QUANTUM OPTICS
Wintersemester 2008/2009

Blatt 1
zur Übung am 20. Oktober 2008

Exercise 1

Estimate the number of photons in the visible regime of the universe. Assume that the volume is homogeneously irradiated by the cosmic microwave background radiation ($T = 2.7$ K). Further assume that the universe is flat within this regime and has a radius of 12 billion light years.
Calculate also the mean energy per photon and the total amount of energy of this radiation. Compare this with the amount of energy arising from matter (you can assume a total mass that corresponds to 10^{80} neutrons).

Exercise 2

Derive Planck's law from the Einstein coefficients A and B by using the Boltzmann distribution.

Exercise 3

It is well known that for a classical electromagnetic field the energy is given by

$$H = \frac{1}{2}\int(\epsilon_0\mathbf{E}^2 + \mu_0\mathbf{H}^2)d^3r. \tag{1}$$

With

$$\begin{aligned}\hat{\mathbf{E}} &= \sum_{\mathbf{k}}\left(\frac{\hbar\omega_{\mathbf{k}}}{2\epsilon_0 V}\right)^{\frac{1}{2}}\mathbf{e}_{\mathbf{k}}(\hat{a}_{\mathbf{k}}e^{-i(\omega_{\mathbf{k}}t-\mathbf{kr})} + c.c.)\\ \hat{\mathbf{H}} &= \frac{1}{\mu_0}\sum_{\mathbf{k}}\left(\frac{\hbar}{2\omega_{\mathbf{k}}\epsilon_0 V}\right)^{\frac{1}{2}}(\mathbf{k}\times\mathbf{e}_{\mathbf{k}})(\hat{a}_{\mathbf{k}}e^{-i(\omega_{\mathbf{k}}t-\mathbf{kr})} + c.c.)\end{aligned} \tag{2}$$

show that for a quantum mechanical field the energy is given by

$$\hat{H} = \sum_{\mathbf{k}}\hbar\omega_{\mathbf{k}}(\hat{a}^+_{\mathbf{k}}\hat{a}_{\mathbf{k}} + \frac{1}{2}). \tag{3}$$

Exercise 4

The completeness of the Fock states (for a given mode $(\mathbf{k}, s)$) is given by

$$\sum_n |n\rangle\langle n| = 1, \tag{4}$$

with the relation $\hat{n}|n\rangle = n|n\rangle$.
The destrcution and annihilation operators $\hat{a}$ and $\hat{a}^\dagger$ fulfill the conditions:

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle \qquad \text{and} \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle. \tag{5}$$

a.) Calculate the expectation values of $\hat{a}$ and $\hat{a}^\dagger$ for the Fock state $|n\rangle$.
b.) Calculate the expectation values (first moments) of the variables $\hat{q}$ and $\hat{p}$ for the Fock state $|n\rangle$.
Notice:

$$\hat{q} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a} + \hat{a}^\dagger\right) \qquad \text{and} \qquad \hat{p} = -i\sqrt{\frac{m\omega\hbar}{2}}\left(\hat{a} - \hat{a}^\dagger\right). \tag{6}$$

c.) Calculate the according second moments, $\langle n|\hat{q}^2|n\rangle$ and $\langle n|\hat{p}^2|n\rangle$.
d.) Which conclusion about the uncertainty relation can be obtained from **b.)** and **c.)**?
e.) *2-mode product states:* Let $|\psi\rangle = \sum_{m,n} C_{m,n}|m,n\rangle$. Under which condition is $|\psi\rangle$ a eigen state of the total photon number operator $\hat{N} = \hat{n}_1 + \hat{n}_2$?