

Quantum Optics

Oliver Benson

July 8, 2008

Contents

1	Introduction	1
2	Quantization of the Electromagnetic Field	9
2.1	Basics	9
2.2	Number States or Fock States	11
2.3	Coherent States	13
2.4	Properties of Coherent States	16
2.5	Thermal State	18
3	Coherence Properties of the Electromagnetic Field	20
3.1	Correlation Functions	20
3.2	Optical Coherence	21
3.3	Photon Bunching and Antibunching	24
3.3.1	Some properties of $g^{(2)}$:	27
3.3.2	$g^{(2)}$ for quantum fields	30
3.4	Quantum Light	31
3.5	Single Photon Sources	34
4	Representations of the Electromagnetic Field	37
4.1	Basics	37
4.2	Glauber-Sudarshan or P-representation	38
4.3	Optical Equivalence Theorem	38
4.4	Wigner Function	40
4.5	Quantum Tomography	42
4.6	Tomography of a Single Photon Fock State	44

5	Photon Pairs	48
5.1	Parametric down-conversion	48
5.2	Non-classical behavior of down-converted light	51
5.3	Generation of entangled states	52
5.4	Two-photon interference experiments	55
6	Entanglement	58
6.1	Basic Principles	58
6.2	EPR Paradoxon	58
6.3	Experimental Tests of Bell's inequality	61
6.3.1	Requirements	61
6.3.2	Entangled State Production	61
6.4	Three experiments	62
6.4.1	Aspect's Experiment:	62
6.4.2	The Experiment by Ou and Mandel	64
6.4.3	Zeilinger's Experiment:	65
6.5	Single Photon Detectors	69
7	Quantized Interaction of Light and Matter	74
7.1	The electron wavefunction	74
7.2	Bloch representation	75
7.3	Interaction of an atom with a classical field	77
7.4	Ramsey fringes	80
7.5	Interaction of an atom with a quantized field	83
7.6	Jaynes-Cummings Model	85
7.7	Wigner-Weisskopf theory of spontaneous emission	88
7.8	Collapse and Revival & Quantum beats	90
7.8.1	Collapse & Revival 90	
7.8.2	Quantum Beats	92

8	System Reservoir Interactions	95
8.1	The master equation	95
8.2	Representations of the master equation	99
8.2.1	Fock representation	99
8.2.2	Fokker-Planck-equation	100
8.3	Quantum Langevin equation	101
8.4	Resonance fluorescence	103
8.4.1	Master equation	103
8.4.2	Spectrum of the fluorescent light	105
8.4.3	Photon correlation	110
9	Quantum Theory of Maser and Laser	113
9.1	The Micromaser	113
9.1.1	Features of the photon statistics	117
9.2	Single mode laser master equation	123
9.3	Laser Photon Statistics and Linewidth	124
10	Cavity QED	128
10.1	Spontaneous decay	128
10.1.1	Method 1: master equation	128
10.1.2	Method 2: Wigner-Weisskopf theory	129
10.1.3	Method 3: Fermi's golden rule	129
10.2	Spontaneous emission in cavities	131
10.2.1	Weak coupling regime	133
10.2.2	Strong coupling regime	134
10.3	The spectrum in the strong coupling regime	135
10.4	Modification of spatial emission pattern	139
10.5	Modification of energy levels (vacuum shift)	143
10.6	Atoms in cavities	146
10.6.1	Generalization of the Jaynes-Cummings-Hamiltonian	146
10.6.2	Important parameters for Cavity-QED	147
10.7	Artificial atoms	153
10.8	Examples and applications of CQED-systems	154