Spontaneous Emission and the Vacuum State of EM Radiation

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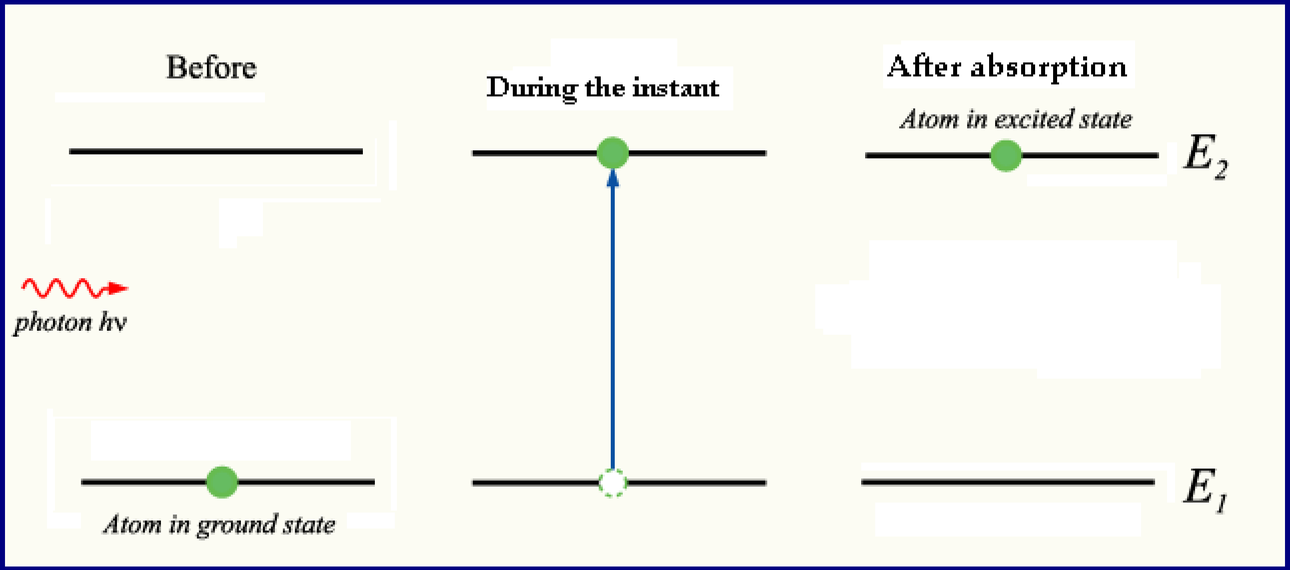
Introduction

The augmentation for the Hamiltonian of a quantum mechanical atom with that of an external classical EM field provides an alley for describing the transitions between energy states of the atom.

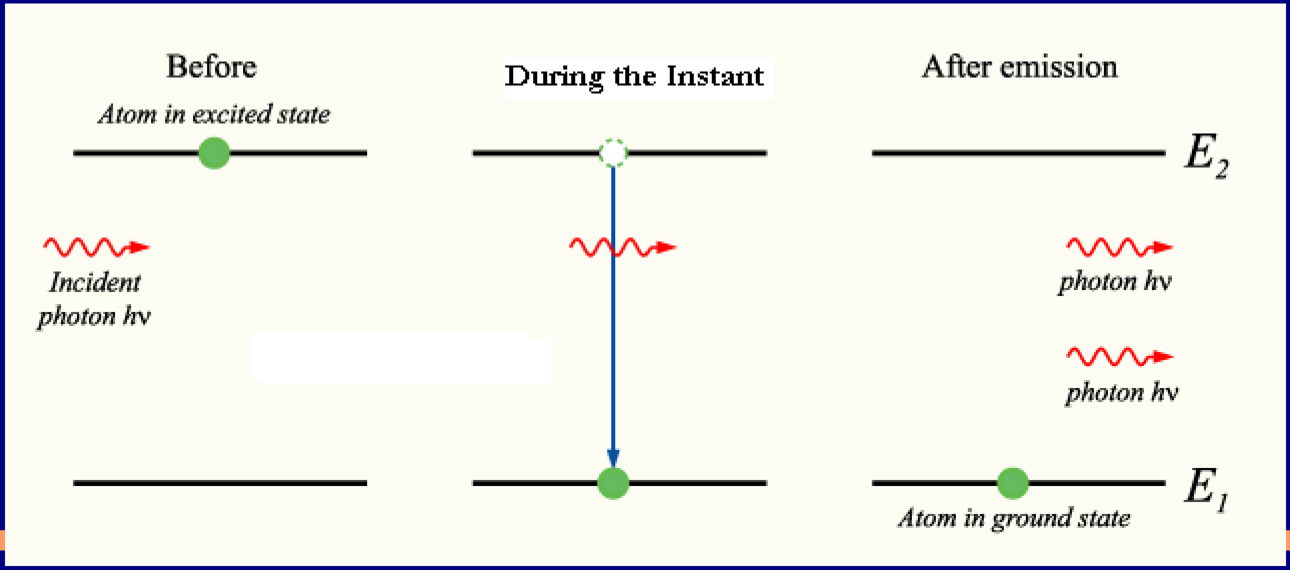
This model supports the following transitions:

- "absorption of photons"
- "stimulated emission of photons"

Optical Absorption



Stimulated Emission



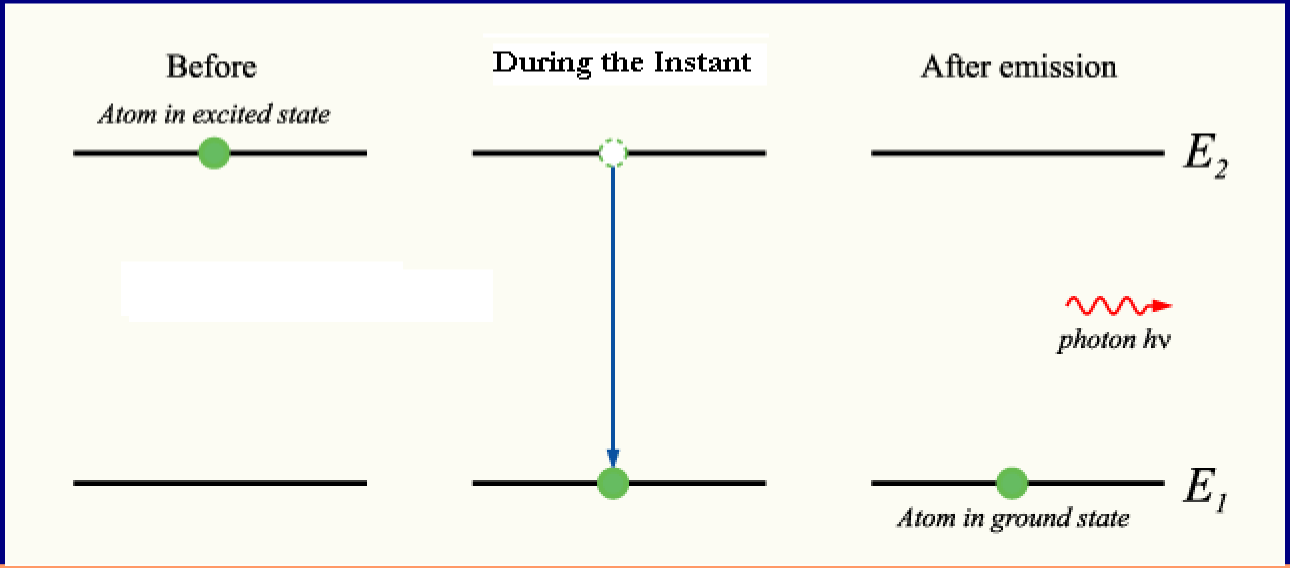
Shortcomings of this model:

The external EM field is indifferent to the transitions of the atom. It behaves an an infinite energy reservoir.

This model does not give an explanation for the spontaneous emission of photons, which occurs even in the absence of an external driving field.

Spontaneous emission

(aka optical decay aka fluorescence)



Atom inside a thermal equilibrium cavity
(remembering Max Planck)

The equilibrium of the atom with the cavity is mediated by radiation only (atom doesn't touch the wall)

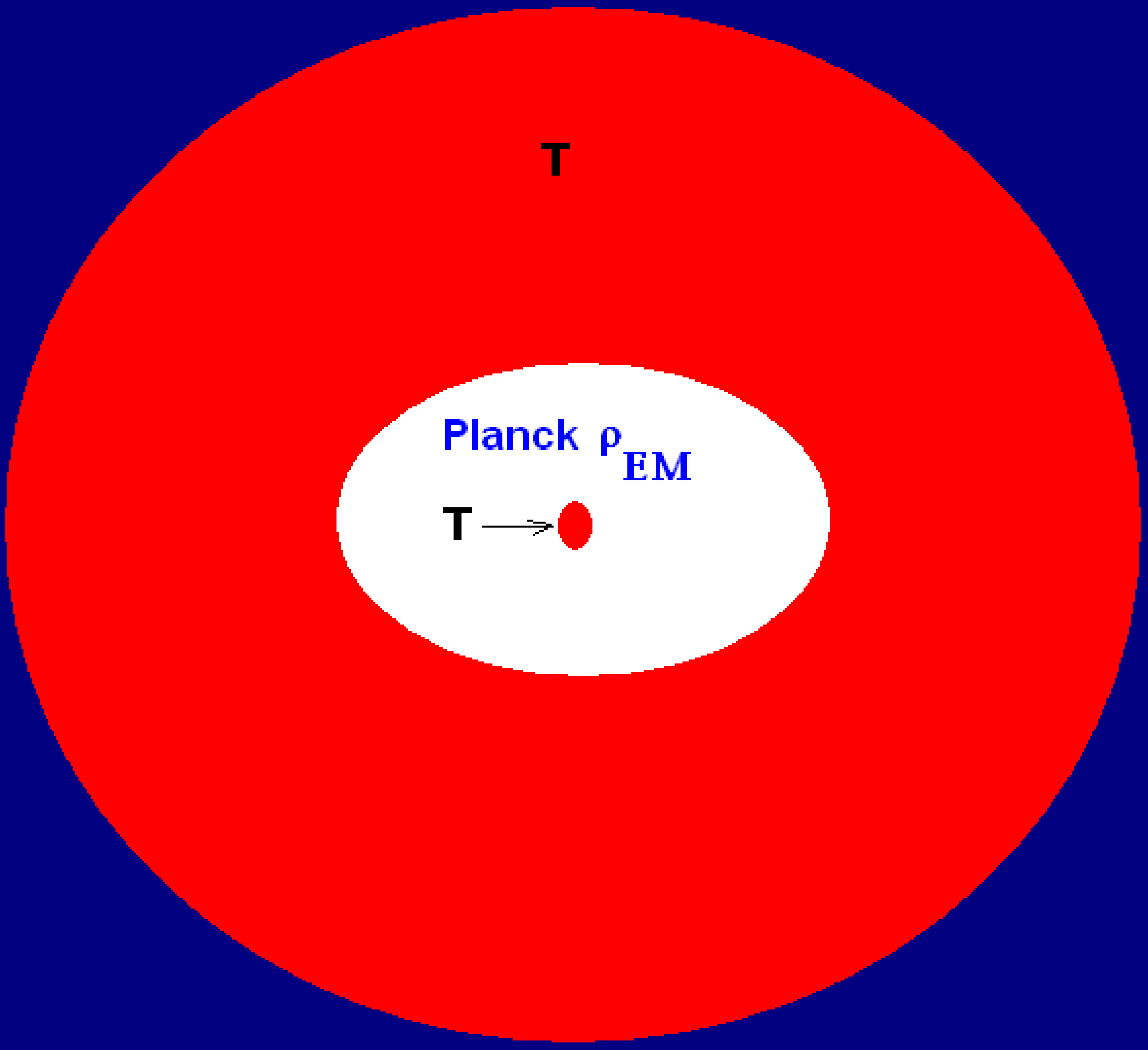
$$\rho_{EM}(\omega)=\frac{\hbar}{\pi^2}c^3\frac{\omega^3}{e^{\hbar\omega/k_B T}-1}$$

EM energy density at the frequency ω inside the thermal cavity

$$\frac{\rho_{11}}{\rho_{22}}=e^{\frac{\hbar\omega_{21}}{k_B T}}$$

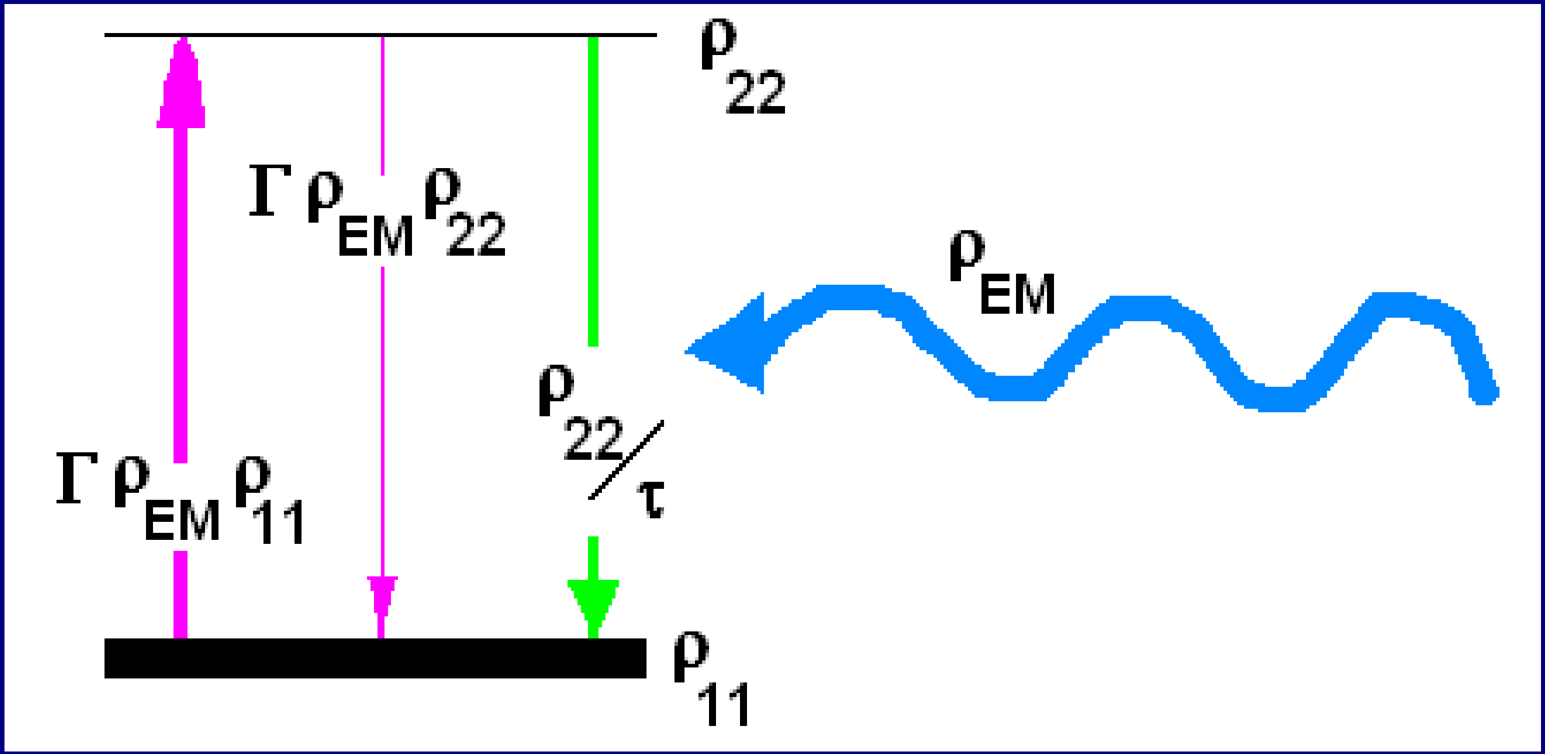
ratio of occupation probabilities for states |1> and |2>

both equations are from thermodynamical considerations ==> both equations must be fulfilled for equilibrium


T
Planck ρ_{EM}
T

Using time-dependent perturbation theory with a classical field in the dipole approximation, one accounts for the transition rates in the presence of radiation. In this case radiation = radiation in thermodynamical cavity

$$I^{\mathrm{stm}} = \Gamma\,\rho_{EM}(\omega_{12})\,\rho_{22} \qquad I^{\mathrm{abs}} = \Gamma\,\rho_{EM}(\omega_{12})\,\rho_{11}$$



$$\Gamma = \frac{\pi}{3\,\epsilon_0\,\hbar^2}\,{d_{12}}^2$$

condition for therm. equilibrium (rate equation):

$$\Gamma^{spn}\rho_{22} + I^{stm} = I^{\mathrm{abs}}$$

$$\Gamma^{spn}\rho_{22}+I^{stm}=I^{\text{abs}}\rightarrow$$

$$(\Gamma^{spn}\rho_{22})+(\Gamma(\frac{\hbar}{\pi^2}c^3\frac{\omega^3}{e^{\hbar\omega/k_BT}-1})\rho_{22})=\Gamma(\frac{\hbar}{\pi^2}c^3\frac{\omega^3}{e^{\hbar\omega/k_BT}-1})\rho_{11}$$

using $\frac{\rho_{11}}{\rho_{22}}=e^{\frac{\hbar\omega_{21}}{k_BT}}$ $\qquad \Gamma=\frac{\pi}{3\epsilon_0\hbar^2}d_{12}{}^2$

$$\rightarrow \quad \Gamma^{spn}\stackrel{\text{def}}{=}1/\tau=\omega_{12}^3\frac{d_{12}^2}{3\pi\epsilon_o\hbar c^3}$$

The request for thermal equilibrium of the atom inside the cavity mediated by EM radiation accounts properly for spontaneous emission, which has the above function.

(second or canonical) Quantisation of EM Field In optical cavity (resonator)

(of Length L, linear polarized light, propagates in x-direction)

classical Hamiltonian

$$\hat{H} = \frac{1}{2}\int_{V_{cavity}} dV\,(\epsilon_0 E_x^{\ 2} + \mu_0 H_y^{\ 2}) = \frac{1}{2}\sum_j (m_j \omega_j^{\ 2} q_j^{\ 2} + \frac{p_j^{\ 2}}{m_j})$$

- each mode is equivalent to a mechanical Harmonic oscillator
- quantization: {p,q} operators; *canonical transformation* of {p,q } to annihilation and creation operators: {a,a⁺}

$$[\hat{q}_j, \hat{p}_i] = i\hbar\,\delta_{ji} \;\;\&\;\; [\hat{q}_j, \hat{q}_i] = [\hat{p}_j, \hat{p}_i] = 0$$

$$\rightarrow$$

$$[\hat{a}_j, \hat{a}_i^{+}] = i\hbar\,\delta_{ji} \;\;\&\;\; [\hat{a}_j, \hat{a}_i] = [\hat{a}_j^{+}, \hat{a}_i^{+}] = 0$$

Quantization in resonator

$$\hat{a}_j e^{-i\nu_j t} = \frac{1}{\sqrt{2 m_j \hbar \nu_j}}(m_j \nu_j \hat{q}_j + i\,\hat{p}_j)$$

$$\hat{a}_j^{+} e^{i\nu_j t} = \frac{1}{\sqrt{2 m_j \hbar \nu_j}}(m_j \nu_j \hat{q}_j - i\,\hat{p}_j)$$

$$\hat{H} = \hbar \sum_j (\hat{a}_j^{+}\hat{a}_j + \frac{1}{2})$$

quantized EM field in optical cavity (cubic cavity $V = L^3$)

classical fields (superposition of plane waves)

$$\hat{\boldsymbol{E}}(\hat{\boldsymbol{r}},t)=\sum_{\vec{k}}(\hat{\epsilon}_{\vec{k}}\tilde{E}_{\vec{k}}\alpha_{\vec{k}}e^{-i\nu_k t+i\vec{k}\vec{r}}+c.c.)$$

For quantizing the radiation field: $\alpha_k \rightarrow \hat{a}_k$
The field can be split into a sum of positive- and negative-frequency-fields:

$$\hat{\boldsymbol{E}}(\vec{\boldsymbol{r}},t)=\hat{\boldsymbol{E}}^{(+)}(\vec{\boldsymbol{r}},t)+\hat{\boldsymbol{E}}^{(-)}(\vec{\boldsymbol{r}},t)$$

$$\hat{\boldsymbol{E}}^{(+)}(\hat{\boldsymbol{r}},t)=\sum_{\vec{k}}(\hat{\epsilon}_{\vec{k}}\tilde{E}_{\vec{k}}\hat{a}_{\vec{k}}e^{-i\nu_k t+i\vec{k}\vec{r}}+c.c.)$$

$$\hat{\boldsymbol{E}}^{(-)}(\hat{\boldsymbol{r}},t)=\sum_{\vec{k}}(\hat{\epsilon}_{\vec{k}}\tilde{E}_{\vec{k}}\hat{a}^{+}_{\vec{k}}e^{-i\nu_k t+i\vec{k}\vec{r}}+c.c.)$$

one of the results: E and Hamilton do not commute ==> noise in measuring E

Vacuum state is not "nothing": |0> photon state

$$\hat{H}|n\rangle = \hbar\omega(a^{+}a + \frac{1}{2})|n\rangle = \hbar\omega(n + \frac{1}{2})|n\rangle \qquad |n\rangle = \frac{(\hat{a}^{+})^{n}}{\sqrt{n!}}|0\rangle$$

$$\langle n|\hat{E}|n\rangle = 0 \; ; \; \langle n|\hat{E}^{2}|n\rangle = 2\left|\tilde{E}^{2}\right|(n + 1/2)$$

$$\langle 0|\hat{E}|0\rangle = 0$$

$$\langle 0|\hat{E}^{2}|0\rangle > 0$$

there is a nonzero variance (fluctuations) for the vacuum state!

The |0> is a non classical field state.
It exists in empty space.
It is sometimes called a virtual photon state.

Conclusion

These vacuum oscillations "stimulate" spontaneous emission!

Two Level atom (quantized) in Quantized (isolated cavity mode) EM Field

(second quantization, canonical quantization, ~ QED)

(rotating wave and dipole approximations)

coupling factor

$$H = \hbar \omega_{FIELD} \hat{a}^{+} \hat{a} + \frac{1}{2} \hbar \omega_{12} (\hat{\sigma}_{11} - \hat{\sigma}_{22}) + \hbar g (\hat{\sigma}_{12} \hat{a} + \hat{a}^{+} \hat{\sigma}_{21})$$

$$with \quad \sigma_{ij} = |i\rangle\langle j|$$

absorption emission

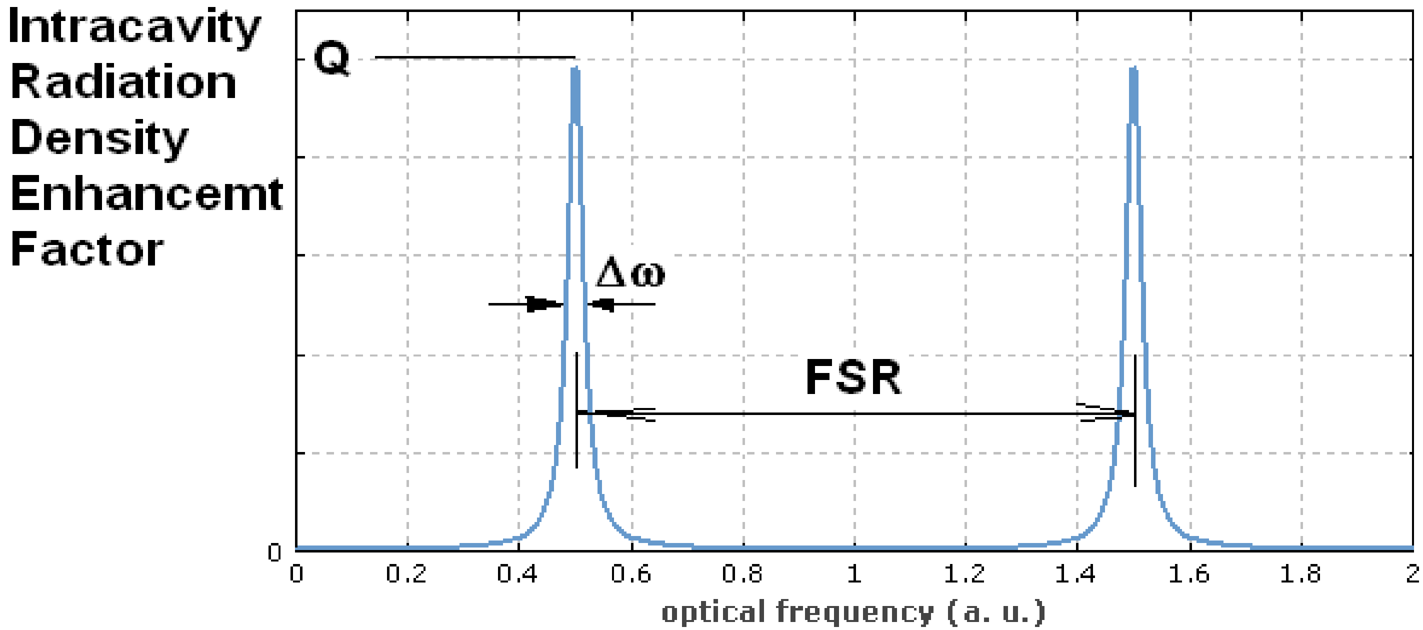
The occupation of the EM field states is energetically balanced by the occupation of the atom's energy states. The total energy of the system is conserved!

Optical cavity

Example: two flat parallel mirrors separated by distance L

The EM field inside the simplest form of a cavity

$$Q = 2\pi \frac{energy\ inside\ cavity}{energy\ lost\ \mathrm{in}\ single\ oscillation} = \frac{FSR}{\Delta\omega} = \frac{\pi\sqrt{R}}{1-R}$$



$$Q = \frac{FSR}{\Delta\omega} = \pi \frac{sqrt(R)}{1-R}$$

FSR = Free Spectral Range = c/2L

The intracavity radiation density enhancements (and suppressions) applies equally and without exception to the vacuum |0> states of EM radiation

The cavity spatial distribution of nodes is imprinted onto the vacuum |0> states as well.

Master Equation of quantized EM field with no photons inside optical cavity

"Much Ado About Nothing"

The Master Equation's density matrix elements for the quantized EM field with no photons (<n>=n=0) inside the cavity is as follows:

$$\frac{d\rho}{dt}_{n\,m} = -\frac{\nu}{2Q(\nu)}\left(n\rho_{m\,n} + m\rho^{+}_{n\,m} - \sqrt{(n+1)(m+1)}\left(\rho_{n+1\;m+1} + \rho^{+}_{n+1\;m+1}\right)\right)$$

$$with\quad \langle n|\hat{\rho}|m\rangle = \rho_{n\,m}$$

Eigenstates of the Hamiltonian of the EM field in the cavity

as a classical comparison (as for example in harmonic oscillators, with arbitrary loss mechanism):

$$\frac{dE}{dt} = -\frac{\omega_0}{Q}E$$

Cavity-Enhanced Spontaneous Emission

- redistribute the number of radiation modes around the resonant frequency of the ensemble of atoms and around the correct spatial positions==> enhanced spontaneous emission

Experiment from Goy, Raimond, Gross, Haroche in 1983:

- Na Rydberg atoms in 23S state sent through vacuum resonator with two spherical mirrors (able to capture the resonant modes in large spatial angle)

- spontaneous emission <=> dipole transition 23S-22P; the Rydberg atoms are easily ionized by static electric field (tunneling over potential barrier) ==> counting of the number of electrons coming to a electron multiplier

- spontaneous emission rate found from number of atoms on the 22P state

Observation of Cavity-Enhanced Single-Atom Spontaneous Emission

P. Goy, J. M. Raimond, M. Gross, and S. Haroche
Laboratoire de Physique de l'Ecole Normale Supérieure, F-75231 Paris Cedex 05, France

(Received 1 April 1983)

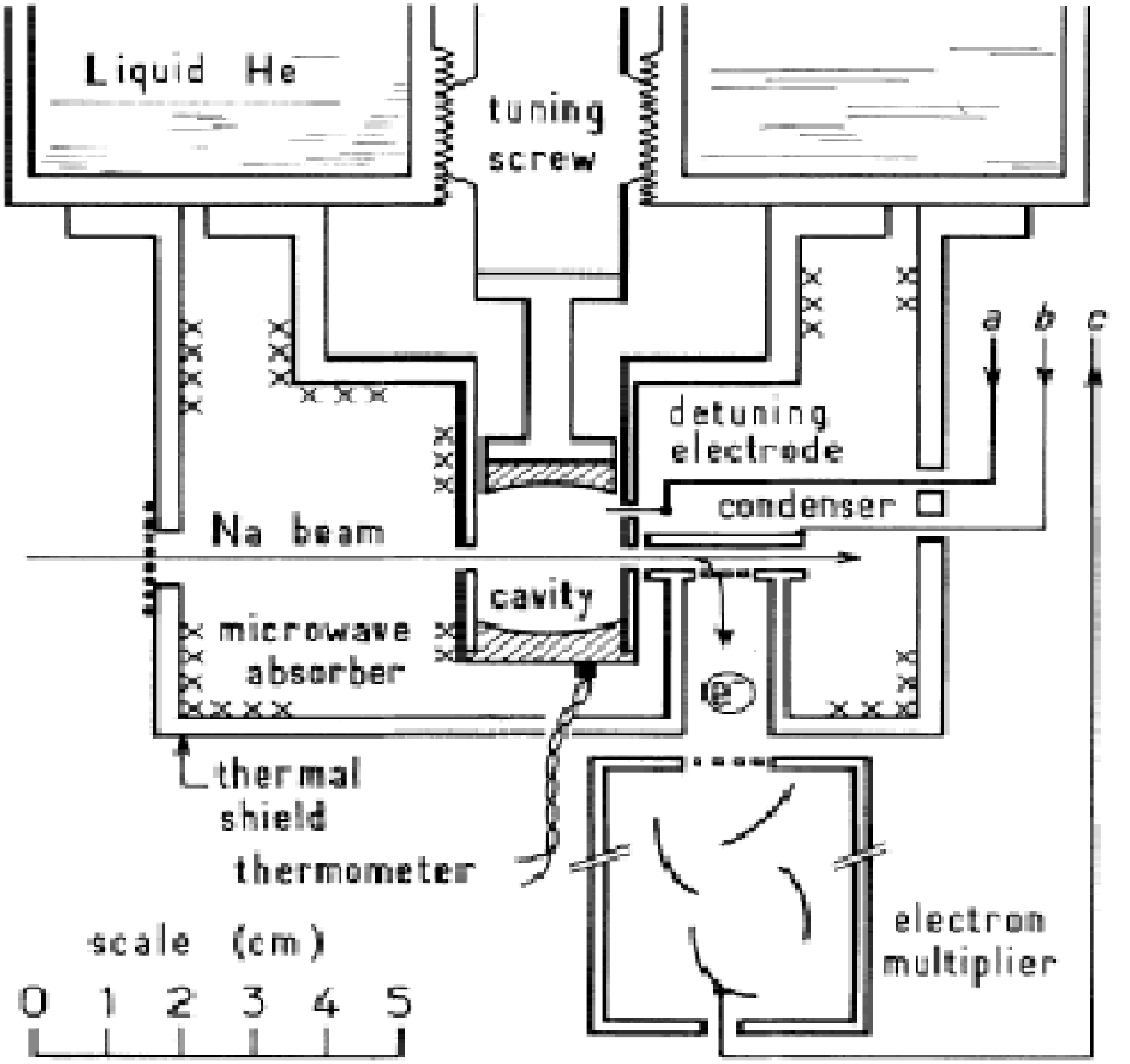


FIG. 1. Experimental arrangement.

Experimental results

free space value of spontaneous emission rate: $\Gamma_0 = 150 \frac{1}{s}$

spontaneous emission rate in cavity: $\Gamma_{cav} = 8 * 10^4 \frac{1}{s}$

Footnote: The Q-factor of the cavity is 10 times too small in order to cause Rabi oscillations (the emitted photon would be stored in the cavity long enough for the atom to reabsorb it.)

Resonant Fluorescence: Case of single two level atom *(exposed to monochromatic pumping laser beam tuned to resonant frequency of atom)*

If atom is only coupled to the vacuum continuum:

$$\rho_{22}(0)=1,\ \ \rho_{11}(0)=\rho_{21}(0)=\rho_{12}(0)=0$$

$$\rho_{22}(t)=e^{-2\gamma t}, \qquad \rho_{11}(t)=1-e^{-2\gamma t}$$

exponential decay of excited-state probability due to spontaneous emission

Now for a driven atom, the solution for the occupation probabilities are very complex-looking. Instead: Graph

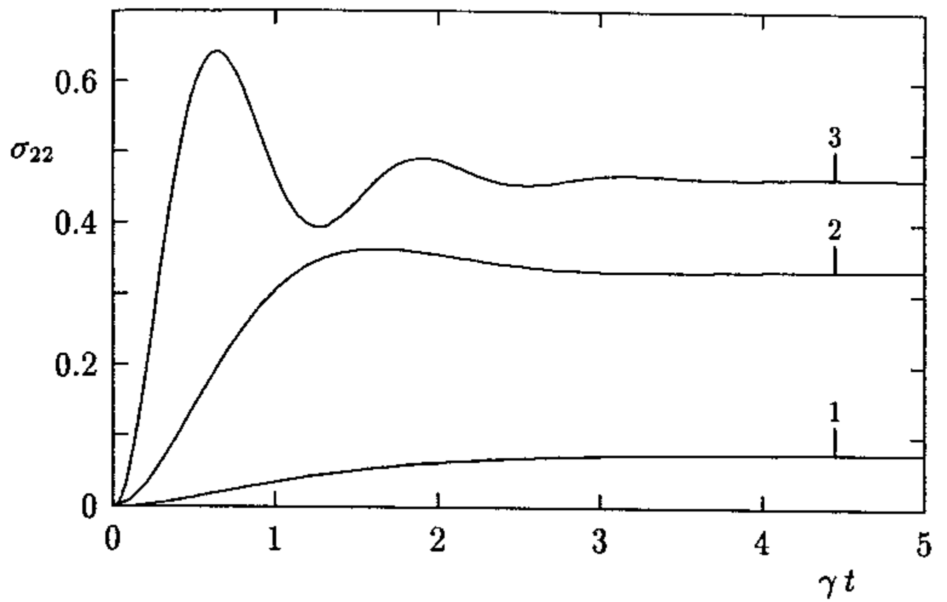


Figure 10.1: The time evolution of the excited-state occupation probability $\sigma_{22}(t)$ of a two-level atom undergoing radiative damping ($\Gamma_1=2\Gamma_2=2\gamma$) and resonantly driven by a monochromatic laser beam is shown for various values of the Rabi frequency: $\Omega_R/\gamma=0.6$ (1); $\Omega_R/\gamma=2$ (2); $\Omega_R/\gamma=5$ (3).

W. Vogel, D.G. Welsch: "Lectures on Quantum Optics". p363

The radiative damping is here due to spontaneous emission. Damping <=> quantum mech. destruction of states

Review/Discussion

- the classical EM field does not explain adequately physics of atoms in radiation
- the quantized EM field (second quantization) accounts for all weaknesses, but introduces new and unexpected result of the vacuum as an active entity
- one can influence of the properties of the vacuum inside optical cavities

Thanks for your attention

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