissonian, i.e. $V_n = \frac{\langle\Psi|V^2|\Psi\rangle}{\langle\Psi|\hat{n}|\Psi\rangle} < 1$?

b) poissonian, i.e. $V_n = 1$?

c) super-poissonian, i.e. $V_n > 1$?