QUANTUM OPTICS
Wintersemester 2008/2009

Blatt 7
zur Übung am 20. Januar 2009

Exercise 1: Fock representation of the master equation

The master equation for the free field is given by

$$\frac{d\hat{\rho}}{dt} = \frac{\gamma}{2}(\bar{n}_R + 1)(\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{\gamma}{2}\bar{n}_R(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger). \tag{1}$$

Transform this master equation to the Fock basis. What is the time evolution

(a) of the diagonal elements $p_n = \rho_{n,n}$?

(b) of $\rho_{n,m}$ for $m, n \gg 1$?

Exercise 2: Fokker-Planck equation

For the Fokker-Planck equation (equation (495) in the script) show

(a) $\partial_t \langle\alpha\rangle_P = -(\gamma/2)\langle\alpha\rangle_P$

(b) $\partial_t \langle\alpha^*\alpha\rangle_P = -\gamma\langle\alpha^*\alpha\rangle_P + \gamma n_R$

and show that

(c) $P(\alpha) = (\pi n_R)^{-1} \exp(-|\alpha|^2/n_R)$ is a steady-state solution.

Exercise 3: Observation of sub-Poissionian photon statistics in the cavity QED microlaser

Read Choi *et al.*, PRL **96**, 093603 (2006) and answer the following questions:

1. What is the advantage of an one-atom-laser ($\nu = 5 \cdot 10^{14}$ Hz) compared to an one-atom-maser ($\nu = 50$ GHz)?
2. Why does an one-atom-maser has to be cooled?
   And down to which temperature? (Hint: how big should $\bar{n}_{th}$ be?)
3. How is the interaction time between atom and cavity adjusted?
   How can the cavity be tuned?
4. What is the evidence for lasing?
5. Why does the gain function of the microlaser differ from the gain function of an usual laser (Fig. 2a)?
6. Why does the photon statistic of the microlaser change from sub-poissionian to super-poissionian?