QUANTUM OPTICS
Sommersemester 2008

Blatt 2
zur Übung am 8. Mai 2008

1. Coherent state in the Fock basis

Given the coherent state $|\alpha\rangle$ which satisfies the eigenvalue equation $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ show that $|\alpha\rangle$ can be expressed in the Fock basis $\{|n\rangle\}$ as

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle.$$

2. Completeness of coherent states

Show that the coherent states $\{|\alpha\rangle\}$ are complete, i. e.

$$\frac{1}{2\pi} \int |\alpha\rangle\langle\alpha| d^2\alpha = \sum_n |n\rangle\langle n| = 1.$$

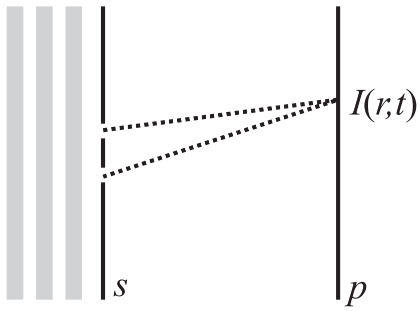
3. Coherent state with an unknown phase

Let $\alpha = |\alpha| e^{i\phi}$, with an unknown phase ϕ and uniformly distributed. Show that

$$\rho = \frac{1}{2\pi} \int_0^{2\pi} |\alpha\rangle\langle\alpha| d\phi = \sum_{n=0}^{\infty} e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} |n\rangle\langle n|,$$

i.e. the phase ignorance washes out the off-diagonal elements.

4. Quantum and classical interference



The state at the double slit s is given by $\frac{1}{\sqrt{2}}(\hat{a}_1^+ + \hat{a}_2^+)|0\rangle$. For the intensity $I(r,t)$ on the plane p show that

$$I(r,t) = \eta(1 + \cos\phi)$$

where η is an amplitude and ϕ some phase. Thus, the result is the same as in the classical case.

5. Variance of a photon field

The variance V^2 of a photon field is given by

$$V^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2.$$

Consider a single-mode field in the state $|\Psi\rangle = c_1|\alpha_1\rangle + ic_2|\alpha_2\rangle$ which is a linear superposition of the coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$, with$\langle\Psi|\Psi\rangle = 1$. All four numbers c_1, c_2, α_1, α_2 are real. Under which conditions is the statistic

a) sub-poissonian, i.e. $V_n = \frac{\langle\Psi|V^2|\Psi\rangle}{\langle\Psi|\hat{n}|\Psi\rangle} < 1$?

b) poissonian, i.e. $V_n = 1$?

c) super-poissonian, i.e. $V_n > 1$?